

A memory frontier for complex synapses





Subhaneil Lahiri and Surya Ganguli Department of Applied Physics, Stanford University, Stanford CA

Background

Storage capacity of synaptic memory

A classical perceptron, when used as a recognition memory device, has a memory capacity proportional to the number of synapses, N.

However, this requires synapses to have a dynamic range also $\propto N$.

If synaptic efficacies are limited to a fixed dynamic range, this introduces a strong tradeoff between learning and forgetting due to new memories overwriting old. If we wish to store new memories rapidly, then memory capacity is $\mathcal{O}(\log N)$.

[Amit and Fusi (1992), Amit and Fusi (1994)]

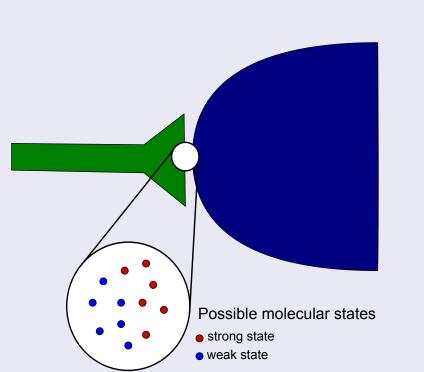
To circumvent this tradeoff, it is essential to enlarge our theoretical conception of a synapse as a single number.

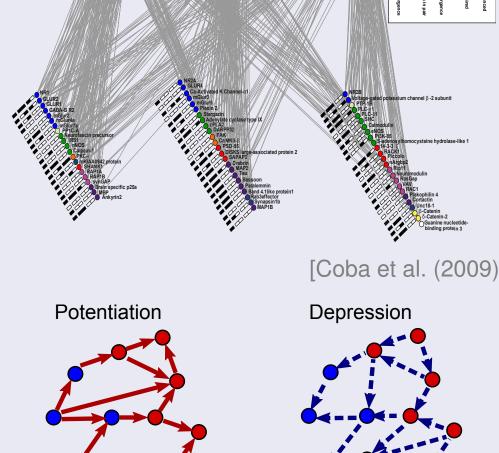
Complex synapses

In reality, a synapse is a complex dynamical system.

We will describe a synapse by stochastic processes on a finite number of states, M.

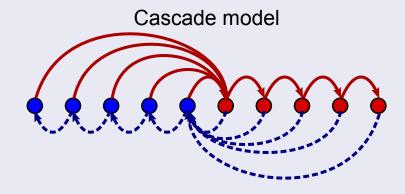
Potentiation and depression cause transitions between these states.





Cascade and serial models

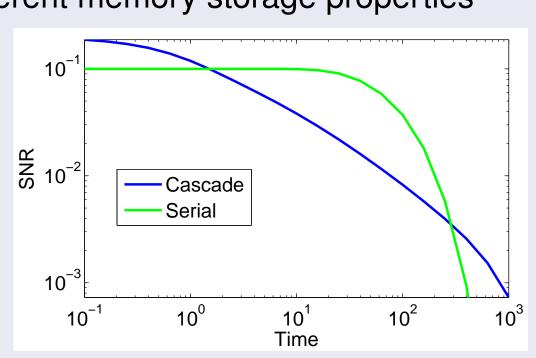
Two example models of complex synapses.



Serial model

[Fusi et al. (2005), Leibold and Kempter (2008)]

These have different memory storage properties



Questions

- Can we understand the space of *all possible* synaptic models?
- How does the structure (topology) of a synaptic model affect its function (memory curve)?
- How does synaptic complexity (number of states) extend the frontiers of possibility for memory?
- Which synaptic state transition topologies maximize measures of memory?

Framework

Synaptic state transition models

We have two Markov processes describing transition probabilities for potentiation, M^{pot}, and depression, M^{dep}. Plasticity events are potentiating with probability f^{pot} and depressing with probability f^{dep} .

After the memory we are tracking, subsequent plasticity events occur at rate r, with transition probabilities $\mathbf{M}^{\text{forget}} = f^{\text{pot}} \mathbf{M}^{\text{pot}} + f^{\text{dep}} \mathbf{M}^{\text{dep}}$

This will eventually return it to the equilibrium distribution, \mathbf{p}^{∞} .

[Fusi et al. (2005), Fusi and Abbott (2007), Barrett and van Rossum (2008)]

Memory curve

We use the ideal observer approach: read synaptic weights directly. This is an upper bound on what could be read from network activity.

The reconstruction probability of a single synapse is:

$$s(t) = f^{\text{pot}}P(\text{strong}, t \mid \text{pot}, 0) + f^{\text{dep}}P(\text{weak}, t \mid \text{dep}, 0)$$

Alternatively, if \vec{w} is an N-element vector of synaptic strengths,

Signal =
$$\langle \vec{w}_{\text{ideal}} \cdot \vec{w}(t) - \vec{w}_{\text{ideal}} \cdot \vec{w}(\infty) \rangle$$
,
Noise = $\sqrt{\text{Var}(\vec{w}_{\text{ideal}} \cdot \vec{w}(\infty))}$.

If we assume there are no correlations between different synapses, the signal-to-noise ratio is:

$$\mathsf{SNR}(t) \sim \sqrt{N}(s(t) - s(\infty)).$$

Upper bounds on performance

Area bound

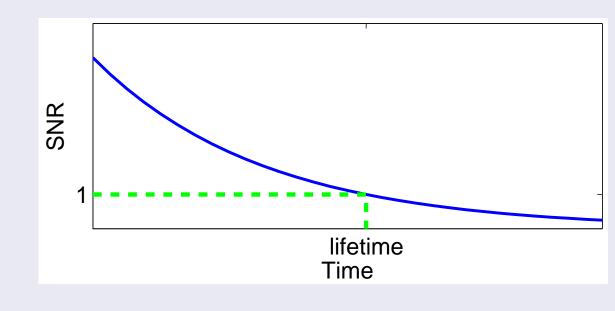
The memory lifetime is bounded by the area under the SNR curve:

SNR(lifetime) = 1 \implies lifetime < A.

We can show that this area has an upper bound:

$$A \leq \sqrt{N}(M-1)/r$$
.

This is saturated by a transition diagram with a linear chain topology.



Proof: Impose an ordering on the states

Let T_{ii} be the mean first passage time from state i to state j. The following quantity

$$\eta = \sum_{j} \mathsf{T}_{ij} \mathsf{p}_{j}^{\infty},$$

is independent of the initial state *i*. It is known as Kemeney's constant.

[Kemeny and Snell (1960)]

We define:

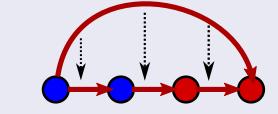
 $\eta_{\pmb{i}}^+ = \sum_{\pmb{j} \in \mathsf{strong}} \mathsf{T}_{\pmb{i}\pmb{j}} \mathsf{p}_{\pmb{j}}^\infty, \qquad \eta_{\pmb{i}}^- = \sum_{\pmb{j} \in \mathsf{weak}} \mathsf{T}_{\pmb{i}\pmb{j}} \mathsf{p}_{\pmb{j}}^\infty.$

These measure "distance" to the strong/weak states. They can be used to arrange the states in an order (increasing η^- or decreasing η^+).

Maximal area

Given any synaptic model, we can construct one with a linear chain topology that has

- the same state order,
- the same equilibrium distribution,



Uses a deformation that reduces "shortcut" transition probabilities and increases the bypassed "direct" ones.

The area of this model is

a larger area.

 $A = \frac{2\sqrt{N}}{r} \sum_{k} \mathbf{p}_{k}^{\infty} |k - \langle k \rangle|.$

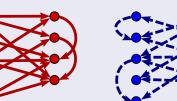
This is maximized when the equilibrium probability distribution is concentrated at both ends.

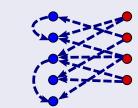
Initial SNR bound

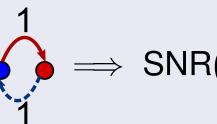
Initial SNR is closely related to equilibrium flux between strong & weak states

$$\mathsf{SNR}(0) \leq \frac{4\sqrt{N}}{r} \mathbf{\Phi}_{-+}.$$

Maximized when potentiation guarantees $\vec{w} \rightarrow$ strong, depression guarantees $\vec{w} \rightarrow \text{weak}$.







 \rightarrow Equivalent to: \Longrightarrow SNR $(t) = \sqrt{N} (4f^{\text{pot}}f^{\text{dep}}) e^{-rt}$.

Maximal initial SNR:

 $SNR(0) \leq \sqrt{N}$.

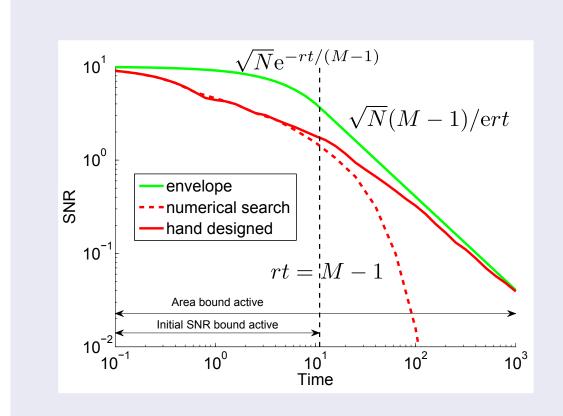
The memory envelope

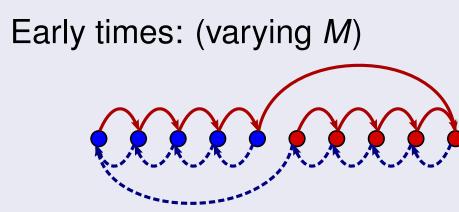
The frontiers of possibility: a maximal SNR curve

Markovian learning and forgetting \implies SNR is a sum of decaying exponentials.

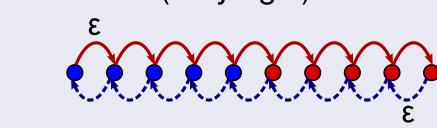
Optimizing the SNR at one time, t_0 , over the space of such curves, subject to upper bounds on initial SNR and area, yields an upper bound on SNR at t_0 for any synaptic model. The resulting optimal memory curve is a single exponential (optimizing at two or more well separated times requires multiple exponentials).

Varying t_0 yields a memory envelope curve with a power law tail.





Late times: (varying ε)



Summary

- We have formulated a general theory of learning and memory with complex synapses.
- We can impose an order on the internal states of a synapse through the theory of first passage times.
- The area under the memory curve of any synaptic transition diagram cannot exceed that of a linear chain with the same equilibrium probability distribution.
- We find a memory envelope: a single curve that cannot be exceeded by the memory curve of any synaptic model.
- Synaptic complexity (M internal states) raises the memory envelope linearly in *n* for times $> \mathcal{O}(M)$.

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Acknowledgements

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