Learning and memory with complex synapses

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Background

Storage capacity of synaptic memory

A classical perceptron, when used as a recognition memory device, has a memory capacity proportional to the number of synapses, N.

However, this requires synapses to have a dynamic range also $\propto N$. If synaptic efficacies are limited to a fixed dynamic range, this introduces a strong tradeoff between learning and forgetting due to new memories overwriting old. If we wish to store new memories rapidly, then memory capacity is $\mathcal{O}(\log N)$.

[Amit and Fusi (1992), Amit and Fusi (1994)]

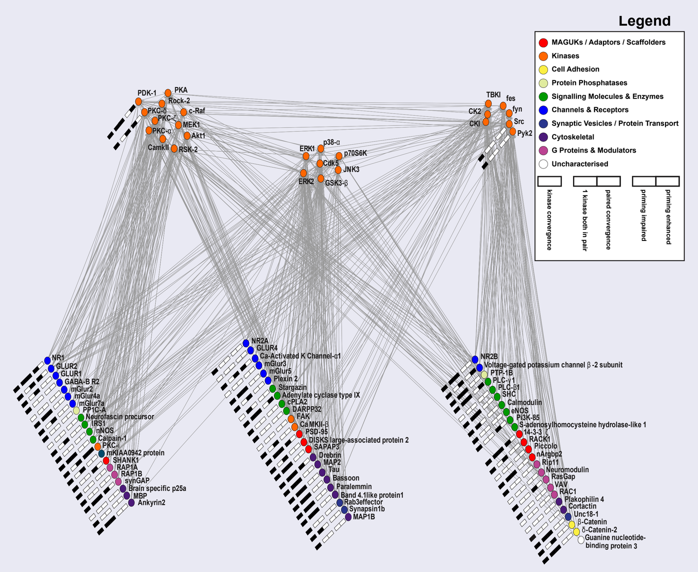
To circumvent this tradeoff, it is essential to enlarge our theoretical conception of a synapse as a single number.

Complex synapses

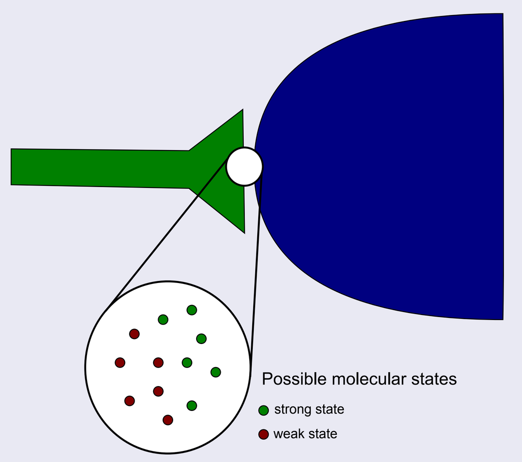
In reality, a synapse is a complex dynamical system.

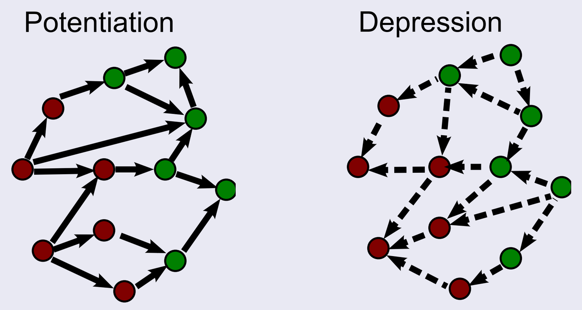
We will describe a synapse by stochastic processes on a finite number of states, n.

Potentiation and depression cause transitions between these states.



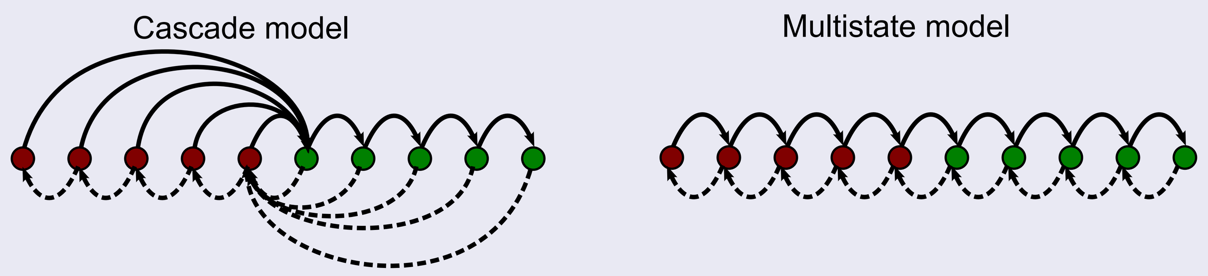
[Coba et al. (2009)]





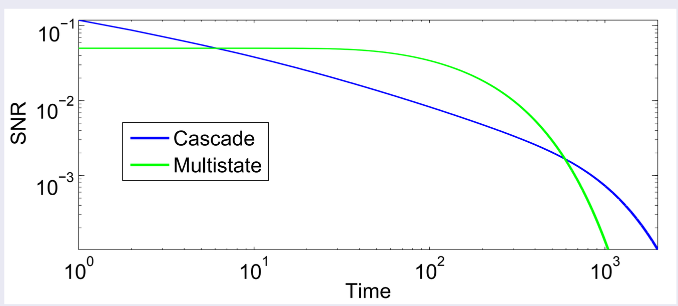
Cascade and multistate models

Two example models of complex synapses.



[Fusi et al. (2005), Amit and Fusi (1994), Fusi and Abbott (2007)]

These have different memory storage properties



Questions

- Can we understand the space of *all possible* synaptic models?
- How does the structure (topology) of a synaptic model affect its function (memory curve)?
- How does synaptic complexity (number of states) extend the frontiers of possibility for memory?
- Which synaptic state transition topologies maximize measures of memory?

Framework

Synaptic state transition models

We have two Markov processes describing transition probabilities for potentiation, $\mathbf{M}^{\text{pot}}$, and depression, $\mathbf{M}^{\text{dep}}$.

Plasticity events are potentiating with probability f^{pot} and depressing with probability f^{dep}.

After the memory we are tracking, subsequent plasticity events occur at rate r, with transition probabilities

$$\mathbf{M}^{\text{forget}} = f^{\text{pot}}\mathbf{M}^{\text{pot}} + f^{\text{dep}}\mathbf{M}^{\text{dep}}.$$

This will eventually return it to the equilibrium distribution, $\mathbf{p}^{\infty}$.

Memory curve

We use the ideal observer approach: read synaptic weights directly. This is an upper bound on what could be read from network activity.
The reconstruction probability of a single synapse is:

$$s(t) = f^{\text{pot}}P(\text{strong}, t|\text{pot}, 0) + f^{\text{dep}}P(\text{weak}, t|\text{dep}, 0)$$

Alternatively, if $\vec{w}$ is an N-element vector of synaptic strengths,

$$\text{Signal} = \langle \vec{w}_{\text{ideal}} \cdot \vec{w}(t) - \vec{w}_{\text{ideal}} \cdot \vec{w}(\infty) \rangle$$
$$\text{Noise} = \text{Var}\left(\vec{w}_{\text{ideal}} \cdot \vec{w}(\infty)\right)$$

If we ignore correlations between different synapses, the signal-to-noise ratio is:

$$\text{SNR}(t) \sim \sqrt{N}(s(t) - s(\infty)).$$

Upper bounds on performance

Area bound

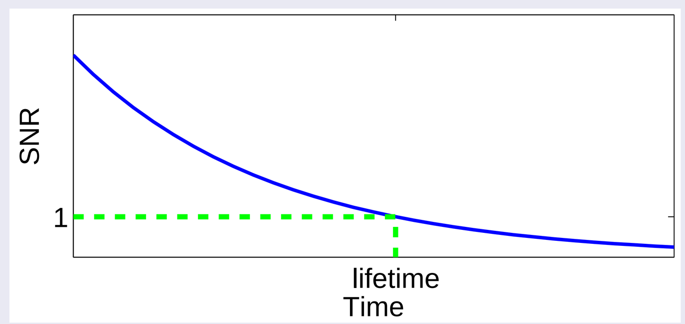
The memory lifetime is bounded by the area under the SNR curve:

$$\text{SNR}(\text{lifetime}) = 1 \qquad \Longrightarrow \qquad \text{lifetime} < A.$$

We can show that this area has an upper bound:

$$A \leq \sqrt{N}(n-1)/r.$$

This is saturated by a transition diagram with a linear chain topology.



Proof: Impose an ordering on the states

Let $\mathbf{T}_{ij}$ be the mean first passage time from state i to state j. The following quantity

$$\eta = \sum_j \mathbf{T}_{ij}\mathbf{p}^{\infty}_j,$$

is independent of the initial state i. It is known as Kemeney's constant. [Kemeny and Snell (1960)]

We define:

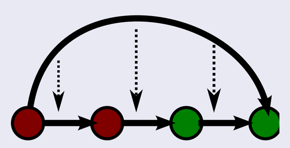
$$\eta^+_i = \sum_{j \in \text{strong}} \mathbf{T}_{ij}\mathbf{p}^{\infty}_j, \qquad \eta^-_i = \sum_{j \in \text{weak}} \mathbf{T}_{ij}\mathbf{p}^{\infty}_j.$$

These measure "distance" to the strong/weak states. They can be used to arrange the states in an order (increasing η^- or decreasing η^+).

Maximal area

Given any synaptic model, we can construct one with a linear chain topology that has

- the same state order,
- the same equilibrium distribution,
- a larger area.



Uses a deformation that reduces "shortcut" transition probabilities and increases the bypassed "direct" ones.

The area of this model is

$$A = \frac{2\sqrt{N}}{r}\sum_k \mathbf{p}^{\infty}_k \left|k - \langle k \rangle\right|.$$

This is maximized when the equilibrium probability distribution is concentrated at both ends.

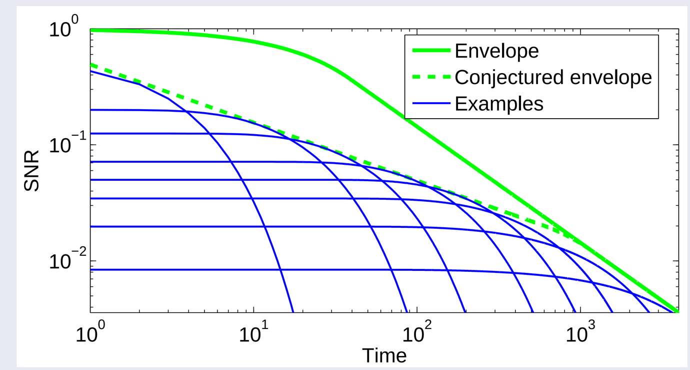
The memory envelope

The frontiers of possibility: a maximal SNR curve

Markovian learning and forgetting $\Longrightarrow$ SNR is a sum of decaying exponentials.

Optimizing the SNR *at one time*, t_0, over the space of such curves, subject to upper bounds on initial SNR and area, yields an upper bound on SNR at t_0 for *any* synaptic model. The resulting optimal memory curve is a single exponential (optimizing at two or more well separated times requires multiple exponentials).

Varying t_0 yields a memory envelope curve with a power law tail.



$$\text{Proven envelope} \sim \sqrt{N}\begin{cases}\mathcal{O}(1) & rt_0 < \mathcal{O}(n),\\ \mathcal{O}\left(\frac{n}{rt_0}\right) & rt_0 > \mathcal{O}(n).\end{cases}$$

$$\text{Conj. envelope} \sim \sqrt{N}\begin{cases}\mathcal{O}\left(\frac{1}{\sqrt{rt_0}}\right) & rt_0 < \mathcal{O}(n^2),\\ \mathcal{O}\left(\frac{n}{rt_0}\right) & rt_0 > \mathcal{O}(n^2).\end{cases}$$

Extra constraint: limits of diffusive learning and forgetting

The envelope above may not be tight. We conjecture an additional constraint, which would yield a tight envelope (dashed line above). Schematically, mode by mode:

$$\text{SNR}(0)\sqrt{\text{time-scale}} \leq \sqrt{N} \cdot \mathcal{O}(1).$$

We have found no model that can exceed this bound. It is saturated by a diffusive chain:

$$\text{SNR}(0) \sim \frac{1}{n}, \qquad \text{time-scale} \sim n^2.$$

Summary

- We have formulated a general theory of learning and memory with complex synapses.
- We can impose an order on the internal states of a synapse through the theory of first passage times.
- The area under the memory curve of any synaptic transition diagram cannot exceed that of a linear chain with the same equilibrium probability distribution.
- We find a memory envelope: a single curve that cannot be exceeded by the memory curve of *any* synaptic model. Synaptic complexity (n internal states) raises the memory envelope linearly in n for times $> \mathcal{O}(n^2)$.
- For times $< \mathcal{O}(n^2)$ we conjecture the close to optimal synaptic model that reaches the envelope exploits deterministic transitions, resulting in diffusive forgetting.

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Acknowledgements

SL and SG thank the Swartz Foundation, Burroughs Wellcome Foundation, Stanford Bio-X Neuroventures and DARPA for funding, and Larry Abbott and Stefano Fusi for useful conversations.