COMPUTER SIMULATION OF THE PROPAGATION AND INTERACTION OF SOLITON SEQUENCES IN NONLINEAR OPTICAL...

Article *in* Journal of Electrical Engineering · January 2001

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Journal of ELECTRICAL ENGINEERING, VOL. 52, NO. 3-4, 2001, 57–62

COMPUTER SIMULATION OF THE PROPAGATION AND INTERACTION OF SOLITON SEQUENCES IN NONLINEAR OPTICAL FIBERS

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The results of the simulation of optical soliton propagation in nonlinear dispersive optical fibers with or without losses are presented. The propagation equation was modeled using the finite-differences-method. The propagation of fundamental soliton, higher order soliton and the sequence of up to five adjacent solitons was observed and analyzed for various initial conditions

K e y w o r d s: soliton propagation, optical fibers

1 INTRODUCTION

Major interest in the research of optical fiber communication systems in the last two decades focuses on combined nonlinearty vs. dispersion effects in fiber waveguides leading to soliton- and soliton-like-pulse propagation. Since the requirements for data transmission are rapidly growing, the existing fiber communication systems are already on the edge of their capacity. The solution to this problem seems to consist in making use of mutual compensation of nonlinear and dispersive effects leading thus to generation and propagation of very stable and narrow pulses called solitons [1, 2], prospective for very long-distance very high-bit-rates transmission. The main limitation of these emerging communication systems will be mostly due to mutual interaction of adjacent closely placed pulses [4, 5] causing thus a limitation of the bit rate. In this work the nature of these interactions is numerically investigated by means of computer simulations of the propagation of either the single- soliton-pulse or the sequences of maximum five adjacent pulses with different amplitudes, phases and distances between them.

In Section 2 the basic propagation equation (nonlinear Schrödinger equation — NLSE) for the propagation of solitons in a fiber and its dimensionless form are given. The numerical technique being used and its advantages are thoroughly described. In Section 3 we concentrate on simulations of single soliton (fundamental and higher order) in a fiber with or without losses. The last Section is devoted to propagation of soliton sequences.

2 THEORETICAL MODEL

To describe the propagation of an optical pulse in the optical waveguiding fiber, one has to start, as in all electromagnetic problems, from the Maxwell equations

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \nabla \times \mathbf{H} = \mathbf{J}_{\mathrm{f}} + \frac{\partial \mathbf{D}}{\partial t} \tag{1a,b}$$

$$\nabla \cdot \mathbf{D} = \rho_{\mathrm{f}} \qquad \nabla \cdot \mathbf{B} = 0 \tag{1c,d}$$

where $\mathbf{E}$ and $\mathbf{H}$ are electric and magnetic field intensities respectively, and $\mathbf{D}$ and $\mathbf{B}$ electric and magnetic flux densities. Furthermore, we use constitutive equations, which relate the field intensities and flux densities, in the form

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} \qquad \mathbf{B} = \mu_0 \mathbf{H} + \mathbf{M} \tag{2a,b}$$

where $\mathbf{P}$ and $\mathbf{M}$ are vectors of induced electric and magnetic polarization and ε_0 and μ_0 the vacuum permittivity and permeability, respectively. In our problem (guided wave in the fiber), there are no free currents $\mathbf{J}_{\mathrm{f}} = 0$), no free charges ($\rho_{\mathrm{f}} = 0$) and the fiber is a nonmagnetic medium ($\mathbf{M} = 0$).Thus, the following wave equation is obtained from (1) and (2)

$$\nabla \times \nabla \times \mathbf{E} = -\frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} - \varepsilon_0 \frac{\partial^2 \mathbf{P}}{\partial t^2}\,. \tag{3}$$

We still need to relate polarization $\mathbf{P}$ and the electric field $\mathbf{E}$. If we restrict ourselves to analysis of microscopically isotropic media, we can divide the induced polarization $\mathbf{P}$ into two parts (linear and nonlinear) where only the linear part has the explicit dependence on spatial coordinates

$$\mathbf{P}(r,t) = \mathbf{P}_{\mathrm{LIN}}(r,t,E) + \mathbf{P}_{\mathrm{NL}}(t,E)\,. \tag{4}$$

Making use of two important approximations — weakly guiding approximation (WGA) and slowly varying envelope approximation (SVEA) one finally arrives at the propagation equation of the pulse envelope in a nonlinear dispersive fiber [3]. WGA-approximation means that

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ISSN 1335-3632

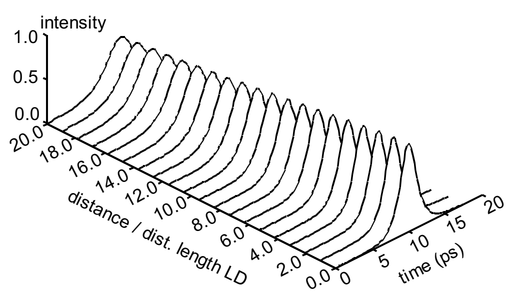

Fig. 1. Fundamental soliton in fiber with losses

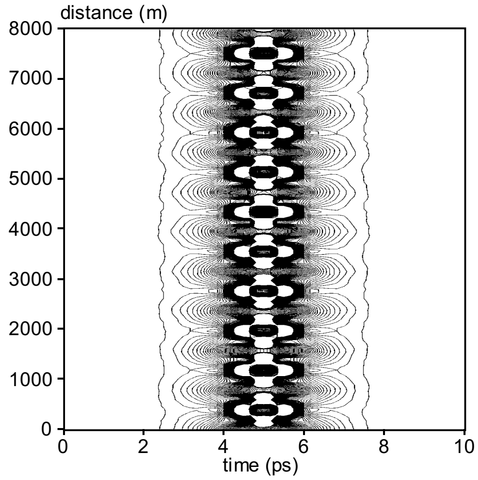

Fig. 2. Third order soliton along ten soliton periods

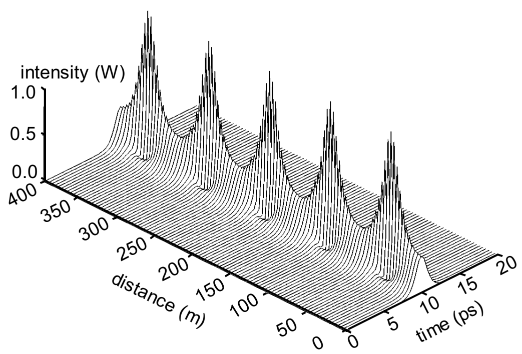

Fig. 3a. Surface map of the fourth order soliton along five soliton periods

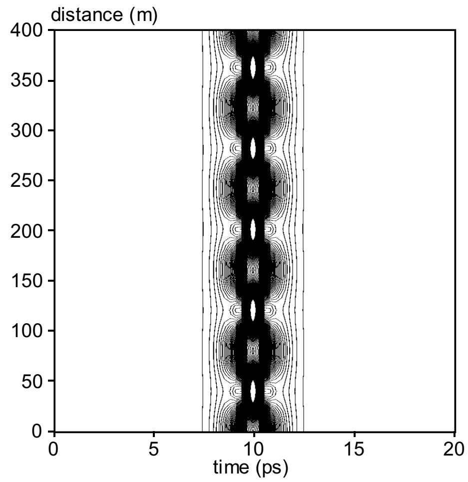

Fig.3b. Contour map of the fourth order soliton along five soliton periods

our fiber is weakly guiding, *ie* that the refractive index difference between the core and the cladding of the fiber is very small. This gives us the approximation

$$\nabla \times \nabla \times \mathbf{E} = \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} \approx -\nabla^2 \mathbf{E}\,. \tag{5}$$

On the other hand SVEA-approximation simplifies the wave-packet equation from being second order in z to the first order in z by the assumption that there are no backward-going waves. It is very useful to notice that the physical requirements for both WGA and SVEA are the same — the induced nonlinear refractive index of the fiber has to be very small. In that case WGA surely holds. It is obvious that the backscattered light is then also negligibly small (SVEA holds).

After all these approximations we get from (3) the partial nonlinear differential equation which describes the propagation of an optical pulse in nonlinear dispersive media with losses (the so-called nonlinear Schrödinger equation — NLSE) in the form

$$j\frac{\partial \Psi}{\partial z} = -j\beta_1 \frac{\partial \Psi}{\partial t} + \frac{1}{2}\beta_2 \frac{\partial^2 \Psi}{\partial t^2} - \frac{j}{2}\alpha\Psi - \gamma|\Psi|^2\Psi \tag{6}$$

where Ψ is the amplitude of the optical pulse envelope, β_1 and β_2 are the first and second coefficients in the Taylor expansion of the propagation constant $\beta(\omega)$ around the carrier frequency ω_0, α the coefficient of fiber losses and γ the nonlinear coefficient. Equation (6) has to be modeled by means of numerical methods in order to simulate the pulse propagation in the fiber in nonlinear regime. The easiest and the quickest way to do it is to divide (6) into the set of two coupled equations [6]. Before doing so, it is convenient to employ the so-called ‘moving frame transformation’

$$T = t - z/v_g = t - \beta_1 z \tag{7}$$

where the reference window is forced to move with the pulse at the group velocity v_g. After implementing transformation (7) in (6) the term containing group velocity $\beta_1 = 1/v_g$) disappears, leading thus to the equation

$$j\frac{\partial \Psi}{\partial z} = \frac{1}{2}\beta_2 \frac{\partial^2 \Psi}{\partial t^2} - \frac{j}{2}\alpha\Psi - \gamma\,|\Psi|^2\,\Psi\,. \tag{8}$$

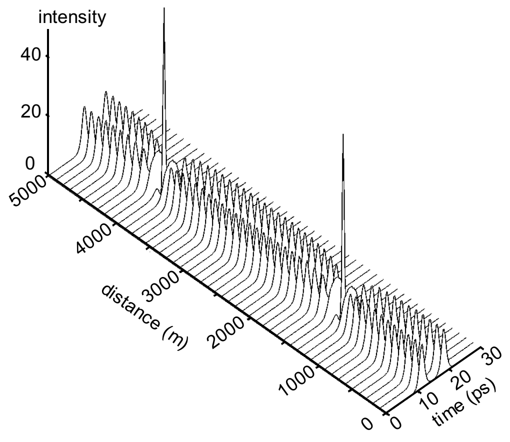

Fig. 4. The sequence of two fundamental solitons

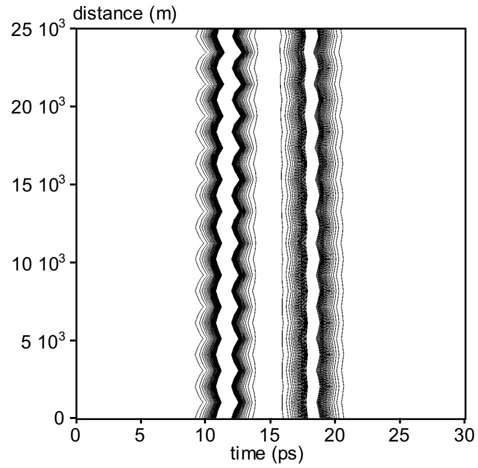

Fig. 5. Two adjacent solitons with unequal amplitudes and equal phase

This equation can now be split into the set of two coupled equations

$$\begin{aligned} \frac{1}{2}\frac{\partial \Psi}{\partial z} &= -j\frac{1}{2}\beta_2\frac{\partial^2\Psi}{\partial T^2} \\ \frac{1}{2}\frac{\partial \Psi}{\partial z} &= j\gamma\left|\Psi\right|^2\Psi - \frac{\alpha}{2}\Psi\,. \end{aligned} \tag{9}$$

The first equation in (9) is commonly solved after discretisation using the direct and inverse discrete Fourier transformation. Here the first equation was solved using the Crank-Nicholson finite differences scheme which is not only unconditionally numerically stable but also wave-power-flux conserving and second order accurate. Since it is an implicit method, the penalty in using it is that one has to solve the tridiagonal system of equations for each step in z-direction. The approximate solution of the second equation is straightforward. Then in the discretised form for $\Psi_i^n = \Psi(z_n, t_i)$ one obtains from (9)

$$\begin{aligned} \Psi_i^{n+1/2} &+ \frac{j\beta_2\Delta_z}{2\Delta_t^2}\left(\Psi_{i-1}^{n+1/2} - 2\Psi_i^{n+1/2} + \Psi_{i+1}^{n+1/2}\right) \\ &= \Psi_i^n - \frac{j\beta_2\Delta_z}{2\Delta_t^2}\left(\Psi_{i-1}^n - 2\Psi_i^n + \Psi_{i+1}^n\right) \\ \Psi_i^{n+1} &= \Psi_i^{n+1/2}\exp\left(\left[2j\gamma\left|\Psi_i^{n+1}\right|^2 + \alpha\right]\Delta_z\right) \end{aligned} \tag{10}$$

where $\Delta_z = z_{n+1} - z_n$ and $\Delta_t = t_{i+1} - t_i$ are equidistant intervals between discretisation points. This is also the core of numerical procedure where the two equations are used alternately. It represents a typical "forward marching" algorithm, *ie* the values in the new layer $n+1$ are calculated from the values in the previous layer n. To solve the system (10) one has to give the initial condition — the temporal distribution of the pulse amplitude at the beginning of the propagation path $\Psi(z,t)\big|_{z=0}$ or in the discrete form $\Psi_i^n\big|_{n=0}$ for all i. The single-pulse envelope in the form a the hyperbolic secant function (for fundamental and higher order single soliton propagation) or a group of hyperbolic secant shaped adjacent pulses with various amplitudes, phases and distances between them has been used as the input amplitude distribution.

The values can be chosen so as to closely represent the realizable systems. Usually a dimensionless form of the NLSE is used where the so-called soliton units are introduced

$$\begin{aligned} &\tau = \frac{T}{T_0}\,; \quad \xi = \frac{z}{L_D}\,; \quad u = NU = N\frac{A_0}{\sqrt{P_0}} \\ &N^2 = \frac{L_D}{L_{NL}} = \frac{\gamma P_0 T_0^2}{|\beta_2|}\,; \;\; L_D = \frac{T_0^2}{|\beta_2|}\,; \;\; L_{NL} = \frac{1}{\gamma P_0} \end{aligned} \tag{11}$$

where L_D and L_{NL} are the dispersion and the nonlinearity length, respectively, N is the order of soliton, $P_0 = |\Psi|^2$ is the peak power of soliton pulse and T_0 is the halfwidth of the pulse defined as the point of decrease of amplitude to $\operatorname{sech}(1) \approx 0.648$ of its maximum value. As the result we obtain the normalized equation

$$j\frac{\partial u}{\partial \xi} + \frac{1}{2}\frac{\partial^2 u}{\partial \tau^2} + j\frac{\alpha}{2}L_D u + N^2\left|u\right|^2 u = 0\,. \tag{12}$$

This equation can again be divided into the set of coupled equations

$$\begin{aligned} &\frac{j}{2}\frac{\partial u}{\partial \xi} + \frac{1}{2}\frac{\partial^2 u}{\partial \tau^2} = 0 \\ &\frac{j}{2}\frac{\partial u}{\partial \xi} + \left(N^2|u|^2 + jL_D\frac{\alpha}{2}\right)u = 0 \end{aligned} \tag{13}$$

and then similarly numerically modeled

$$\begin{aligned} j\frac{u_i^{n+1} - u_i^n}{\Delta_\xi} &+ \frac{u_{i-1}^{n+1} - 2u_i^{n+1} + u_{i+1}^{n+1}}{2\Delta_\tau^2} \\ &+ \frac{u_{i-1}^n - 2u_i^n + u_{i+1}^n}{2\Delta_\tau^2} = 0 \\ u_i^{n+2} &= u_i^{n+1}\exp\left(\left[2jN^2\left|u_i^{n+1}\right|^2 - L_D\alpha\right]\Delta_\xi\right)\,. \end{aligned} \tag{14}$$

No matter which equation (8) or (12) is modeled, the input pulse was always taken in the form of hyperbolic

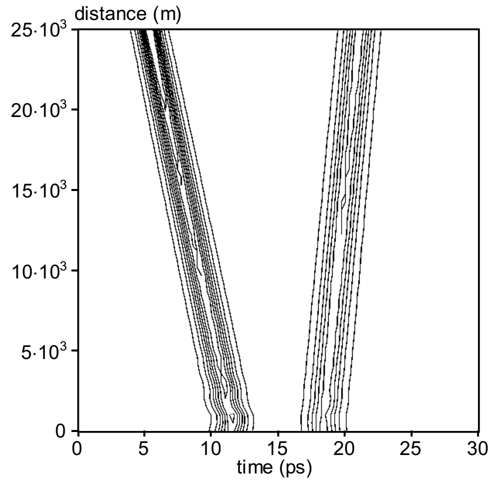

Fig. 6. Two adjacent solitons with unequal amplitudes and unequal phase

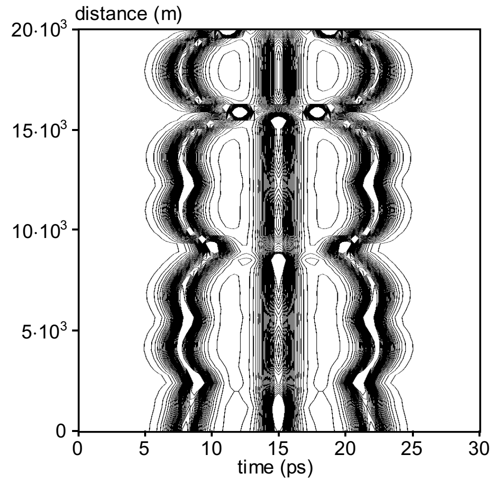

Fig. 7. Three soliton pulses with equal amplitudes and unequal phase

secant $\Psi(0,T) = A_0 \operatorname{sech}(T/T_0)$ for (8), or $u(0,\tau) = N \operatorname{sech}(\tau)$ for (12).

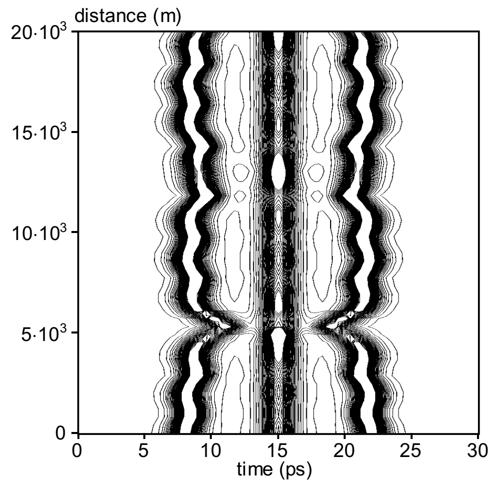

Fig. 8. Three soliton pulses with unequal amplitude and equal phase

3 SIMULATION OF FUNDAMENTAL AND HIGHER ORDER SOLITONS

The results of simulations will be shown in several examples. With the normalized equation (12) we observed the propagation of fundamental soliton ($N = 1$) with halfwidth $T_0 = 2$ ps in the fiber with losses $\Gamma = \alpha L_D/2 = 0.03$. Results are given in Fig. 1. for the propagation length of $20\, L_D$. As can be seen from the results, good quantitative judgement can be obtained in this way.

To model situations occurring in nonlinear fibers realistic parameters have to be taken for simulations in (8). As an example, the fiber with no losses ($\alpha = 0$) and fiber parameters $\beta_2 = -20\,\mathrm{ps^2 km^{-1}}$ and $\gamma = 4\,\mathrm{W^{-1}km^{-1}}$ has been taken. The input pulse is in the form $\Psi(0,T) = A_0 \operatorname{sech}(T/T_0)$, 1 ps wide ($T_0 = 1$ ps). We have simulated the propagation of the third order soliton for propagation distance of ten soliton periods (Fig. 2) and fourth order soliton propagation for five soliton periods (Fig. 3a — surface plot) and (Fig. 3b — contour map). Soliton period $z_0 = \pi L_D/2 = \pi T_0^2/2\beta_2$ gives the distance on which the higher order solitons again achieve their initial hyperbolic secant shape. This periodicity is easily seen in both Fig. 2 and Fig. 3b. For the simulations of higher order solitons the initial peak powers are calculated from $N^2 = \gamma P_0 T_0^2/\beta_2$ and for the required initial pulse amplitudes $A_0 = \sqrt{P_0}$ one obtains $A_0 = 6.708\,\mathrm{W}^{1/2}$ and $A_0 = 8.944\,\mathrm{W}^{1/2}$ for the third and fourth order solitons, respectively.

4 SIMULATION OF SOLITON SEQUENCES

The input amplitude distribution for two, three, four or five adjacent solitons is given by

$$\begin{aligned}\Psi(0,\tau) &= r_1 \operatorname{sech}[r_1(\tau - q_0)] \exp(j\Theta_1) \\ &+ r_2 \operatorname{sech}[r_2(\tau + q_0)] \exp(j\Theta_2)\end{aligned} \quad (15)$$

$$\begin{aligned}\Psi(0,\tau) &= r_1 \operatorname{sech}[r_1(\tau - 2q_0)] \exp(j\Theta_1) \\ &+ r_2 \operatorname{sech}(r_2\tau) \exp(j\Theta_2) \\ &+ r_3 \operatorname{sech}[r_3(\tau + 2q_0)] \exp(j\Theta_3)\end{aligned} \quad (16)$$

$$\begin{aligned}\Psi(0,\tau) &= r_1 \operatorname{sech}[r_1(\tau - 3q_0)] \exp(j\Theta_1) \\ &+ r_2 \operatorname{sech}[r_2(\tau - q_0)] \exp(j\Theta_2) \\ &+ r_3 \operatorname{sech}[r_3(\tau + q_0)] \exp(j\Theta_3) \\ &+ r_4 \operatorname{sech}[r_4(\tau + 3q_0)] \exp(j\Theta_4)\end{aligned} \quad (17)$$

$$\begin{aligned}\Psi(0,\tau) &= r_1 \operatorname{sech}[r_1(\tau - 4q_0)] \exp(j\Theta_1) \\ &+ r_2 \operatorname{sech}[r_2(\tau - 2q_0)] \exp(j\Theta_2) \\ &+ r_3 \operatorname{sech}(r_3\tau) \exp(j\Theta_3) \\ &+ r_4 \operatorname{sech}[r_4(\tau + 2q_0)] \exp(j\Theta_4) \\ &+ r_5 \operatorname{sech}[r_5(\tau + 4q_0)] \exp(j\Theta_5)\end{aligned} \quad (18)$$

where $\tau = T/T_0$, r_i, Θ_i, $i = 1,2,3,4,5$ are relative amplitudes and phases of each pulse and q_0 is the distance

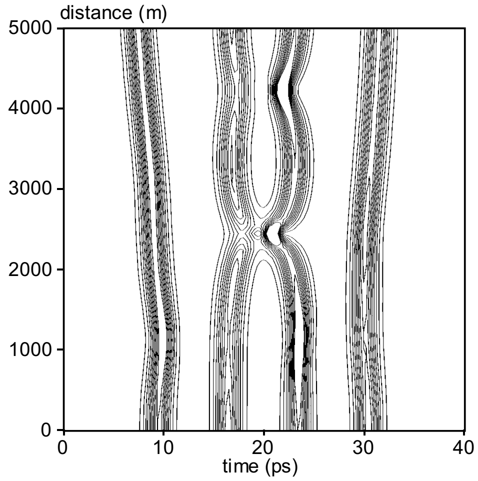

Fig. 9. Four adjacent solitons with equal amplitude and unequal phase

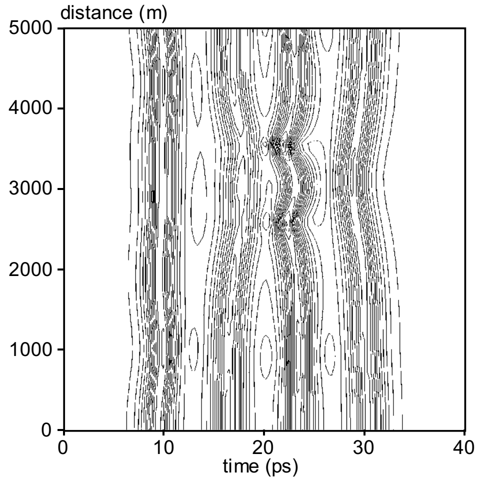

Fig. 10. Four adjacent solitons with unequal amplitude and equal phase

between the centers of adjacent pulses in half widths of the pulse T_0.

When two adjacent soliton pulses propagate in a nonlinear dispersive fiber and are close enough, they will periodically collapse to a single pulse and break again to the original form. The period of this breakdown is given by

$$z_p = \frac{\pi}{2} L_D e^{q_0} = \frac{\pi}{2} \frac{T_0^2}{|\beta_2|} e^{q_0} . \tag{19}$$

It is obvious that if the distance between the pulses were large enough, the interaction would be negligible. On the contrary, if we choose as the input amplitude distribution two fundamental solitons with $T_0 = 1$ ps in distance $q_0 = 3.5$ propagating in the fiber with $\beta_2 = -20\,\mathrm{ps^2km^{-1}}$ and $\gamma = 1.3\,\mathrm{W^{-1}km^{-1}}$, the period of their collapse will be $z_p = 2.6$ km (Fig. 4).

Having performed a number of simulations with various relative amplitudes, phases and distances between the pulses, one arrives at the following conclusion — for the shorter propagation distances the best way to stabilize the pulses is to choose small relative phases between them (for example $30°$ or $45°$) whereas for longer distances (calculated up to 20 km) the best way to stabilize the pulse propagation is to use slightly different amplitudes of the pulse (10 % difference is sufficient). For two adjacent solitons the above statements are illustrated in Fig. 5 and Fig. 6. In Fig. 5 solitons co-propagate 25 km in the fiber. They have different amplitudes ($r_1 = 1$, $r_2 = 1.1$) and their stabilization is easily seen. If we now add also the phase difference ($\Theta_2 = \pi/6$), we see (Fig. 6) that the propagation is corrupted and pulses will separate.

Propagation of three neighbouring ($q_0 = 3.5$) pulses along the fiber with parameters $\beta_2 = -20\,\mathrm{ps^2km^{-1}}$ and $\gamma = 4\,\mathrm{W^{-1}km^{-1}}$ is shown in Fig. 7 and in Fig. 8. Here 20 km propagation of equal amplitude pulses with relative phases $\Theta_1 = 0$, $\Theta_2 = \pi/4$ and $\Theta_3 = 0$ is shown in Fig. 7. The propagation is stabilized compared to the case of equal amplitude and phase pulses

Even better results were obtained for 20 km propagation of different-amplitude pulses ($r_1 = 1.1$, $r_2 = 1$, $r_3 = 1.1$) having equal phases (Fig. 8)

The sequence of four adjacent soliton pulses with equal amplitude and unequal phase ($\Theta_1 = 0$, $\Theta_2 = \pi/4$, $\Theta_3 = 0$ and $\Theta_4 = \pi/4$) propagating along short distance of 5 km is shown in Fig. 9 for fiber parameters equal to three soliton case. Propagation is again stabilized compared to the case of equal amplitude and phase. Still, the best way to stabilize the propagation for longer distances is to choose different amplitudes ($r_1 = 1.1$, $r_2 = 1$, $r_3 = 1.1$, $r_4 = 1$) and equal phases (5 km propagation is shown in Fig. 10).

With equal fiber parameters also the propagation of the five adjacent solitons was modeled. Their behavior, if all pulses have the same amplitude and phase is shown in Fig. 11. The best way to stabilize them — the unequal amplitude and equal phase case is shown in Fig. 12. The amplitudes $r_1 = 1.1$, $r_2 = 1$, $r_3 = 1.1$, $r_4 = 1$ and $r_5 = 1.1$ were chosen for simulation of 15 km propagation path.

5 CONCLUSIONS

The behavior of soliton pulses and their sequences during propagation in optical fiber waveguides were investigated. A sequence of adjacent solitons shows detrimental periodical collapse of two adjacent pulses for all cases with equal input amplitudes and phases when the pulses are close enough (usual value $q_0 = 3.5$). This behavior can be suppressed by different phases of adjacent input pulses (for small propagation distances) or by different amplitudes of the pulses, which stabilizes the propagation behavior of the sequence for longer distances (in our examples distances of 15–20 km were shown). If one combines both different phases and amplitudes for longer distance propagation then either the detrimental collapse of adjacent pulses was obtained or they separate quite quickly one from another.

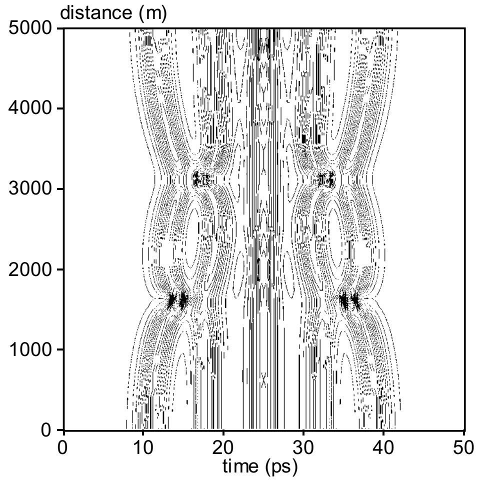

Fig. 11. Five solitons with equal amplitude and phase

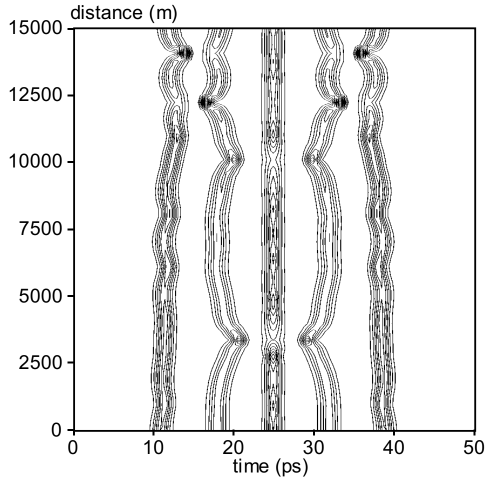

Fig. 12. Five solitons with unequal amplitude and equal phase

Generally, the best solution how to achieve the highest bit-rates of such systems lies in slightly different amplitudes of adjacent pulses (about 10 %). Larger amplitude differences do not result in a notable improvement of propagation behavior. We also noticed that there is a difference in the behavior of uneven sequence of pulses depending of whether the leading pulse has the amplitude 1.1 or 1. Better results were obtained with sequences beginning with 10 % higher amplitude. The numerical simulations have been performed for sequences of up to five adjacent solitons, but can be easily extended for longer sequences merely by changing the form of initial pulse distribution.

Acknowledgement

This work has been done as part of the VEGA grant No. 1/8264/01 of the Slovak governmental grant agency. The stay of Ms. Zentner with the Slovak University of Technology was made possible through the CEEPUS project CZ-103. The financial support of both institutions is kindly acknowledged.

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Received 16 February 2001

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