1. 第一步, 记

$$S\left(x_0,\cdots,x_n,\cdots;y_0,\cdots,y_n,\cdots\right):=x\oplus y,\ \forall x:=\left(x_n\right)_{n\geq 0},\ y:=\left(y_n\right)_{n\geq 0}\in\mathcal{W}\left(\kappa\right).$$

由于

$$\left\{\tau_e\left({x_n}^{p^{-m}}\right)\right\}_{n\geq 0},\ \left\{\tau_e\left({y_n}^{p^{-m}}\right)\right\}_{n\geq 0}\in\mathcal{W}\left(\Re\right),$$

我们记

$$S'\begin{pmatrix}\tau_e\left({x_0}^{p^{-m}}\right),\cdots,\tau_e\left({x_n}^{p^{-m}}\right),\cdots;\\ \tau_e\left({y_0}^{p^{-m}}\right),\cdots,\tau_e\left({y_n}^{p^{-m}}\right),\cdots\end{pmatrix}:=\left\{\tau_e\left({x_n}^{p^{-m}}\right)\right\}\oplus\left\{\tau_e\left({y_n}^{p^{-m}}\right)\right\}.$$

由此成立

$$W_m^{(\Re)}\left(S'\begin{pmatrix}\tau_e\left({x_0}^{p^{-m}}\right),\cdots,\tau_e\left({x_n}^{p^{-m}}\right),\cdots;\\ \tau_e\left({y_0}^{p^{-m}}\right),\cdots,\tau_e\left({y_n}^{p^{-m}}\right),\cdots\end{pmatrix}\right)\tag{4.3.2.16a}$$

$$=W_m^{(\Re)}\left(\left\{\tau_e\left({x_n}^{p^{-m}}\right)\right\}\right)+W_m^{(\Re)}\left(\left\{\tau_e\left({y_n}^{p^{-m}}\right)\right\}\right).\tag{4.3.2.16b}$$

2. 第二步, 针对式 *(4.3.2.16a)* 的出现, 我们考虑

$${S_n}'\begin{pmatrix}\tau_e\left({x_0}^{p^{-m}}\right),\cdots,\tau_e\left({x_n}^{p^{-m}}\right),\cdots;\\ \tau_e\left({y_0}^{p^{-m}}\right),\cdots,\tau_e\left({y_n}^{p^{-m}}\right),\cdots\end{pmatrix}\in\Re\tag{4.3.2.17}$$

与

$$\tau_e\left(S_n\left({x_0}^{p^{-m}},\cdots,{x_n}^{p^{-m}},\cdots;{y_0}^{p^{-m}},\cdots,{y_n}^{p^{-m}},\cdots\right)\right)\in\Re\tag{4.3.2.18}$$

的关系. 基于推论*4.2.2.9*, 我们知道式 *(4.3.2.17)*, 式 *(4.3.2.18)* 都是整系数多项式. 而由于模去理想映射 ϕ 满足

$$\phi\left(\boldsymbol{z}\in\Re\right)=\phi\left(\sum_{i=1}^{z}\mathbf{1}\in\Re\right)=\sum_{i=1}^{z}\phi\left(\mathbf{1}\in\Re\right)=\sum_{i=1}^{z}1\in\kappa=z,\ \forall z\in\mathbb{Z}.$$

因此由 ϕ 保持加法和乘法的性质, 成立

$$\begin{aligned}&\phi\left({S_n}'\left(\tau_e\left({x_0}^{p^{-m}}\right),\cdots,\tau_e\left({x_n}^{p^{-m}}\right),\cdots;\tau_e\left({y_0}^{p^{-m}}\right),\cdots,\tau_e\left({y_n}^{p^{-m}}\right),\cdots\right)\right)\\ =&S_n\begin{pmatrix}\phi\left(\tau_e\left({x_0}^{p^{-m}}\right)\right),\cdots,\phi\left(\tau_e\left({x_n}^{p^{-m}}\right)\right),\cdots;\\ \phi\left(\tau_e\left({y_0}^{p^{-m}}\right)\right),\cdots,\phi\left(\tau_e\left({y_n}^{p^{-m}}\right)\right),\cdots\end{pmatrix}\\ =&S_n\left({x_0}^{p^{-m}},\cdots,{x_n}^{p^{-m}},\cdots;{y_0}^{p^{-m}},\cdots,{y_n}^{p^{-m}},\cdots\right)\\ =&\tau_e\left(S_n\left({x_0}^{p^{-m}},\cdots,{x_n}^{p^{-m}},\cdots;{y_0}^{p^{-m}},\cdots,{y_n}^{p^{-m}},\cdots\right)\right).\end{aligned}$$

因此成立

$$\begin{aligned}&{S_n}'\left(\tau_e\left({x_0}^{p^{-m}}\right),\cdots,\tau_e\left({x_n}^{p^{-m}}\right),\cdots;\tau_e\left({y_0}^{p^{-m}}\right),\cdots,\tau_e\left({y_n}^{p^{-m}}\right),\cdots\right)\\ \equiv\ &\tau_e\left(S_n\left({x_0}^{p^{-m}},\cdots,{x_n}^{p^{-m}},\cdots;{y_0}^{p^{-m}},\cdots,{y_n}^{p^{-m}},\cdots\right)\right)\mod\boldsymbol{p}\Re.\end{aligned}$$