Learning and memory with complex synapses

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Background

Storage capacity of synaptic memory

A classical perceptron, when used as a recognition memory device, has a memory capacity proportional to the number of synapses N.

However, this requires synapses to have a dynamic range also $\propto N$. If synaptic efficacies are limited to a fixed dynamic range, this introduces a strong tradeoff between learning and forgetting due to new memories overwriting old. If we wish to rapidly store new memories, then memory capacity is $\mathcal{O}(\log N)$.

[Amit and Fusi (1992), Amit and Fusi (1994)]

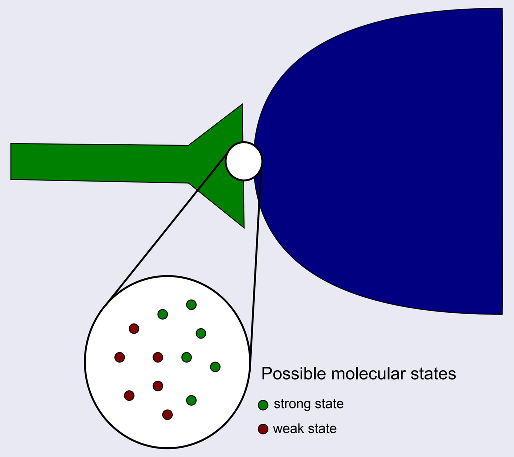
To circumvent this tradeoff, it is essential to enlarge our theoretical conception of a synapse as a single number.

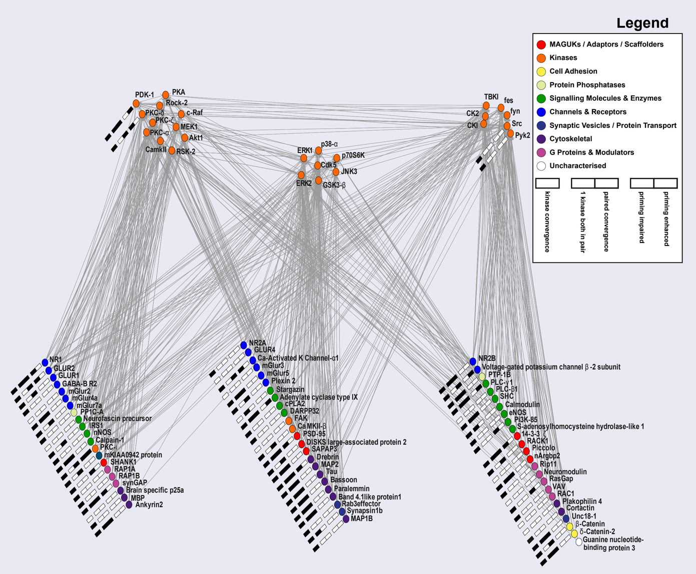
Complex synapses

In reality, a synapse is a complex dynamical system.

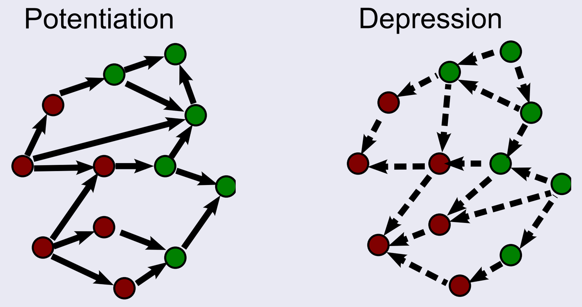
We will describe a synapse by stochastic processes on a finite number of states, n.

Potentiation and depression cause transitions between these states.



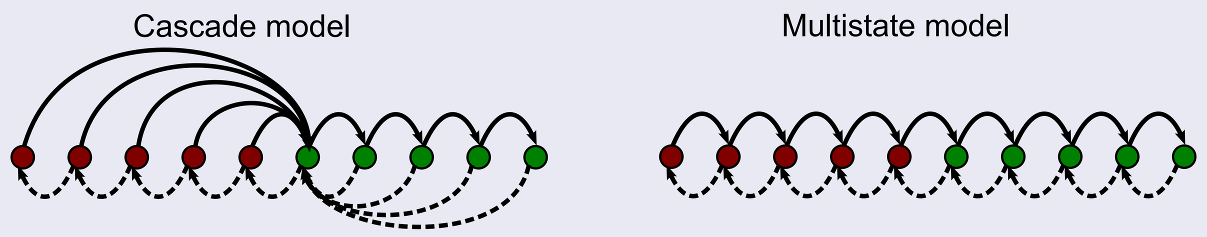


[Coba et al. (2009)]



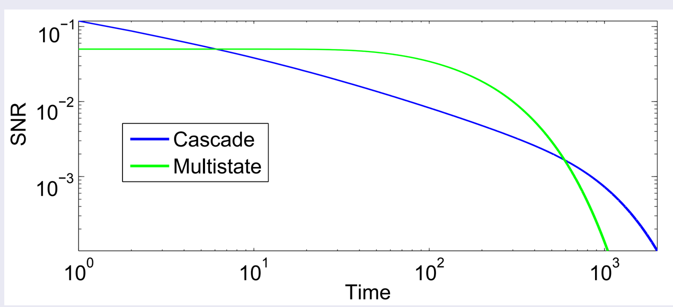
Cascade and multistate models

Two example models of metaplasticity in complex synapses.



[Fusi et al. (2005), Fusi and Abbott (2007)]

These have different memory storage properties



Questions

- Can we understand the space of *all possible* synaptic models?
- How does the structure (topology) of a synaptic model affect its function (memory curve)?
- How does synaptic complexity (number of states) extend the frontiers of possibility for memory?
- Which molecular network topologies maximize measures of memory?

Framework

Synaptic state transition models

We have two Markov processes describing transition probabilities for potentiation, $\mathbf{M}^{\text{pot}}$, and depression, $\mathbf{M}^{\text{dep}}$.

Plasticity events are potentiating with probability f^{pot} and depressing with probability f^{dep}.

After the memory we are tracking, subsequent plasticity events occur at rate r, with transition probabilities

$$\mathbf{M}^{\text{forget}} = f^{\text{pot}}\mathbf{M}^{\text{pot}} + f^{\text{dep}}\mathbf{M}^{\text{dep}}.$$

This will eventually return it to the equilibrium distribution, $\mathbf{p}^{\infty}$.

Memory curve

We use the ideal observer approach: read synaptic weights directly. This is an upper bound on what could be read from network activity.
Reconstruction probability of a single synapse:

$$s(t) = f^{\text{pot}}P(\text{strong}, t|\text{pot}, 0) + f^{\text{dep}}P(\text{weak}, t|\text{dep}, 0)$$

Alternatively, if $\vec{w}$ is an N-element vector of synaptic strengths,

$$\text{Signal} = \langle \vec{w}_{\text{ideal}} \cdot \vec{w}(t) - \vec{w}_{\text{ideal}} \cdot \vec{w}(\infty) \rangle$$
$$\text{Noise} = \text{Var}\left(\vec{w}_{\text{ideal}} \cdot \vec{w}(\infty)\right)$$

If we ignore correlations between different synapses, signal-to-noise ratio:

$$\text{SNR}(t) \sim \sqrt{N}(s(t) - s(\infty)).$$

Upper bounds on performance

Area bound

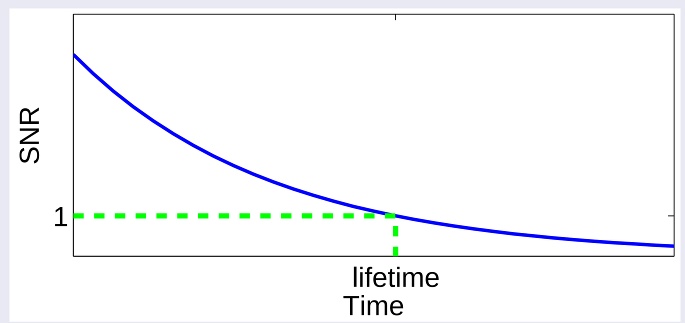
We can show that the area under the SNR curve is bounded:

$$A \leq \sqrt{N}(n-1)/r.$$

This is saturated by a molecular network with the multistate topology.

This leads to a bound on the memory lifetime of *any* synaptic model:

$$\text{SNR}(\text{lifetime}) = 1 \qquad \Longrightarrow \qquad \text{lifetime} < A.$$



Proof: Impose an ordering on the states

Let $\mathbf{T}_{ij}$ be the mean first passage time from state i to state j. The following quantity

$$\eta = \sum_j \mathbf{T}_{ij}\mathbf{p}^{\infty}_j,$$

is independent of the initial state i. It is known as Kemeney's constant. [Kemeny and Snell (1960)]

We define:

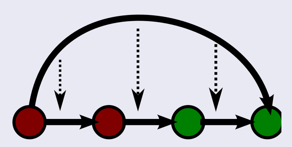
$$\eta^+_i = \sum_{j\in\text{strong}} \mathbf{T}_{ij}\mathbf{p}^{\infty}_j, \qquad \eta^-_i = \sum_{j\in\text{weak}} \mathbf{T}_{ij}\mathbf{p}^{\infty}_j.$$

These measure "distance" to the srong/weak states. They can be used to arrange the states in an order (increasing η^- or decreasing η^+).

Maximal area

Given any molecular network, we can construct one with the multistate topology that has

- same state order,
- same equilibrium distribution,
- larger area.



Uses a deformation that reduces "shortcut" transition probabilities and increases the bypassed "direct" ones.

The area of this model is

$$A = \frac{2\sqrt{N}}{r}\sum_k \mathbf{p}^{\infty}_k \left|k - \langle k \rangle\right|.$$

The maximum is when all probability is at ends.

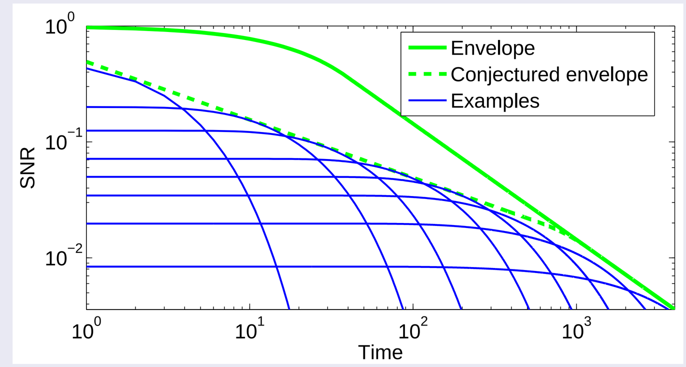
Envelope memory curve

Maximal SNR curve

Markov process $\Longrightarrow$ SNR is a sum of exponentials.

If we find the maximum sum of exponentials *at one time*, subject to upper bounds on initial SNR and area, we get an upper bound on SNR at that time. The resulting curve is always a single exponential.

If we vary the time at which we find the optimum, we get an envelope curve with a power law tail.



Extra constraint: limits of diffusive forgetting

The envelope above may not be tight.

We can get a tight envelope – one that can be saturated at any single time by some model – if we add one more constraint.

Schematically, mode by mode:

$$\text{SNR}(0)\sqrt{\text{time-scale}} \leq \sqrt{N}\cdot\mathcal{O}(1).$$

We have found no model can exceed this. It is saturated by a diffusive chain:

$$\text{SNR}(0) \sim \frac{1}{n}, \qquad \text{time-scale} \sim n^2.$$

Maximum lifetime

We can use the envelope to get a stricter bound on the lifetime of a memory

$$\text{Envelope}(\text{max lifetime}) = 1, \qquad \text{max lifetime} = \frac{\sqrt{N}(n-1)}{\mathrm{e}r}.$$

Summary

Summary to appear here.

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Acknowledgements

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