Lecture 3

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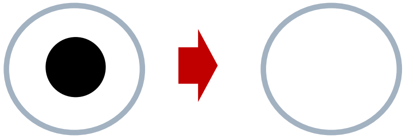
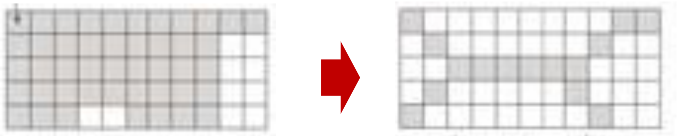
Mathematical Morphology

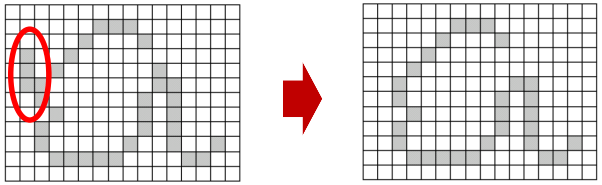
- Mathematical morphology works on shape

- Shape: prime carrier of information in machine vision

- Morphological operations:
  - simplify image data
  - preserve essential shape characteristics,
  - eliminate irrelevancies







Binary Morphology

- Set theory: language of binary mathematical morphology
- Sets in mathematical morphology: represent shapes

- Category
  - Dilation, erosion: primary morphological operations
  - Opening, closing: composed from dilation, erosion

- Related to shape representation, decomposition, primitive extraction

Binary Morphology

- 以二值化影像型態學為例，何時使用型態學處理?
  - 去除雜訊、簡化影像資料、保留重要區塊

灰階影像　二值化影像　型態學　連通元件

Binary Morphology

- A：輸入影像
- B：結構元素**(structuring element)**
- 逐列逐行掃描輸入影像每一像素點，將其鄰近區域與結構元素，進行集合運算操作
- 結構元素又稱為核心**(kernel)**、遮罩**(mask)**、濾波器**(filter)**

0	1	0
1	1	1
0	1	0

B

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

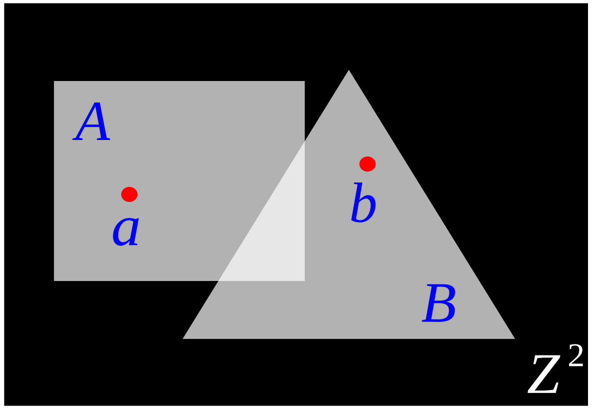
A

Set Theory

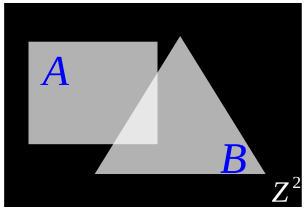
- 定義集合 A, B 在歐式二維空間中 Z^2 **(Euclidean 2-space)**

- 若 $a=(a_1,a_2)$ 是 A 的元素，
  則 $a \in A$

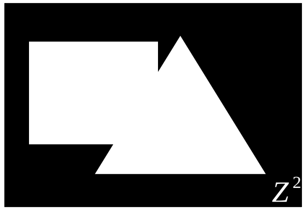
- 若 b 不是 A 的元素，
  則 $b \notin A$



Set Theory

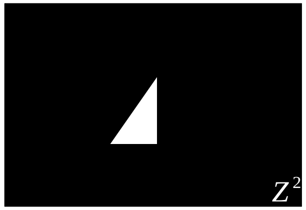


- 聯集 $A \cup B$
  - 包含 A 與 B 的所有元素

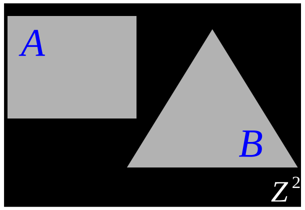


$A \cup B$

- 交集 $A \cap B$
  - 同時屬於 A 與 B 的元素
  - 若沒有交集，即 $A \cap B = \varnothing$ 為空集合

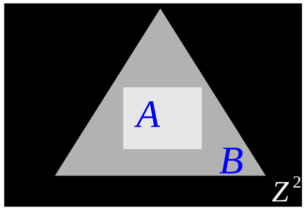


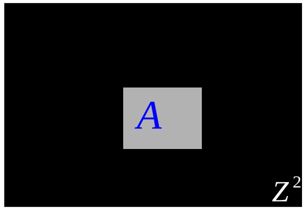
$A \cap B$



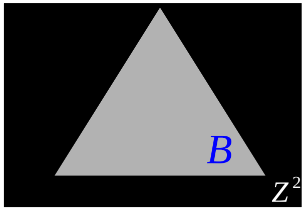
$A \cap B = \varnothing$

- A 為 B 的子集合 $A \subseteq B$ ，則
  - A 的所有元素完全包含於 B
  - $A \cap B = A$
  - $A \cup B = B$





$A \cap B$



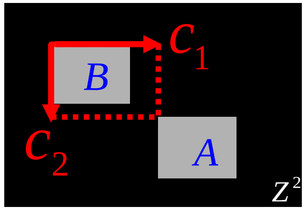
$A \cup B$

Set Theory

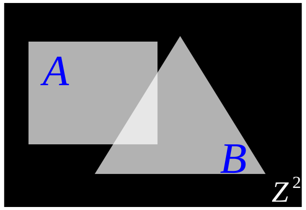
- 集合的數學描述式
- 例：$A = \{ a \mid a = b + c, \text{ for } b \in B \}$

此集合的元素　屬於此集合之元素須滿足的條件　大括號內為集合的描述

- A 集合的元素 a ，為將 B 集合的所有元素 b 位移 $c = (c_1, c_2)$



Set Theory



- A 的補集 A^c
  - 不屬於 A 的元素

$$A^c = \{ w \mid w \notin A \}$$

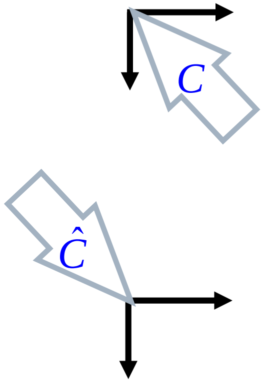
A^c

- A 與 B 的差集 $A - B$
  - 屬於 A ，但不屬於 B 的元素

$$A - B = \{ w \mid w \in A, w \notin B \} = A \cap B^c$$

$A - B$

Set Theory

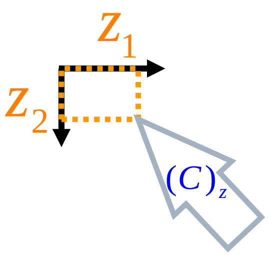


- 集合的反射 $\hat{C}$
  - 元素的座標乘上負號 $(x, y) \rightarrow (-x, -y)$

$$\hat{C} = \{ w \mid w = -c, \quad \text{for} \quad c \in C \}$$

- 集合的平移 $(C)_z$
  - 將 C 集合的所有元素位移 $z = (z_1, z_2)$
  - 元素的座標位移 $(x, y) \rightarrow (x + z_1, y + z_2)$

$$(C)_z = \{ w \mid w = c + z, \quad \text{for} \quad c \in C \}$$



Binary Dilation

- Combine two sets by vector addition of set elements

- Dilation of A by B

Euclidean N-space: E^N

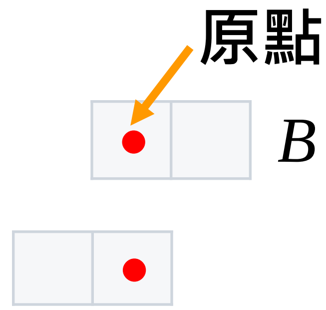
$$A \oplus B$$

- Addition commutative ➔ dilation commutative

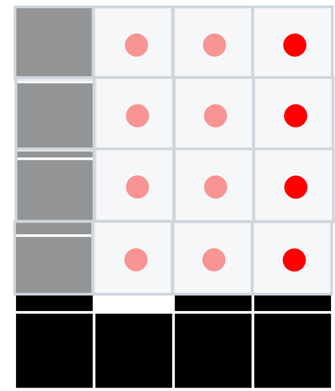
$$\mathrm{A} \oplus \mathrm{B} = \mathrm{B} \oplus \mathrm{A}$$

- A: referred as set (image)
- B: structuring element (kernel)

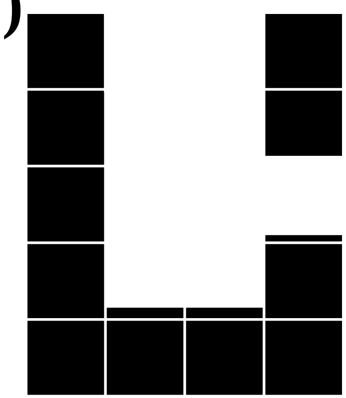
Binary Dilation



B



A



$A \oplus B$

- 膨脹的數學描述式

$$A \oplus B = \left\{ z \mid (\hat{B})_z \cap A \neq \varnothing \right\}$$

  - B 對於其原點反射得 $\hat{B}$ ，並平移此反射 z 單位 (逐列逐行掃描 平移 z)
  - 其位移與 A 重疊至少一個元素
  - 滿足以上敘述之 z 點，即屬於 $A \oplus B$
  - 又可描述為 $A \oplus B = \left\{ z \mid \left[(\hat{B})_z \cap A \right] \subseteq A \right\}$

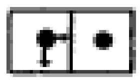
Binary Dilation

- Example

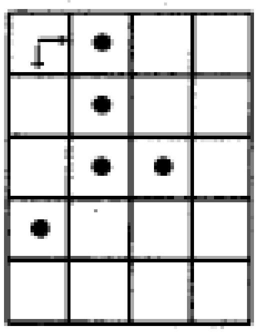
$$A_t = \{c \in E^N \mid c = a + t \text{ for some } a \in A\}$$

$$A = \{(0,1),(1,1),(2,1),(2,2),(3,0)\}$$

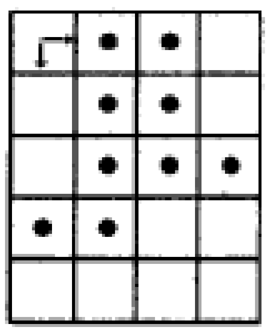
$$B = \{(0,0),(0,1)\}$$



B



A

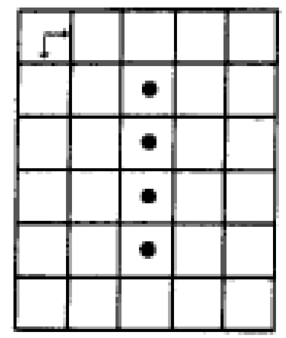


$A \oplus B$

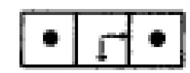
$$A \oplus B = \{(0,1),\ (1,1),\ (2,1),\ (3,0)\ (0,2),\ (1,2),\ (2,2),\ (2,3),\ (3,1)\}$$

- Example
  - Dilation by kernel without origin:

    might not have common pixels with A



A



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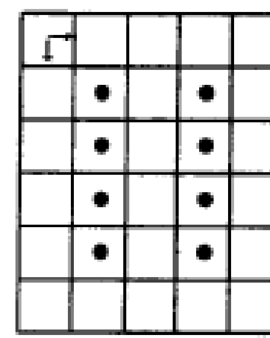


$A \oplus B$

Binary Dilation

- Dilating A by kernel with origin guaranteed to contain A

- Addition associative → dilation associative

$$(A \oplus B) \oplus C = A \oplus (B \oplus C)$$

- Dilation distributes over union

$$(B \cup C) \oplus A = (B \oplus A) \cup (C \oplus A)$$

- Dilating by union of two sets: the union of the dilation

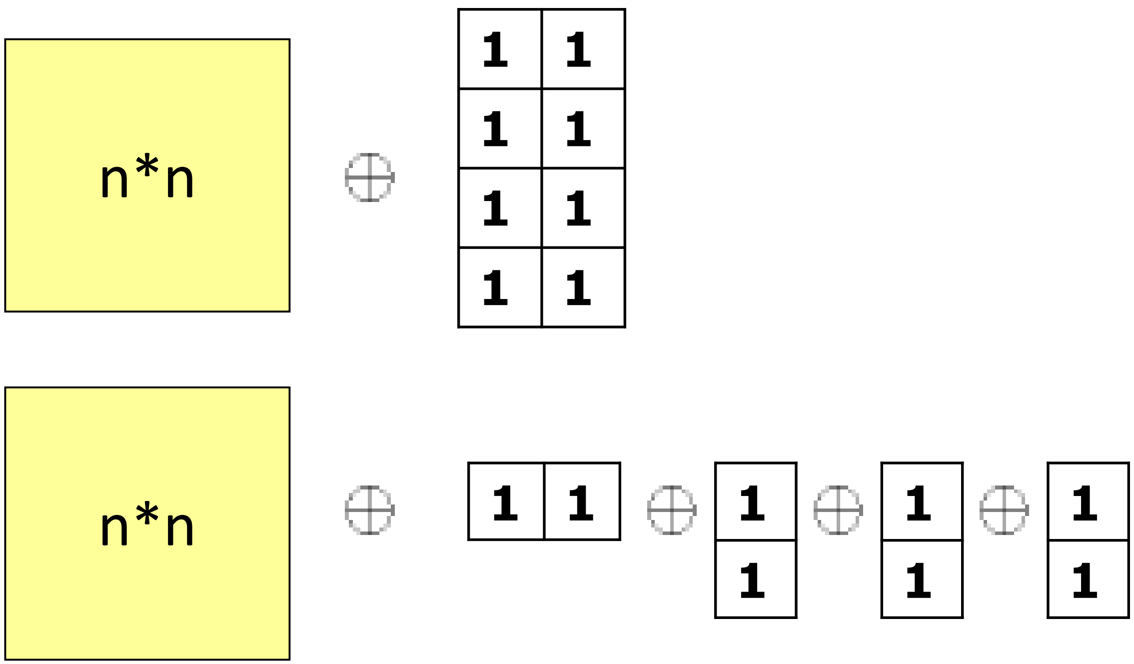
$$A \oplus (B \cup C) = (A \oplus B) \cup (A \oplus C)$$

Binary Dilation

- Decompose kernels to make dilations/erosions fast

- Example: dilate n*n image by 2^M kernel

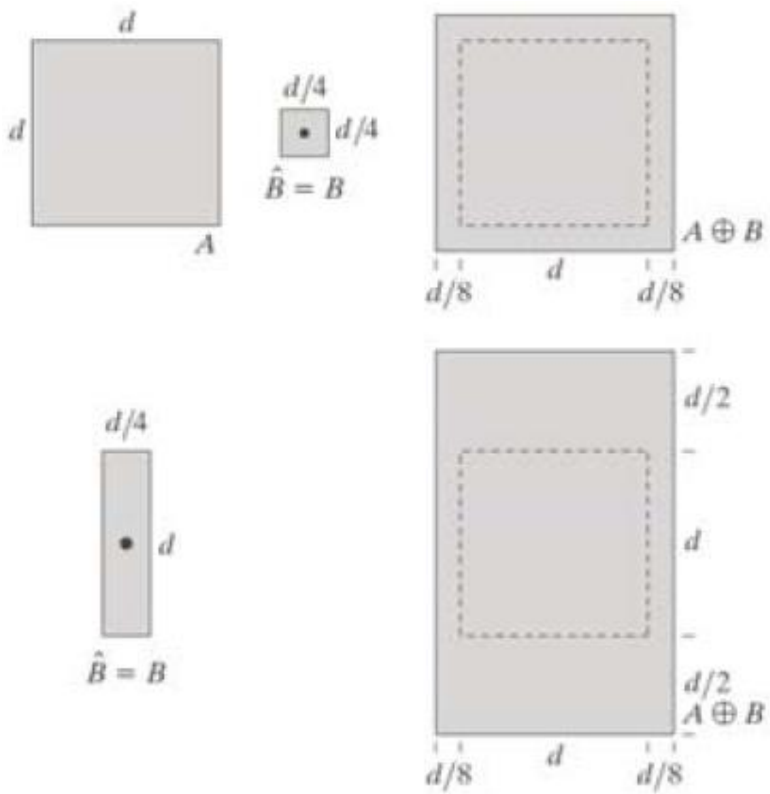
computational complexity

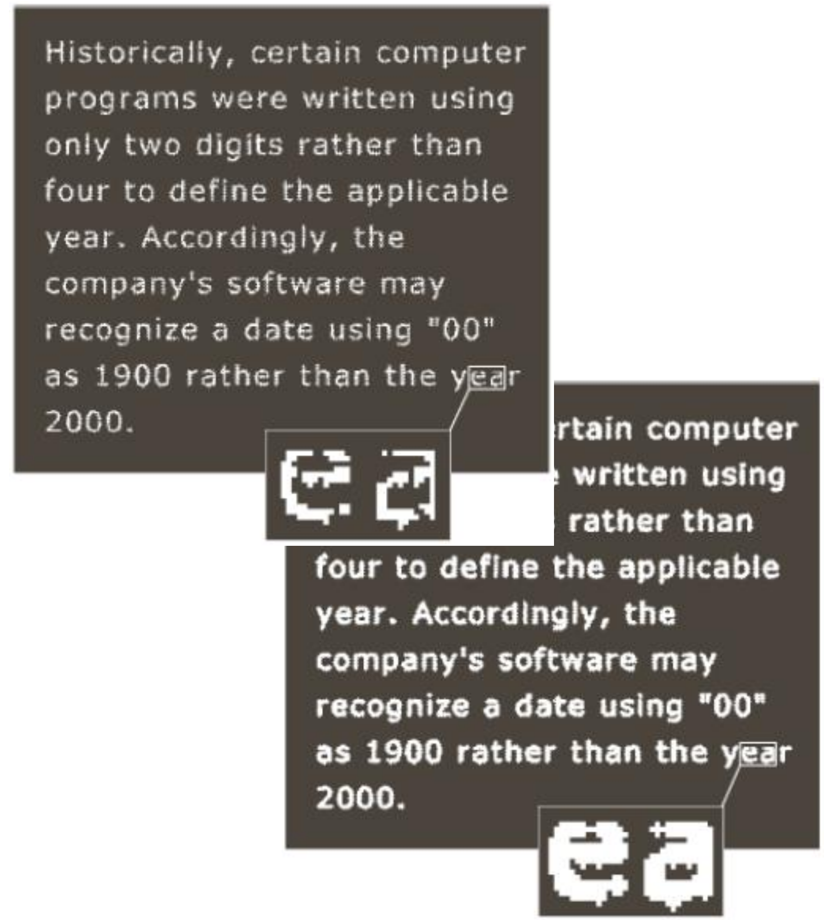


⇨ $n*n*2^M$

⇨ $n*n*2*M$

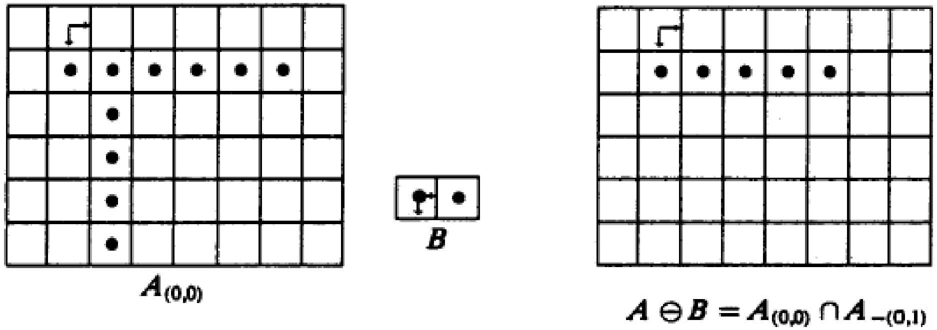
Binary Dilation





Binary Erosion

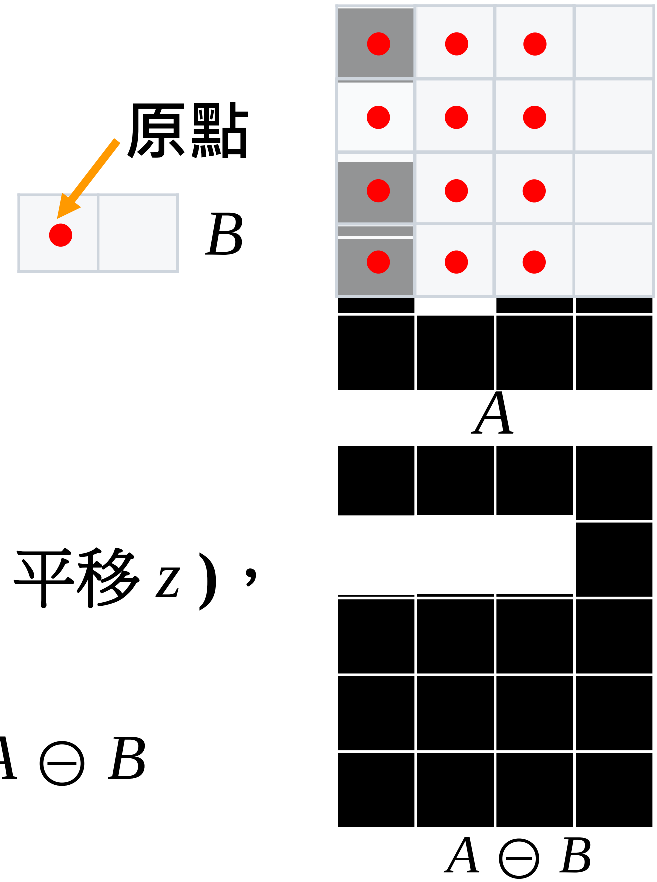
- Erosion of A by B: $A \ominus B$



- 侵蝕的數學描述式

$$A \ominus B=\left\{ z \mid (B)_z \subseteq A \right\}$$

  - A 被 B 侵蝕，
    使得 B 位移 z 後 (逐列逐行掃描 平移 z)，
    可被包含在 A 中
  - 滿足以上敘述之 z 點，即屬於 $A \ominus B$



Binary Erosion

- Eroding A by kernel without origin can have nothing in common with A

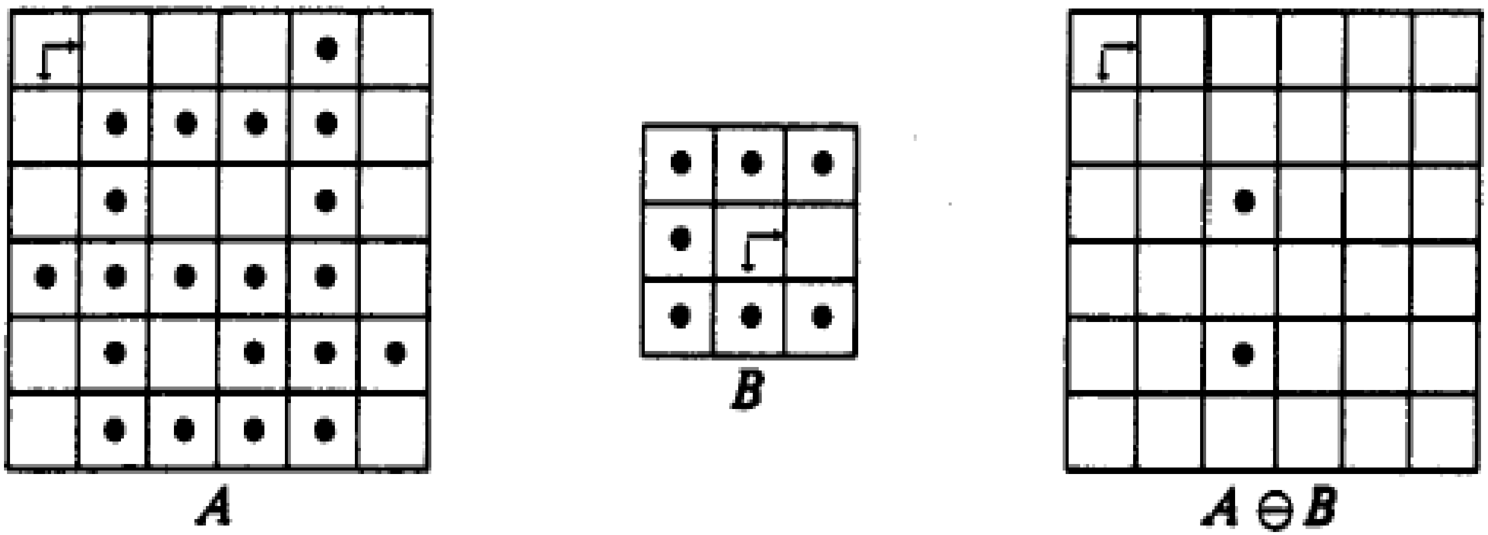


Figure 5.5 Erosion of a set A by a structuring element B that does not contain the origin. As a result, no point of the erosion is guaranteed to be in common with A. However, some translations of $A \ominus B$ are contained in A.

Binary Erosion

- Erosion of intersection of two sets: intersection of erosions $(A \cap B) \ominus K = (A \ominus K) \cap (B \ominus K)$

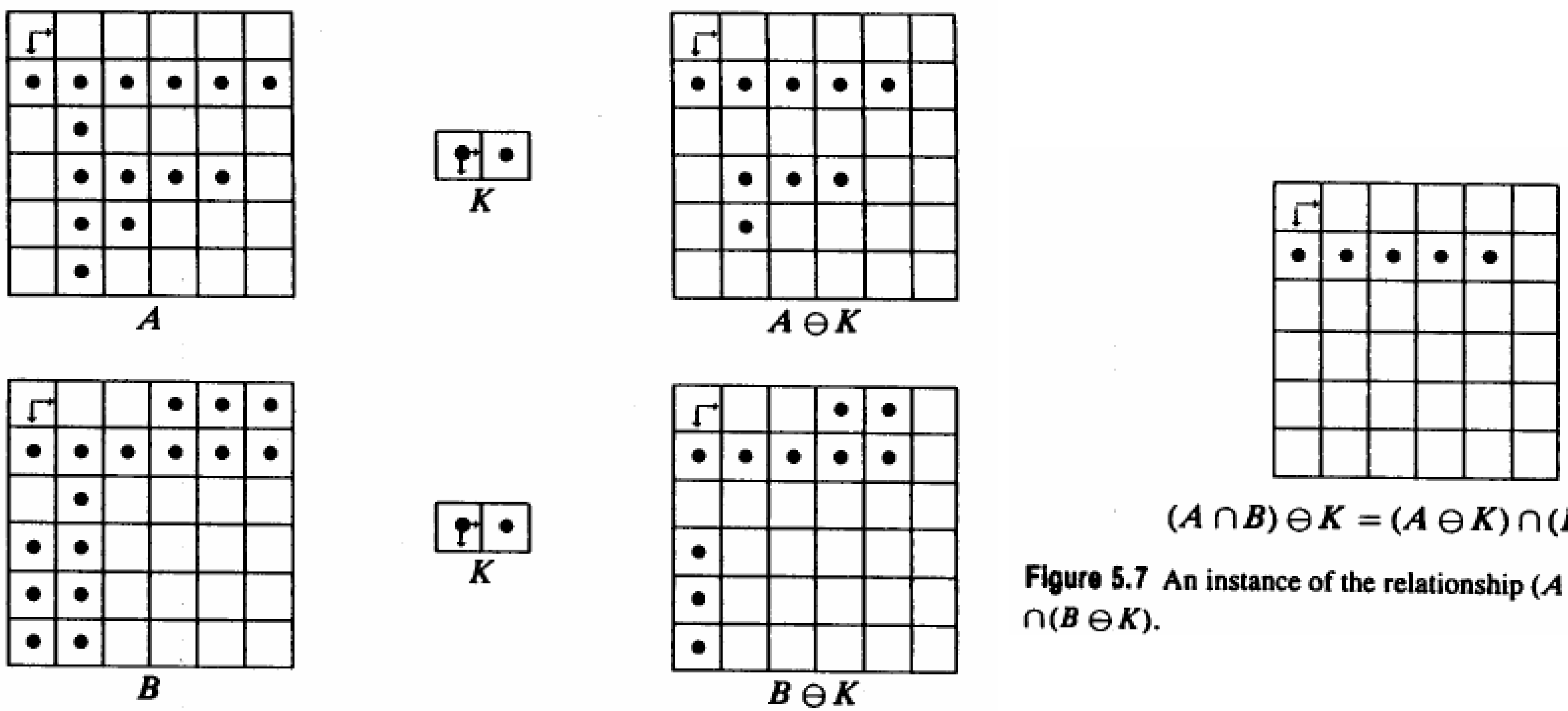


Figure 5.7 An instance of the relationship $(A \cap B) \ominus K = (A \ominus K) \cap (B \ominus K)$.

Binary Erosion

- Erosion of kernel of intersection of two sets: contains union of erosions

$$A \ominus (B \cap C) \supseteq (A \ominus B) \cup (A \ominus C)$$

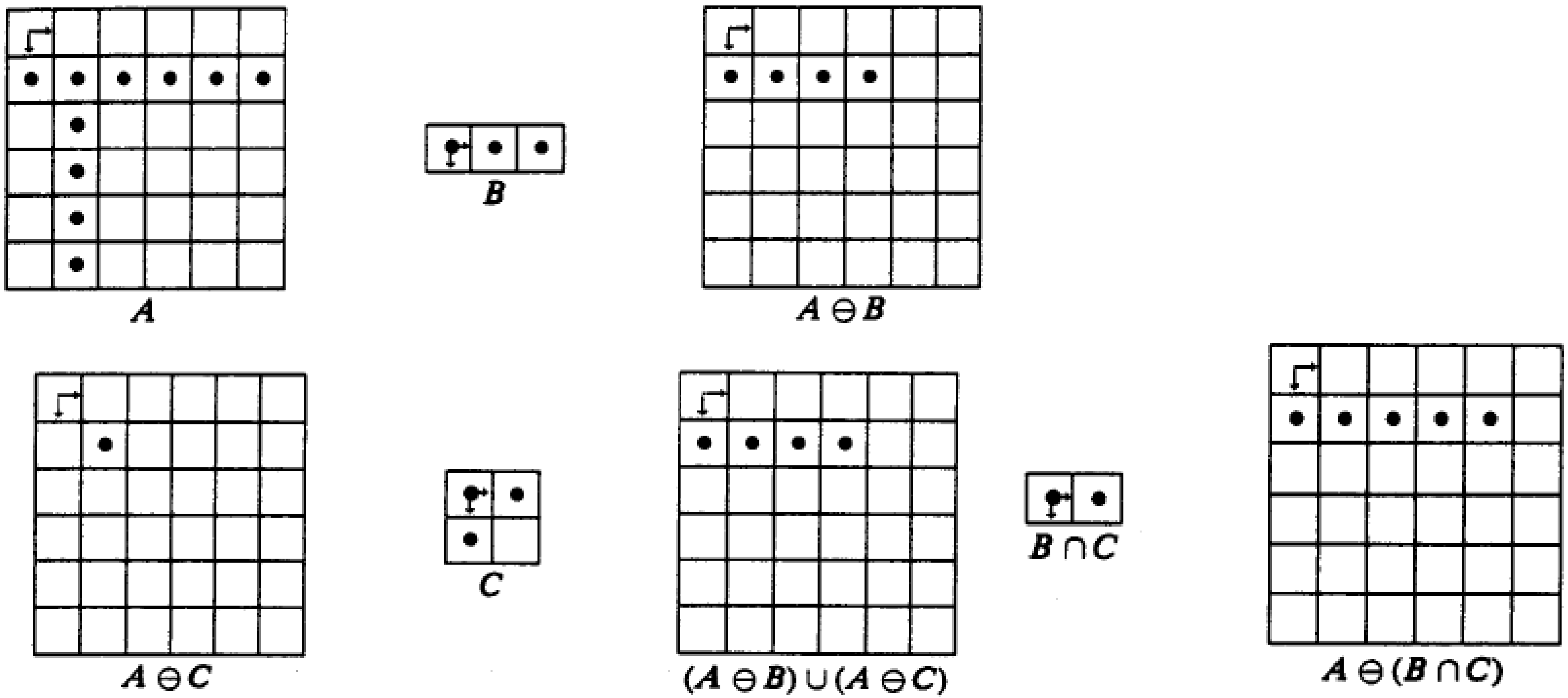


Figure 5.8 An instance in which $A \ominus (B \cap C)$ is a proper superset of $(A \ominus B) \cup (A \ominus C)$, thereby showing that the general relation $A \ominus (B \cap C) \supseteq (A \ominus B) \cup (A \ominus C)$ cannot be made any stronger.

Binary Erosion

- Erosion Dilation Duality $(A \ominus B)^c = A^c \oplus \check{B}$

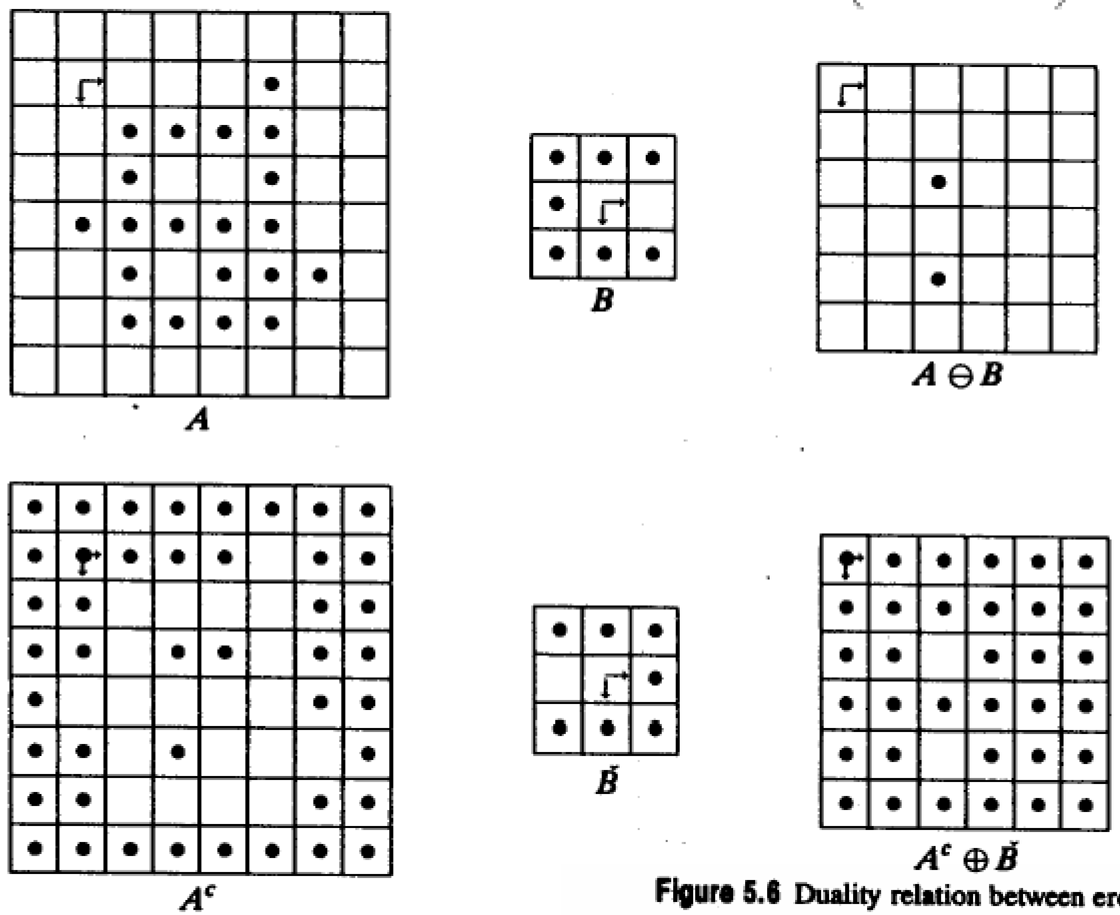


Figure 5.6 Duality relation between erosion and dilation. The set A eroded by B is the complement of the set A^c dilated by $\check{B}$. By convention, we understand that for the complemented set A^c or $A^c \oplus \check{B}$, all pixels outside the area illustrated are binary-1 pixels.

Binary Erosion

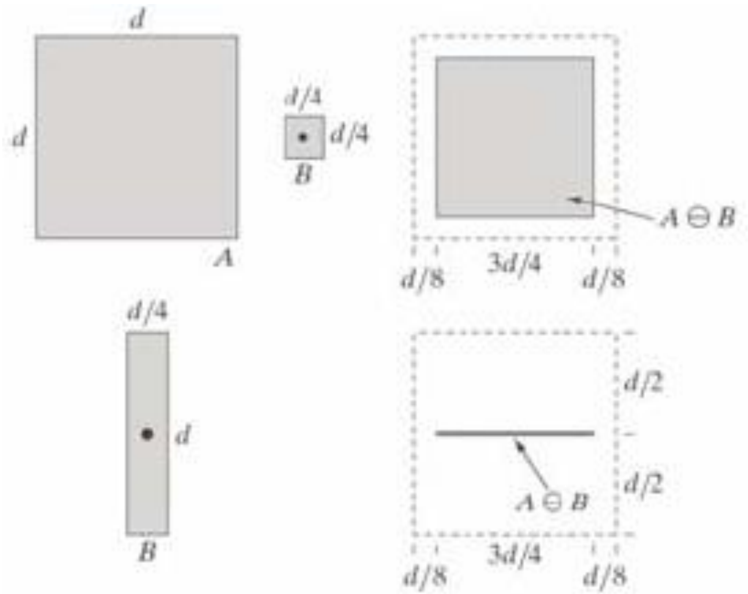
- Chain rule for erosion holds when kernel decomposable through dilation

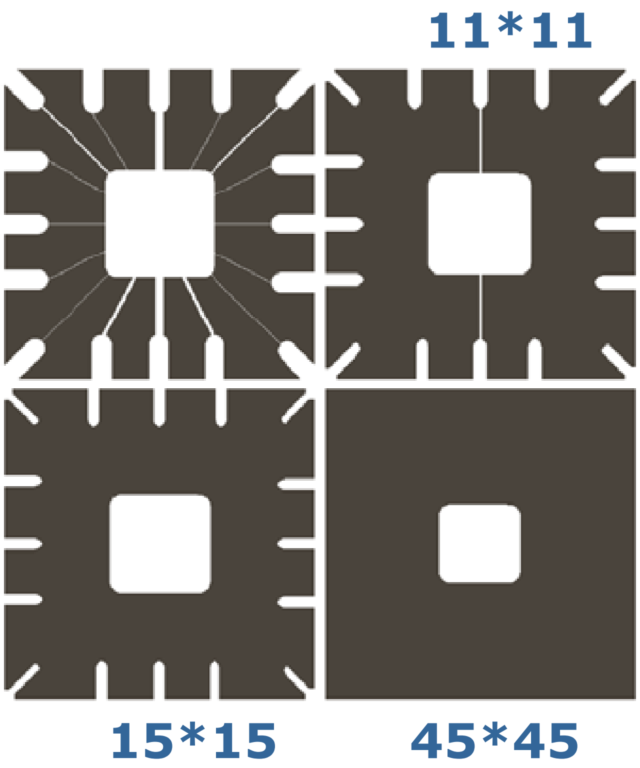
$$A \ominus (B \oplus C) = (A \ominus B) \ominus C$$

- Duality does not imply cancellation on morphological equalities

$$(B \ominus C) \oplus C \neq B$$

Binary Erosion





Binary Morphology

- Opening of image B by kernel K

$$B \circ K = (B \ominus K) \oplus K$$

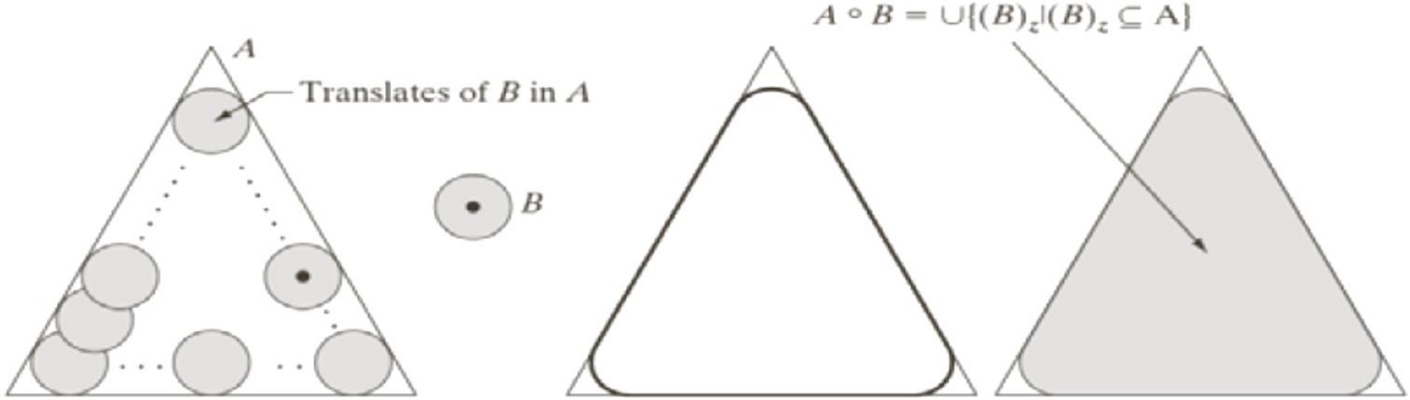
- Closing of image B by kernel K

$$B \bullet K = (B \oplus K) \ominus K$$

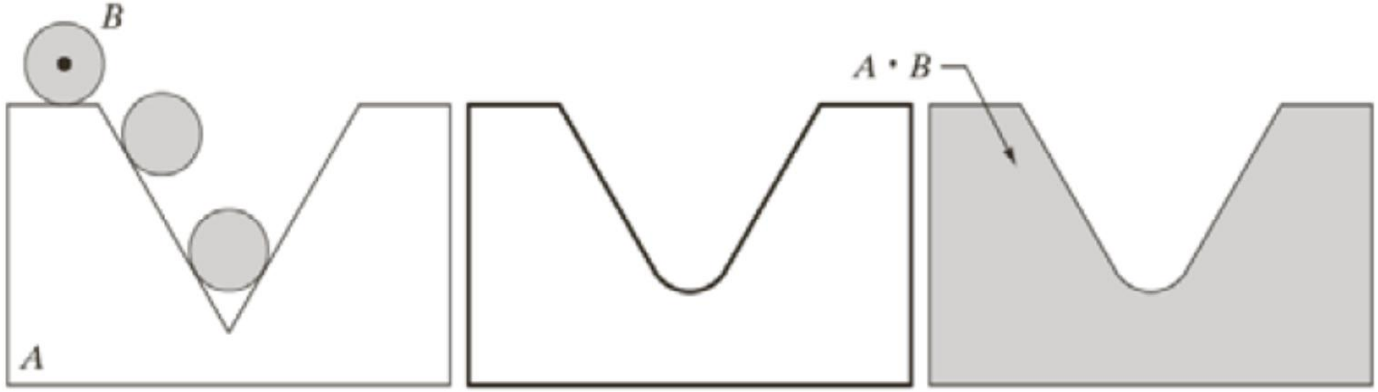
- Opening with disk kernel
  - smoothes contours, breaks narrow isthmuses
  - eliminates small islands, sharp peaks, capes
- Closing by disk kernel
  - smoothes contours, fuses narrow breaks, long, thin gulfs
  - closing with disk kernel: eliminates small holes, fill gaps on the contours

Binary Morphology

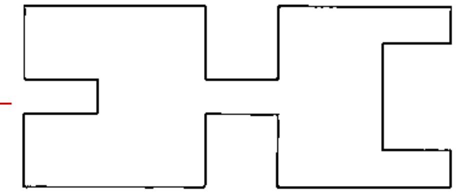
- Opening
  erosion, then dilation



- Closing
  dilation, then erosion

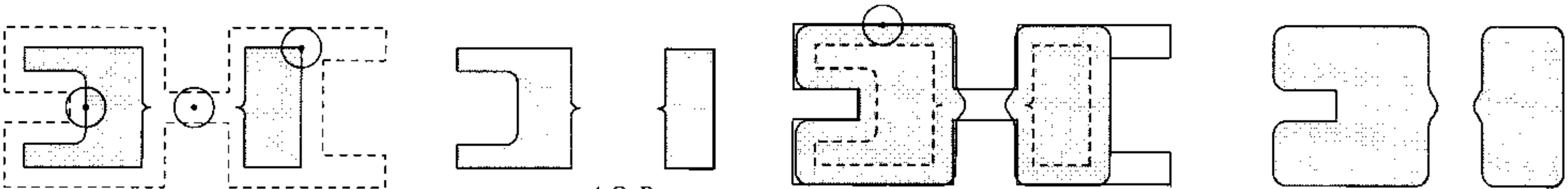


Binary Morphology



A

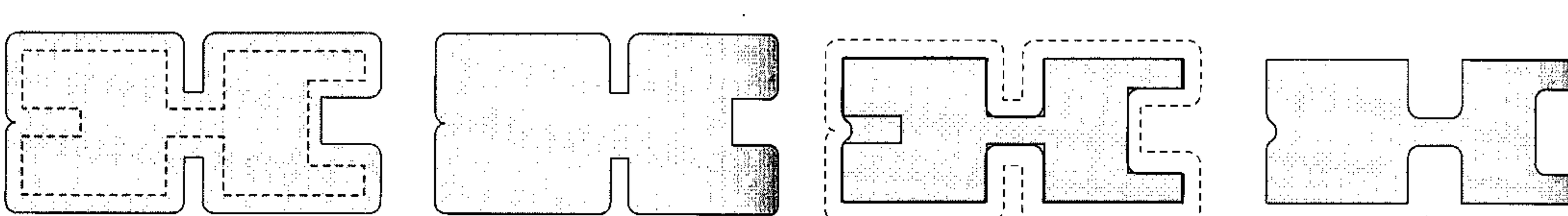
- Opening
  erosion, then dilation



$A \ominus B$

$A \circ B = (A \ominus B) \oplus B$

- Closing
  dilation, then erosion



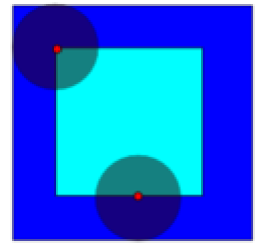
$A \oplus B$

$A \bullet B = (A \oplus B) \ominus B$

Binary Morphology

- Erosion

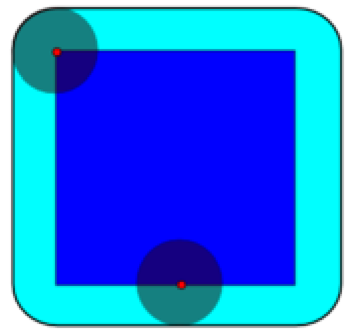
$$A \ominus B = \{z \in E | B_z \subseteq A\}$$



Original: dark
Result: light

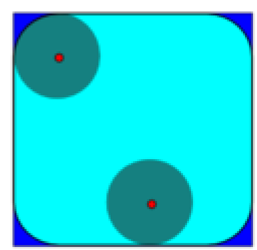
- Dilation

$$A \oplus B = \bigcup_{b \in B} A_b$$



- Opening

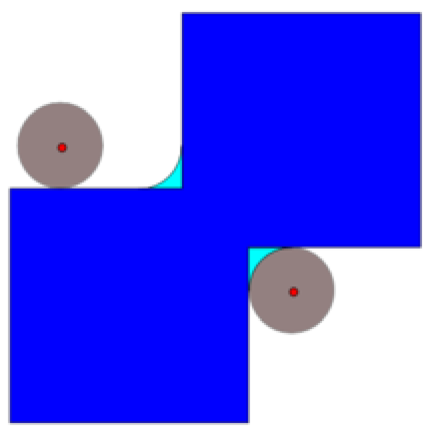
$$A \circ B = (A \ominus B) \oplus B$$



- Closing

$$A \bullet B = (A \oplus B) \ominus B$$

Original: dark
Result: light + dark



Binary Morphology

dilation

binary image

erosion

opening

closing

Applications of Binary Morphology

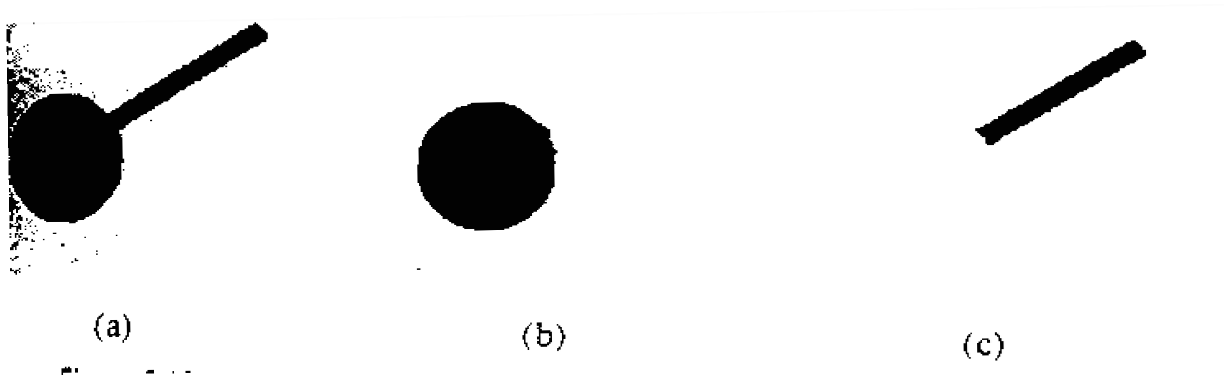


Figure 5.16 Extraction of the body and handle of a shape F by opening with L for the body and taking the residue of the opening for the handle; (a) F, (b) $F \circ L$, and (c) $F - F \circ L$.

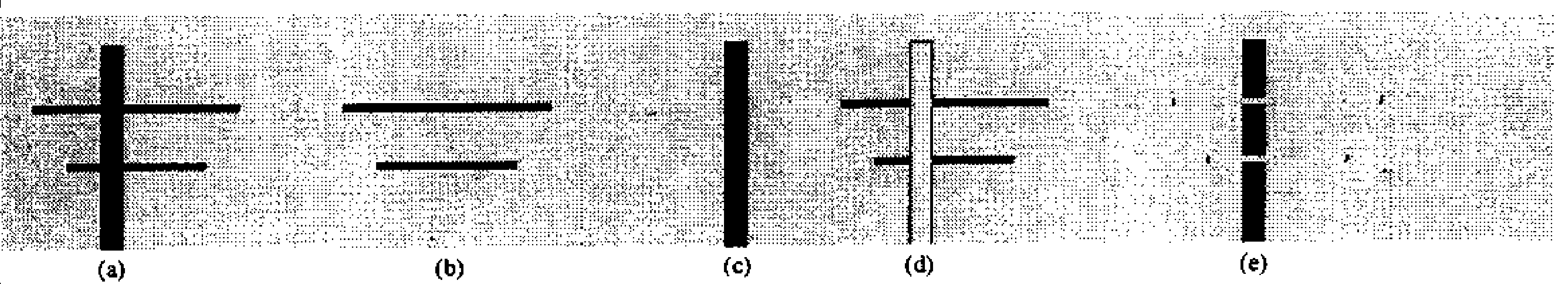
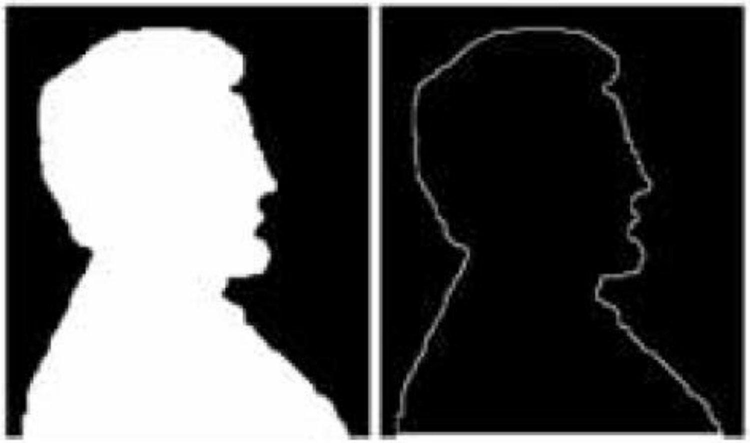


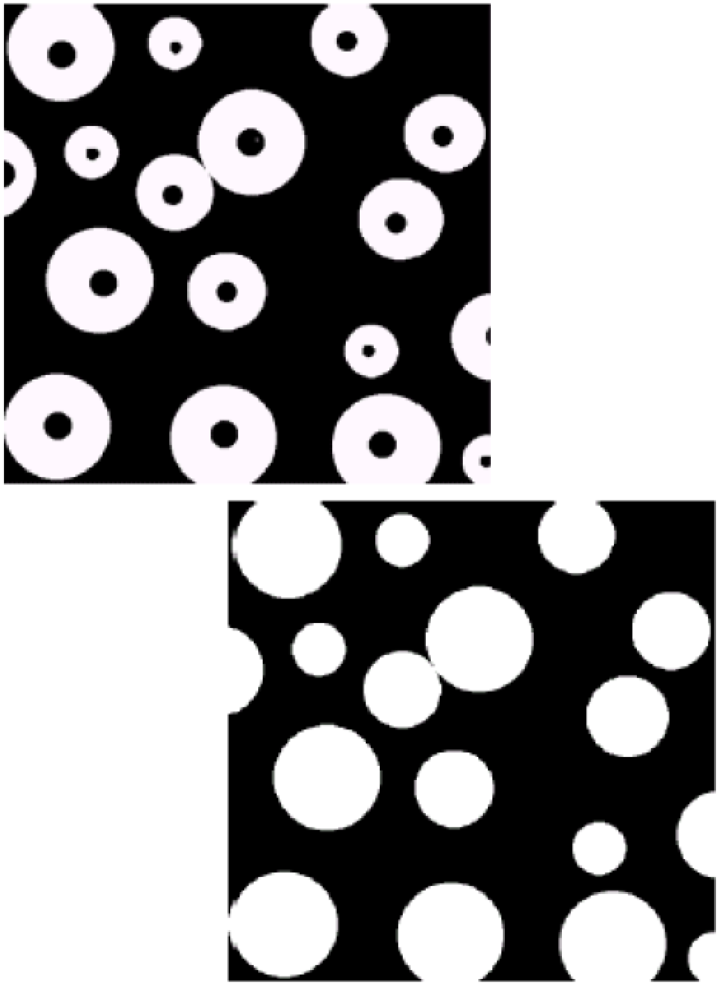
Figure 5.17 Extraction of the trunk and arms of a shape F by opening with vertically and horizontally oriented structuring elements.

Applications of Binary Morphology

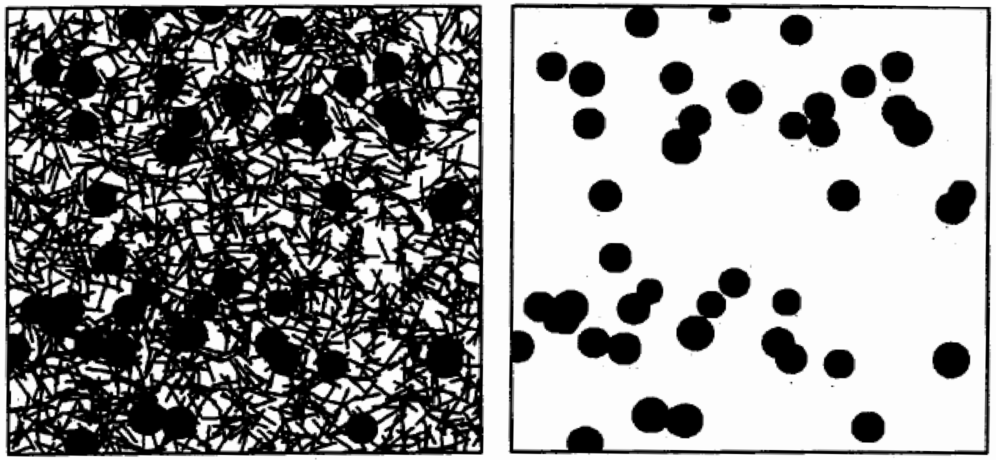
- Boundary extraction



- Region filling

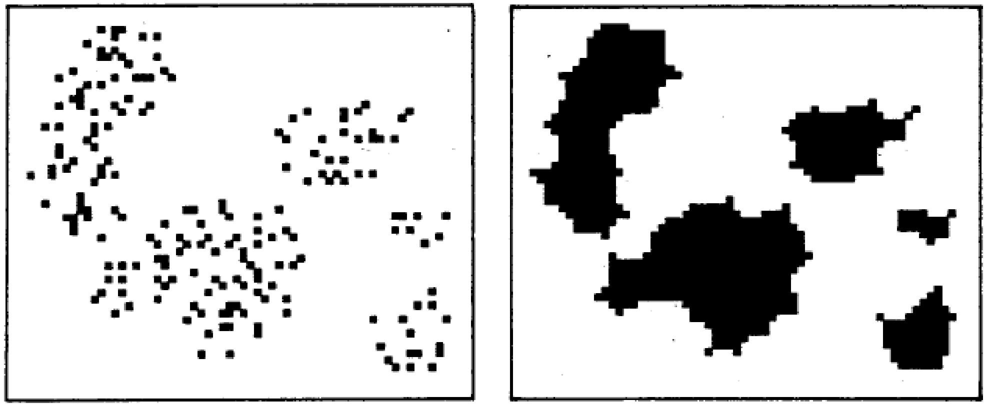


Applications of Binary Morphology



(a) (b)

Figure 5.19 (a) A binary image. (b) Opening of (a) with a disk structuring element.

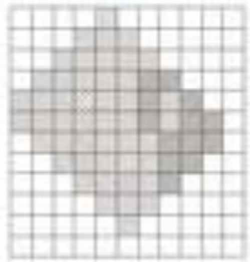


(a) (b)

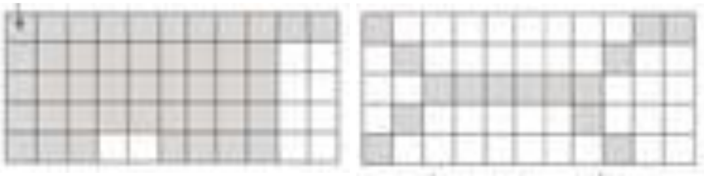
Figure 5.23 (a) A binary image with five clusters of points. Points within each cluster satisfy the partition property with distance ρ_0, and the clusters are farther from each other than $2\rho_0$ pixels. (b) The image of (a) closed by a disk with a radius just greater than $2\rho_0$.

Applications of Binary Morphology

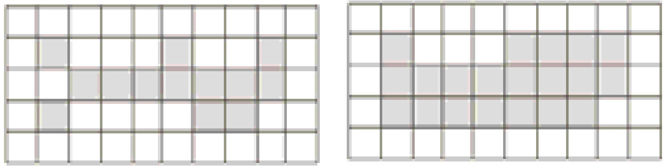
- Extract connected component
- Convex Hull



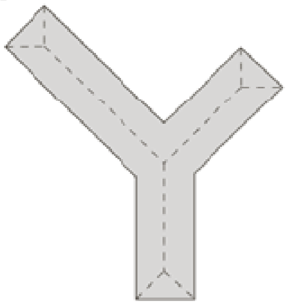
- Thinning



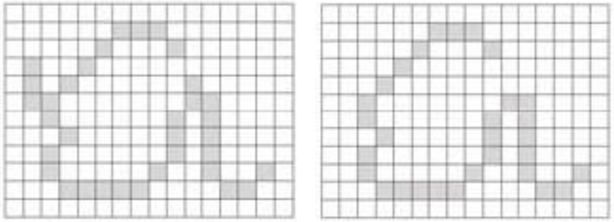
- Thickening



- Skeleton

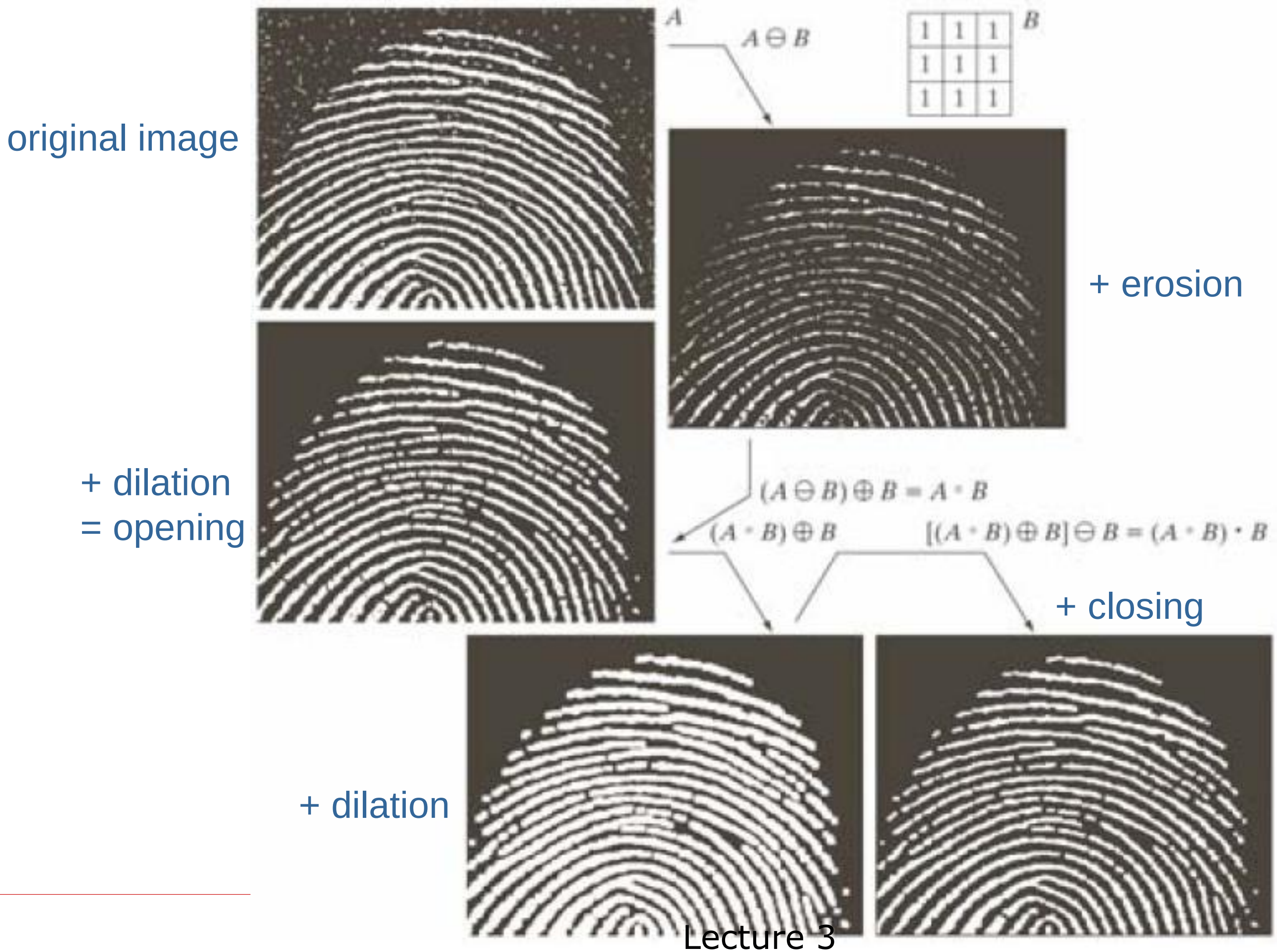


- Pruning



Binary Morphology

- Noise removal

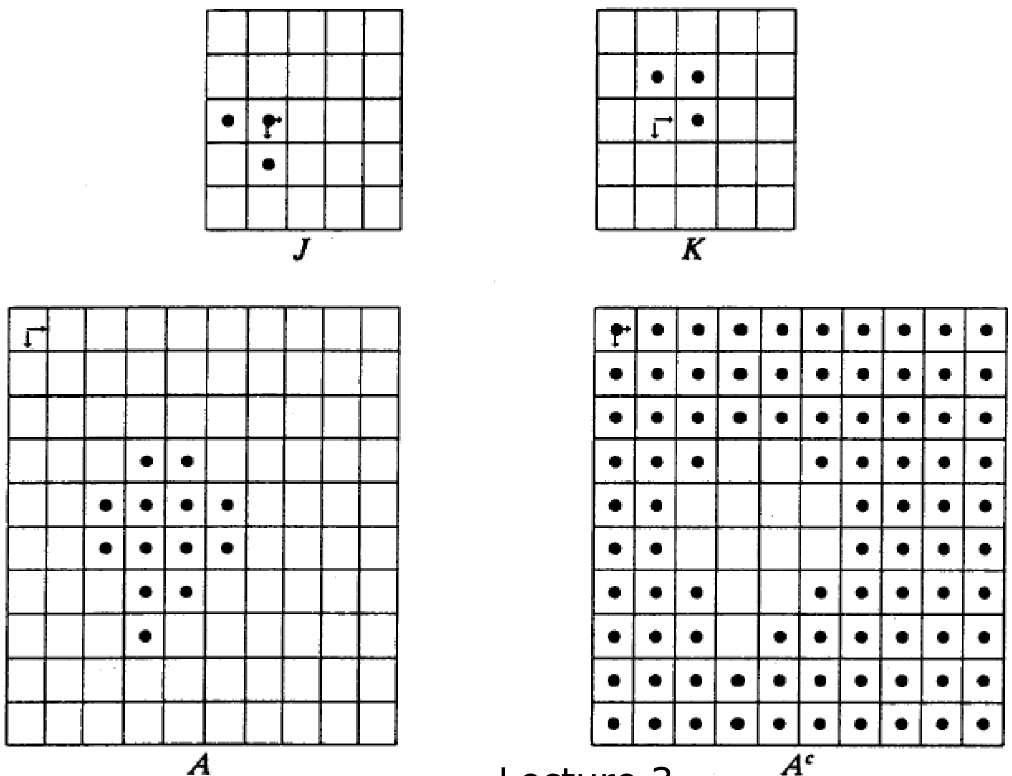


Hit-and-Miss Transform

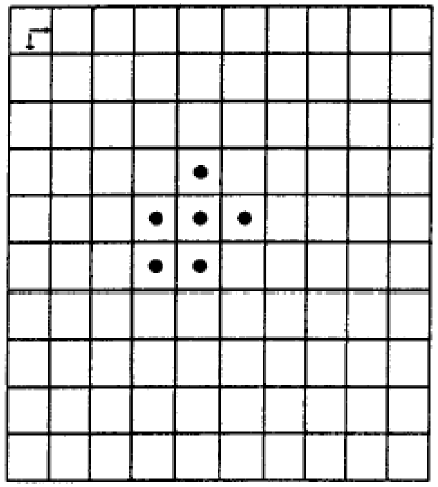
- Selects corners, isolated points, border points
- Intersection of erosions

$$A \otimes (J, K) = (A \ominus J) \cap (A^c \ominus K)$$

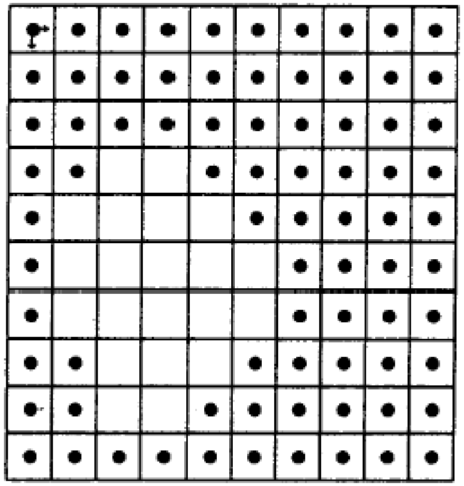
- Example: find upper right-hand corner



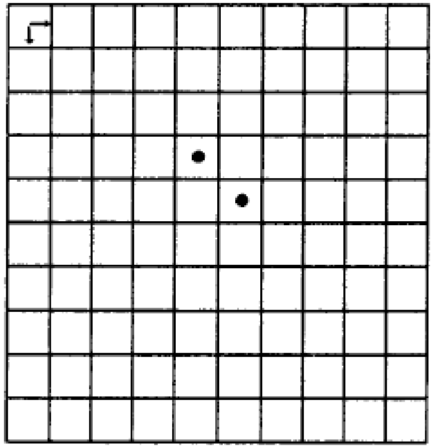
Hit-and-Miss Transform



$A \ominus J$



$A^c \ominus K$



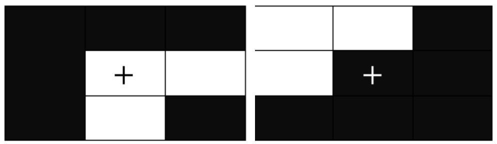
$$A \otimes (J, K) = (A \ominus J) \cap (A^c \ominus K)$$

Figure 5.11 Location of upper right-hand corner points by the hit-and-miss transform.

Binary Morphology

- Hit-and-miss (Hit-or-miss)
  using kernel J 和原始影像做erosion；
  using kernel K 和（255－原始影像）做erosion；
  然後再將兩者的結果取交集

J K

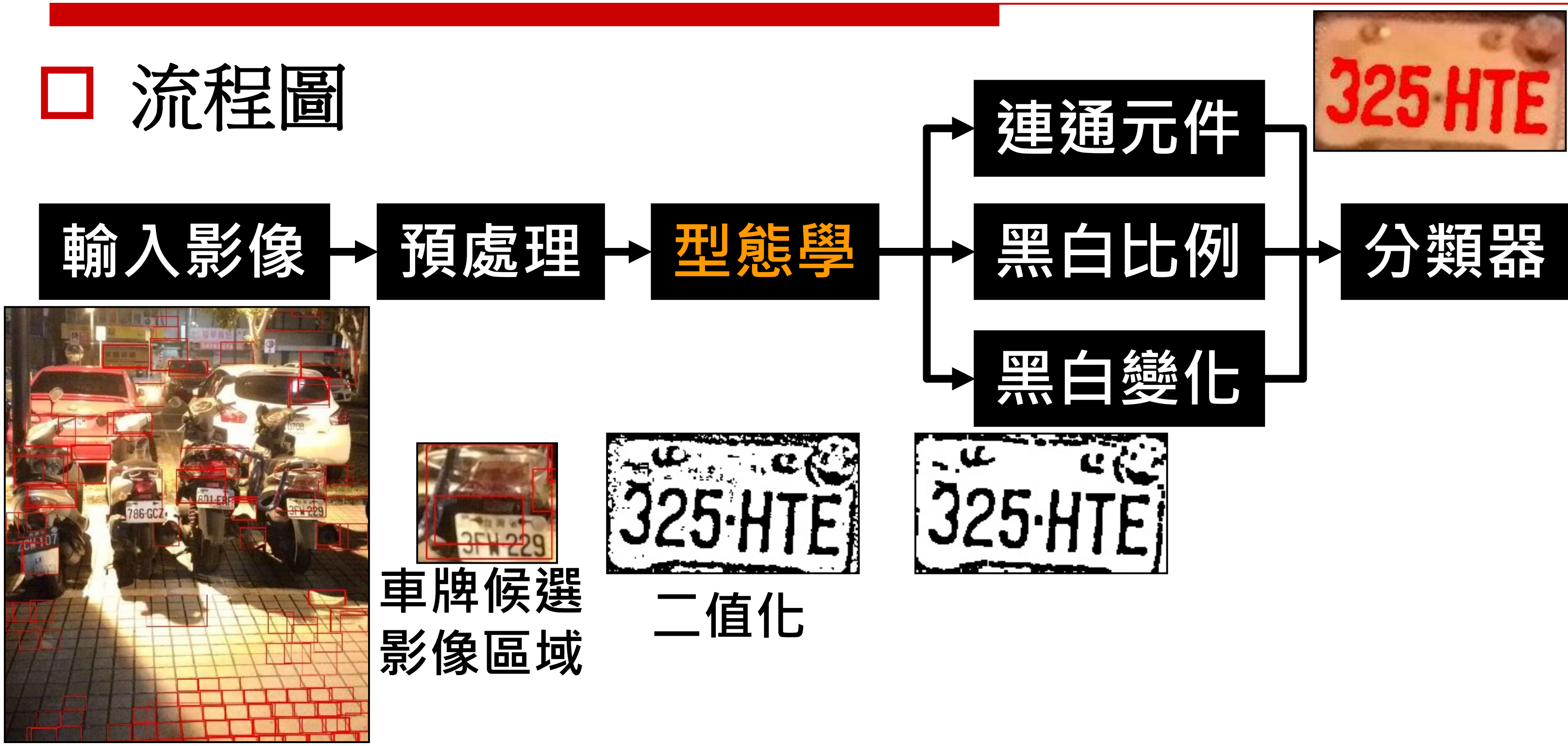


J K

original image

車牌影像偵測

□ 流程圖



- 去除二值化影像中車牌的雜訊，提升偵測率
- 斷開處理 (侵蝕後 再膨脹)
- 原圖大小：**1280*800**
- 車牌候選影像區域, 最小：**200*100**、最大：**300*200**
- 結構元素：正方形 **3*3**