# Optimal synaptic strategies for different timescales of memory

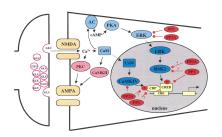
Subhaneil Lahiri and Surya Ganguli

Stanford University, Applied Physics

February 26, 2016

## What is a synapse?

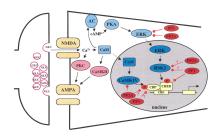
#### Experimenters



[Klann (2002)]

## What is a synapse?

#### Experimenters



[Klann (2002)]

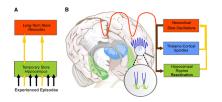
#### **Theorists**

 $W_{ij}$ 

## Timescales of memory

## Memories stored in different places for different timescales

[Squire and Alvarez (1995)] [McClelland et al. (1995)]



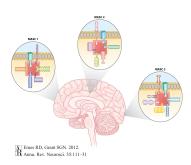
[Born and Wilhelm (2012)]

Also: Cerebellar cortex  $\rightarrow$  nuclei.

[Attwell et al. (2002)]

[Cooke et al. (2004)]

Different synapses have different molecular structures.



[Emes and Grant (2012)]

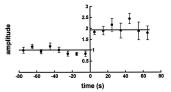
## Storage capacity of synaptic memory

A classical perceptron has a capacity  $\propto$  N, (# synapses).

Requires synapses' dynamic range also  $\propto N$ .

With discrete, finite synapses:

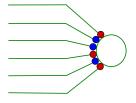
- ⇒ new memories overwrite old,
- ⇒ stability-plasticity dilemma.

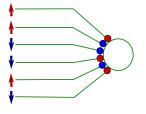


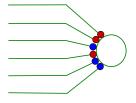
[Petersen et al. (1998), O'Connor et al. (2005)]

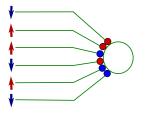
When we store new memories rapidly, memory capacity  $\sim \mathcal{O}(\log N)$ .

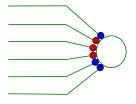
[Amit and Fusi (1992), Amit and Fusi (1994)]

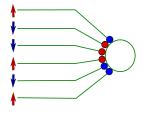


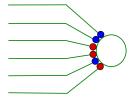




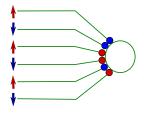








Synapses given a sequence of patterns (pot & dep) to store

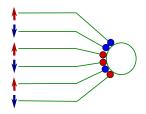


Later: presented with a pattern. Has it been seen before?

Compare  $\vec{s} \cdot \vec{w}(t)$  to threshold.

[Sommer and Dayan (1998)]

Synapses given a sequence of patterns (pot & dep) to store



Later: presented with a pattern. Has it been seen before?

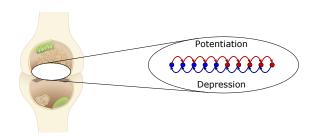
Compare  $\vec{s} \cdot \vec{w}(t)$  to threshold.

[Sommer and Dayan (1998)]

$$\mathsf{SNR}(t) = \frac{\langle \vec{s} \cdot \vec{w}(t) \rangle - \langle \vec{s} \cdot \vec{w}(\infty) \rangle}{\sqrt{\mathsf{Var}(\vec{s} \cdot \vec{w}(\infty))}}, \qquad \overline{\mathsf{SNR}}(\tau) = \int \!\! \mathrm{d}\tau \, \frac{\mathsf{e}^{-t/\tau}}{\tau} \, \mathsf{SNR}(t).$$



- $\bullet \ \ Internal \ functional \ state \ of \ synapse \rightarrow synaptic \ weight.$
- weakstrong
- $\bullet \ \ \mathsf{Candidate} \ \, \mathsf{plasticity} \ \, \mathsf{events} \, \to \, \mathsf{transitions} \ \, \mathsf{between} \ \, \mathsf{states} \\$



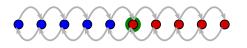
States: #AMPAR, #NMDAR, NMDAR subunit composition, CaMK II autophosphorylation, activating PKC, p38 MAPK,...

[Fusi et al. (2005), Fusi and Abbott (2007), Barrett and van Rossum (2008)]

- $\bullet \ \, \text{Internal functional state of synapse} \to \text{synaptic weight}. \\$
- weak
- $\bullet \ \, \mathsf{Candidate} \ \mathsf{plasticity} \ \mathsf{events} \to \mathsf{transitions} \ \mathsf{between} \ \mathsf{states} \\$

strong

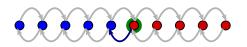
Potentiation event



- $\bullet \ \, \text{Internal functional state of synapse} \to \text{synaptic weight}. \\$
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- ullet Candidate plasticity events o transitions between states

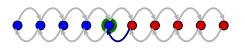
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Potentiation event



- ullet Internal functional state of synapse o synaptic weight.
- weak
- ullet Candidate plasticity events o transitions between states
- strong

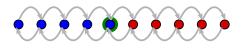
#### Potentiation event



- $\bullet$  Internal functional state of synapse  $\to$  synaptic weight.
- weak
- $\bullet \ \, \text{Candidate plasticity events} \, \to \, \text{transitions between states} \\$

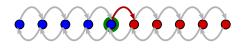
strong

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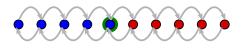
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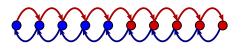
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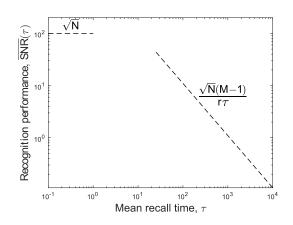
strong

#### Potentiation



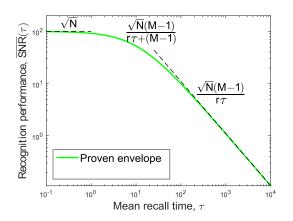
Depression

## Proven envelope: memory frontier



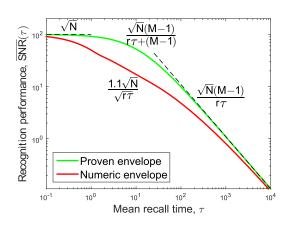
## Proven envelope: memory frontier

Upper bound on memory curve at any timescale.



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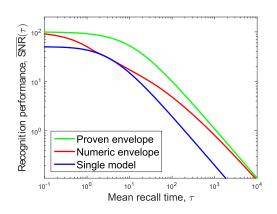
Upper bound on memory curve at any timescale.



Serial topology:

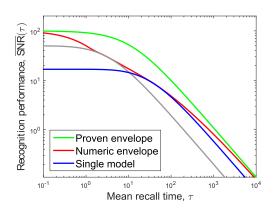


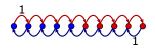
## Models that maximise memory for one timescale



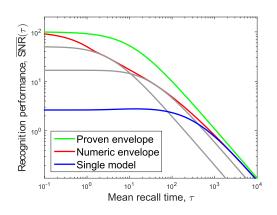


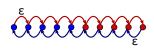
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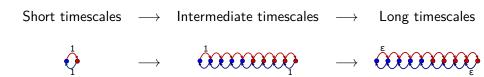


## Models that maximise memory for one timescale

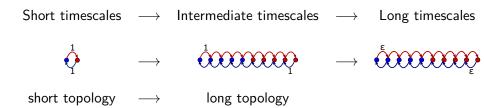




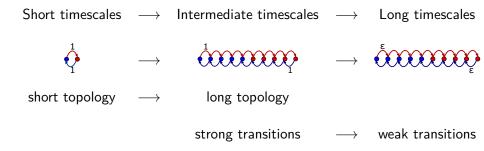
## Synaptic structures for different timescales of memory



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## Proposed Experimental design

Subject a synapse to a sequence of candidate plasticity events. Observe the changes in synaptic efficacy.



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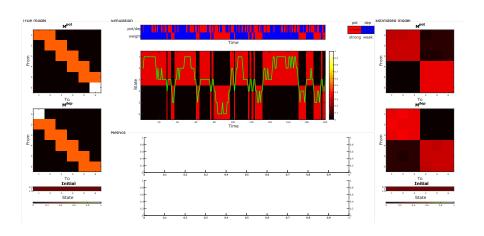
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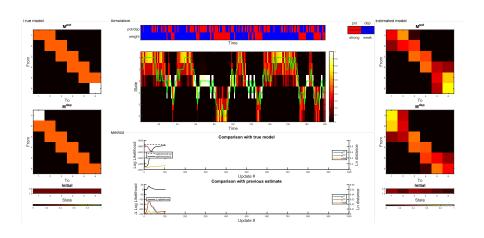


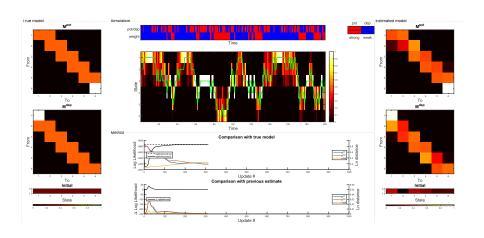
#### **EM** algorithms:

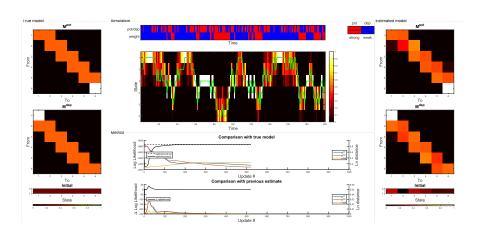
Sequence of hidden states  $\to$  estimate transition probabilities Transition probabilities  $\to$  estimate sequence of hidden states

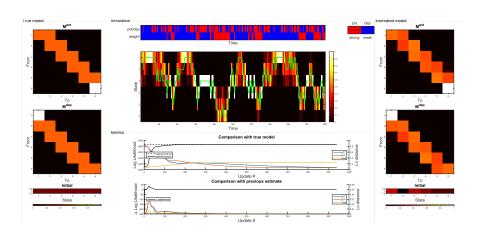
[Baum et al. (1970), Rabiner and Juang (1993), Dempster et al. (2007)]











## Summary

- We have formulated a general theory of learning and memory with complex synapses.
- We find a memory envelope: a single curve that cannot be exceeded by the memory curve of any synaptic model.
- We understood which types of synaptic structure are useful for storing memories for different timescales.

## Acknowledgements

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### References I



Eric Klann.

"Metaplastic Protein Phosphatases".

Learning and Memory, 9(4):153-155, (2002), http://learnmem.cshlp.org/content/9/4/153.full.pdf+html.





Larry R Squire and Pablo Alvarez.

"Retrograde amnesia and memory consolidation: a neurobiological perspective".

Current Opinion in Neurobiology, 5(2):169-177, (April, 1995).





James L McClelland, Bruce L McNaughton, and Randall C O'Reilly.

"Why there are complementary learning systems in the hippocampus and neocortex: Insights from the successes and failures of connectionist models of learning and memory.", 1995.



### References II



Jan Born and Ines Wilhelm.

"System consolidation of memory during sleep.".

Psychological research, 76(2):192-203, (mar, 2012).





Phillip J.E. Attwell, Samuel F. Cooke, and Christopher H. Yeo.

"Cerebellar Function in Consolidation of a Motor Memory".

Neuron, 34(6):1011-1020, (jun, 2002).





Samuel F Cooke, Phillip J E Attwell, and Christopher H Yeo.

"Temporal properties of cerebellar-dependent memory consolidation.".

The Journal of neuroscience: the official journal of the Society for Neuroscience, 24(12):2934–41, (mar, 2004).





### References III



Richard D. Emes and Seth G.N. Grant.

"Evolution of Synapse Complexity and Diversity".

Annual Review of Neuroscience, 35(1):111-131, (2012).





Carl C. H. Petersen, Robert C. Malenka, Roger A. Nicoll, and John J. Hopfield.

"All-or-none potentiation at CA3-CA1 synapses".

Proc. Natl. Acad. Sci. U.S.A., 95(8):4732-4737, (1998) .





Daniel H. O'Connor, Gayle M. Wittenberg, and Samuel S.-H. Wang.

"Graded bidirectional synaptic plasticity is composed of switch-like unitary events".

Proc. Natl. Acad. Sci. U.S.A., 102(27):9679-9684, (2005) .





### References IV



D. J. Amit and S. Fusi.

"Constraints on learning in dynamic synapses".

Network: Computation in Neural Systems, 3(4):443–464, (1992).





D. J. Amit and S. Fusi.

"Learning in neural networks with material synapses".

Neural Computation, 6(5):957-982, (1994).





Friedrich T Sommer and Peter Dayan.

"Bayesian retrieval in associative memories with storage errors.".

IEEE transactions on neural networks / a publication of the IEEE Neural Networks Council, 9(4):705–13, (jan, 1998).

















### References V



S. Fusi, P. J. Drew, and L. F. Abbott.

"Cascade models of synaptically stored memories".

Neuron, 45(4):599-611, (Feb, 2005).





S. Fusi and L. F. Abbott.

"Limits on the memory storage capacity of bounded synapses".

Nat. Neurosci., 10(4):485-493, (Apr., 2007).





A. B. Barrett and M. C. van Rossum.

"Optimal learning rules for discrete synapses".

PLoS Comput. Biol., 4(11):e1000230, (Nov, 2008).



### References VI



LE Baum, T Petrie, George Soules, and Norman Weiss.

"A maximization technique occurring in the statistical analysis of probabilistic functions of Markov chains".

The annals of mathematical statistics, 41(1):164–171, (1970).













Lawrence R Rabiner and Biing-Hwang Juang.

Fundamentals of speech recognition, volume 14 of Signal Processing.

Prentice Hall, Inc., Upper Saddle River, NJ, USA, 1993.

ISBN 0-13-015157-2.















### References VII



A. P. Dempster, N. M. Laird, and D. B. Rubin.

"Maximum Likelihood from Incomplete Data via the EM Algorithm".

Journal of the Royal Statistical Society. Series B (Methodological), (October, 2007).















Finite markov chains.

Springer, 1960.



### Technical detail: ordering states

Let  $T_{ij}$  = mean first passage time from state i to state j. Then:

$$\eta = \sum_j \mathbf{T}_{ij} \mathbf{p}_j^{\infty},$$

is independent of the initial state i (Kemeney's constant).

[Kemeny and Snell (1960)]

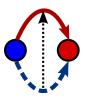
We define:

$$\eta_i^+ = \sum_{j \in \mathsf{strong}} \mathbf{T}_{ij} \mathbf{p}_j^\infty, \qquad \eta_i^- = \sum_{j \in \mathsf{weak}} \mathbf{T}_{ij} \mathbf{p}_j^\infty.$$

They can be used to arrange the states in an order (increasing  $\eta^-$  or decreasing  $\eta^+$ ).

## Technical detail: upper/lower triangular

With states in order:





Endpoint: potentiation goes right, depression goes left.

