Econ 126: Computational Macroeconomics
Winter 2020: Jenkins

Final Exam
Due: 12:30pm, March 20, 2020

Instructions: Please read these instructions carefully. There are 70 points

1. *Submission*: You will submit two files.
   (a) A .doc, .docx, or .pdf formatted file, with your answers to exam questions. This document must be well organized and easy to read.
   (b) An .html version of the Jupyter Notebook file containing all code that you used to generate images and statistics for your exam. Code must be well-organized and thoroughly documented with comments explaining what your code is doing.
2. *Collaboration*: You may collaborate with others in the class subject to two requirements:
   (a) At the top of the exam document (the Word or pdf file), you must list the names of everyone in the class that you worked with.
   (b) Everyone must prepare their own code. If any two Notebooks are deemed sufficiently similar, then the authors of both Notebooks will receive point reductions up to the entire point value of the exam. Make sure that your comments documenting the code are in your own words.
3. *Deadline*: Exams will not be accepted after the deadline indicated above.

Questions begin on the next page.

1 Centralized RBC Model with Stochastic Government Consumption

In this problem, you will simulate the dynamic equilibrium of a centralized RBC model without labor and with stochastic government consumption. As usual, the model features an infinitely-lived household that chooses consumption and capital accumulation to maximize the present value of its lifetime utility. What's new is that we'll assume that there is a government sector that consumes a stochastic quantity of goods each period. A key product of this modeling approach is that it allows us to model how fluctuations in government consumption affect the business cycle.

1.1 The Model

1.1.1 Household Sector

A representative household lives for an infinite number of periods. The expected present value of lifetime utility to the household from consuming $C_0, C_1, C_2, \ldots$ is denoted by U_0:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \log(C_t), \tag{1}$$

where $0 < \beta < 1$ is the household's subjective discount factor. E_0 denotes the expectation with respect to all information available as of date 0.

The household enters period 0 with capital $K_0 > 0$. Production in period t is according to a standard production function that has decreasing returns in capital K_t:

$$F(A_t, K_t) = A_t K_t^{\alpha} \tag{2}$$

where TFP A_t is stochastic:

$$\log A_{t+1} = \rho_A \log A_t + \epsilon_{t+1}^A \tag{3}$$

Each period the government collects a lump-sum tax T_t from the household. The household's resource constraint in each period t is therefore:

$$C_t + K_{t+1} + T_t = A_t K_t^{\alpha} + (1-\delta)K_t, \tag{4}$$

where δ is the rate of capital depreciation.

In period 0, the household solves:

$$\begin{aligned} \max_{C_0, K_1} \; & E_0 \sum_{t=0}^{\infty} \beta^t \log(C_t) \\ \text{s.t.} \quad & C_t + K_{t+1} + T_t = A_t K_t^{\alpha} + (1-\delta)K_t \end{aligned} \tag{5}$$

which, as usual, can be written as a choice of K_1 only:

$$\max_{K_1} E_0 \sum_{t=0}^{\infty} \beta^t \log(A_t K_t^{\alpha} + (1-\delta)K_t - K_{t+1} - T_t) \tag{6}$$

1.1.2 Government Sector

Each period the government consumes G_t units of goods. G_t evolves according to the following process:

$$\log G_{t+1} = (1-\rho_G)\log \bar{G} + \rho_G \log G_t + \epsilon_{t+1}^G \tag{7}$$

By assumption, the government always runs a balanced budget so:

$$T_t = G_t \tag{8}$$

1.1.3 Investment and Output

When the household chooses K_{t+1}, it implicitly chooses investment I_t which is defined by:

$$I_t = K_{t+1} - (1-\delta)K_t \tag{9}$$

and output Y_t which is defined by:

$$Y_t = A_t K_t^{\alpha} \tag{10}$$

1.1.4 Goods Market Clearing

In equilibrium, the quantity of goods produced Y_t has to equal the demand for those goods $C_t + I_t + G_t$:

$$Y_t = C_t + I_t + G_t \tag{11}$$

We call Equation (11) the goods market clearing condition and it represents the aggregate resource constraint for the economy.

1.1.5 Equilibrium

The model has 7 endogenous variables: A_t, G_t, K_t, C_t, T_t, Y_t, I_t. Equilibrium is described by:

1. The household's first-order condition for K_{t+1} (the Euler equation)
2. The TFP evolution equation: Equation (3)
3. The government consumption evolution equation: Equation (7)
4. The government budget constraint: Equation (8)

5. The capital evolution equation: Equation (9)

6. The production function: Equation (10)

7. The goods market clearing equation: Equation (11)

1.1.6 Calibration

Assume the following values for the model's parameters:

Parameter	Value	Description
β	0.99	household's subjective discount factor
α	0.35	Cobb-Douglas production function parameter
δ	0.025	capital depreciation rate
ρ_A	0.75	autocorrelation of tfp
σ_A	0.006	s.d. of TFP shock
$\bar{G}$	–	steady state government consumption
ρ_G	0.9	autocorrelation of government consumption
σ_G	0.015	s.d. of government consumption shock

1.2 Exercises

1. (8 points) Download two series from FRED[1]:

   – Government Consumption Expenditures and Gross Investment (Series ID: GCE)

   – Gross Domestic Product (Series ID: GCE)

   Find the average of the ratio of government consumption to GDP for the US for all dates available.[2] Report this value in your exam document.

2. (4 points) Solve for the household's first-order condition for K_{t+1}. Include this equation in your exam document and be able to explain the intuition behind it.

3. (6 points) Use Python to compute the steady state values of A_t, K_t, Y_t, and I_t. You will have to do this manually. Since you don't know $\bar{G}$ yet, you can't use `linearsolve` for this step. Report the computed steady state values of A_t, K_t, Y_t, and I_t in your exam document.

4. (6 points) Use the average ratio of government consumption to GDP for the US to *calibrate* $\bar{G}$:

$$\bar{G} \quad = \quad \bar{Y} \times \Big[\text{Avg. G-to-Y ratio}\Big] \tag{12}$$

[1] https://fred.stlouisfed.org/

[2] Compute the ratio for each date *first*, then compute the average of the ratio.

Then use Python to compute the steady state values of C_t and T_t. Report the computed steady state values of G_t, C_t, and T_t in your exam document.

5. (6 points) Compute the impulse responses for all of the model's endogenous variables for 41 periods following a one percentage point increase in government consumption in period 5.[3] Create a set of clear, easy to read figures that depict the impulse responses and include them in your exam document. Units of plotted quantities should be percent deviations from steady state.[4] Describe in words why the behavior of each variable in the simulated impulse responses.

6. (10 points) Make sure all code used to generate results for this problem is well-organized and thoroughly documented.

[3]Note that like A_t, G_t is a state (or predetermined) variable.
[4]I.e., multiply the simulated impulse responses by 100.

2 Prescott and Summers

Answer the following.

1. (6 points) To what does Prescott solely attribute macroeconomic fluctuations? Explain what this means in your own words.

2. (8 points) Consider the Euler equation from Prescott's RBC model:

$$\frac{1}{C_t} = \beta E_t \left[\frac{\alpha A_{t+1} K_{t+1}^{\alpha-1} L_{t+1}^{1-\alpha} + 1 - \delta}{C_{t+1}} \right] \tag{13}$$

   Explain in words why the left-hand side represents the marginal cost to the household of increasing K_{t+1} and explain why the right-hand side represents the marginal benefit.

3. (6 points) Summers' fourth critique of Prescott's work "is that it ignores the fact that partial breakdowns in the exchange mechanism are almost surely dominant factors in cyclical fluctuations." Explain what Summers means by *partial breakdowns in the exchange mechanism* and provide an example from reality that is not explained by Prescott's RBC model.

3 Real business cycle and new-Keynesian models

Answer the following.

1. (5 points) *Briefly* describe a fundamental similarity between the modeling approaches of real business cycle (RBC) and new-Keynesian (NK) models.

2. (5 points) *Briefly* describe a fundamental difference between the modeling approaches of RBC and NK models.