

## Lecture 6

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# Image Stitching

- Obtain a mosaic image with wider angle view by combining multiple images.

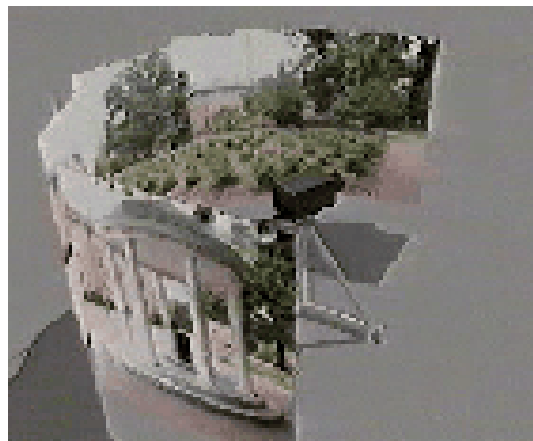
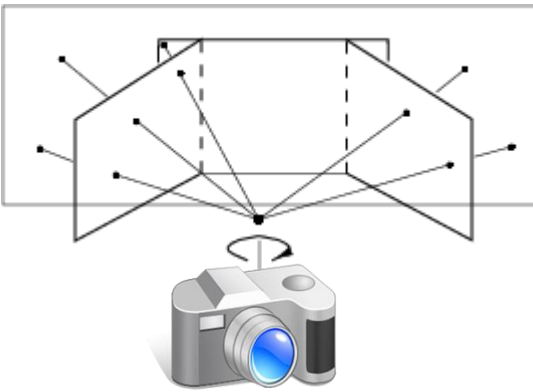


image from S. Seitz

Panoramas Mosaic : 360° field of view

# Image Stitching

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□ Stitching = alignment + blending

geometrical  
registration

photometric  
registration

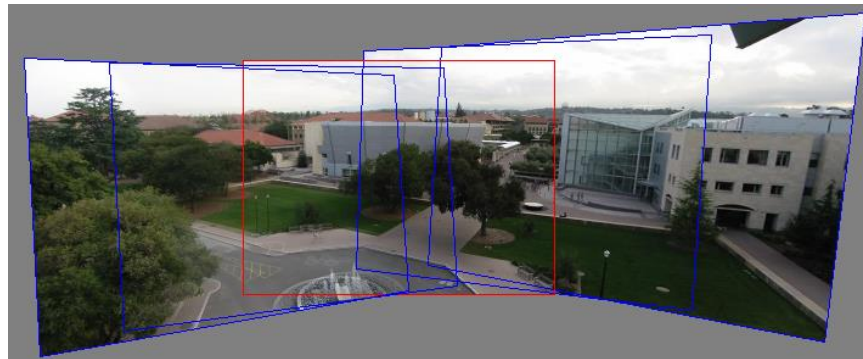
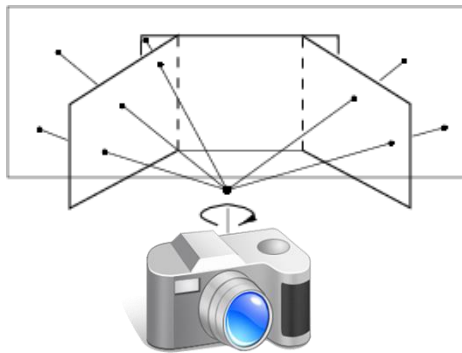


# Stitch Together (a Panorama)

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## □ Basic Procedure

- Take a sequence of images from the same position
  - Rotate the camera about its optical center
- Compute transformation between second image and first (with cylindrical projection)
- Transform the second image to overlap with the first
- Blend the two together to create a mosaic
- (If there are more images, repeat)

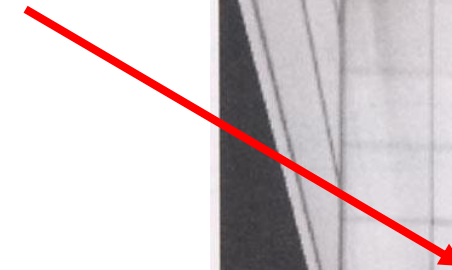
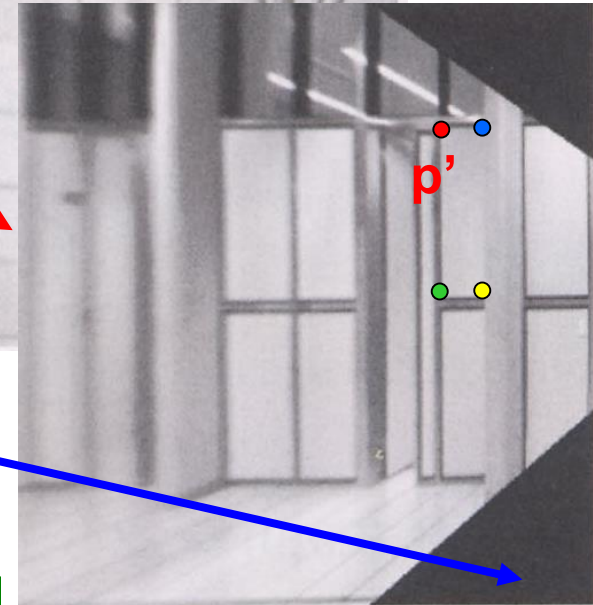
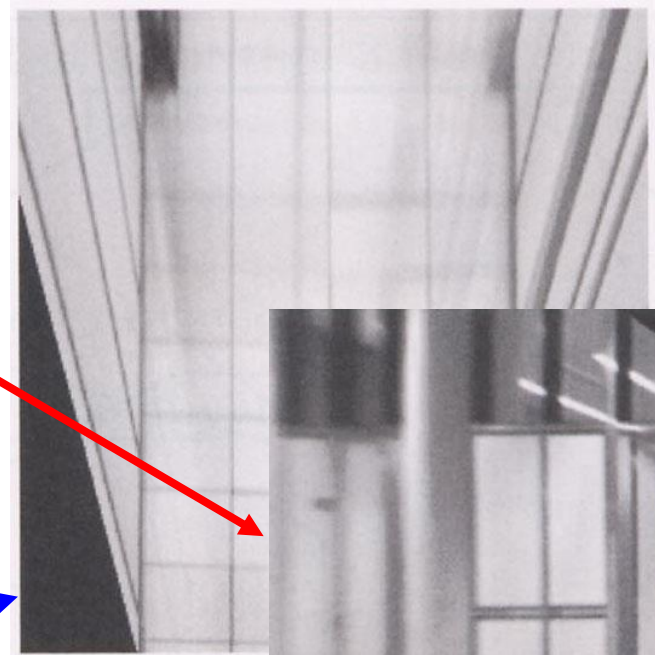
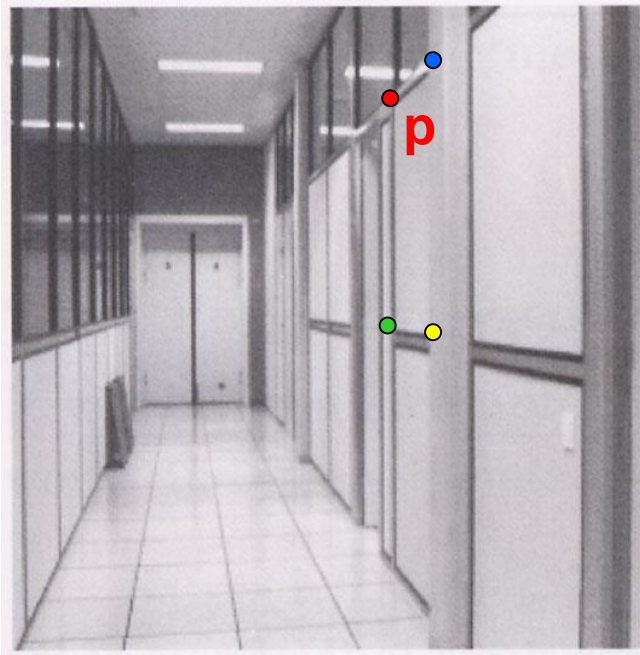


cylindrical projection





# Image Warping with Homographies



black area  
where no pixel  
maps to

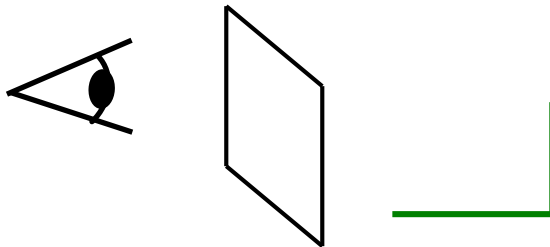
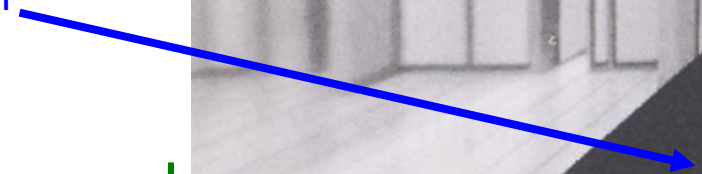


image plane in front

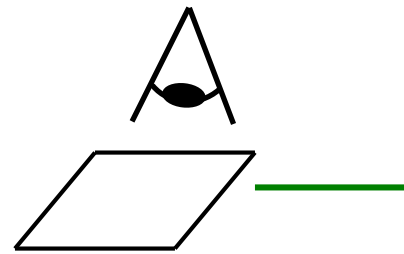
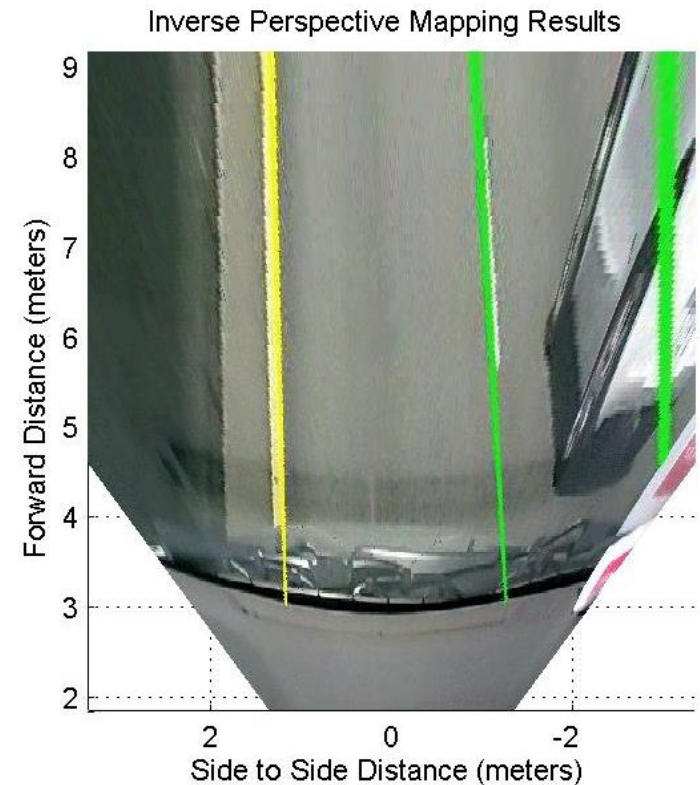
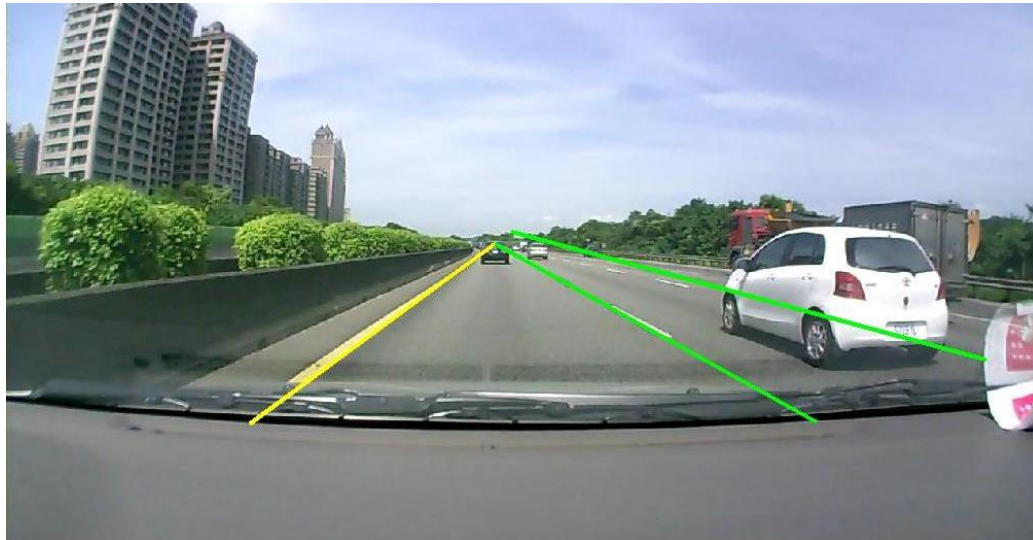


image plane below

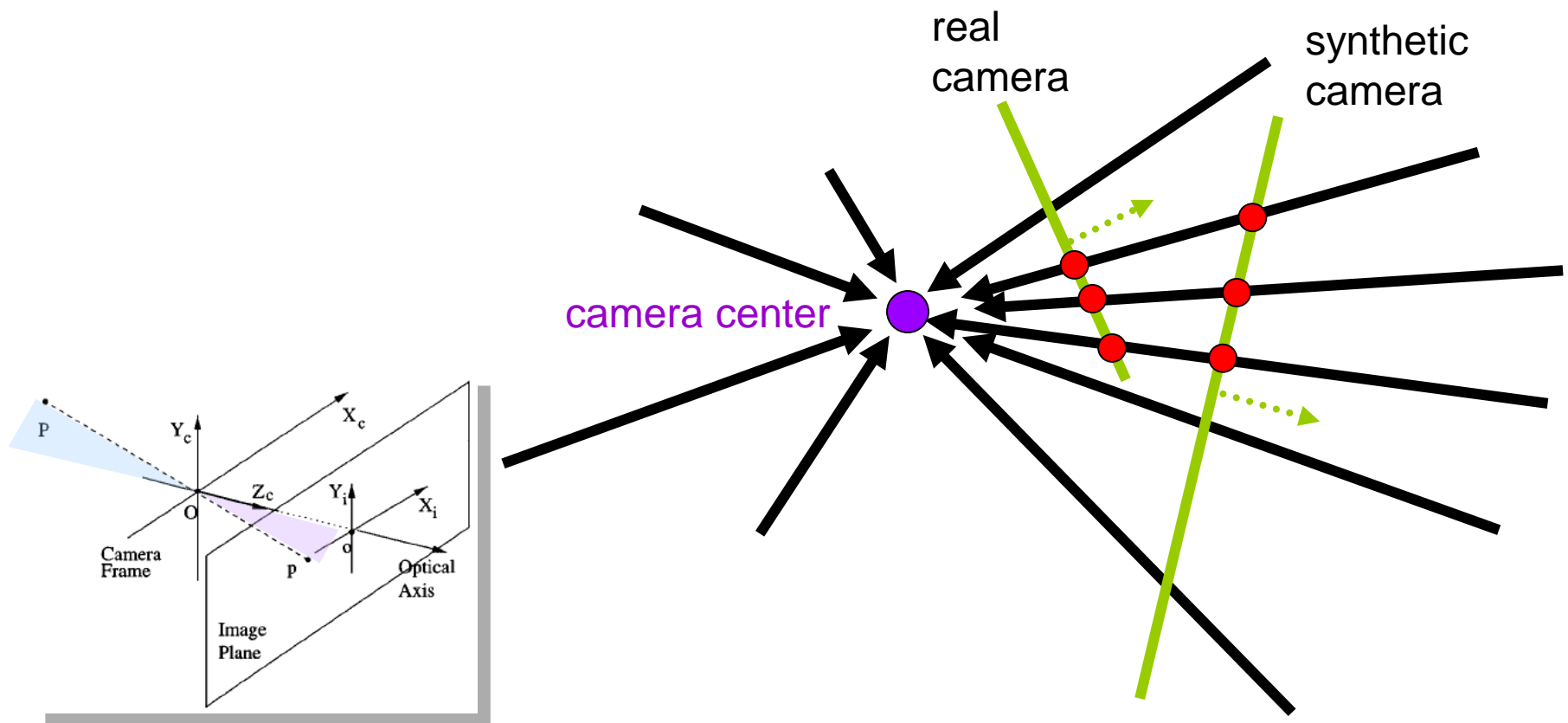
# Image Warping with Homographies

- Inverse perspective mapping (IPM)
  - Lane/vehicle detection and tracking



# Stitch Together a Panorama

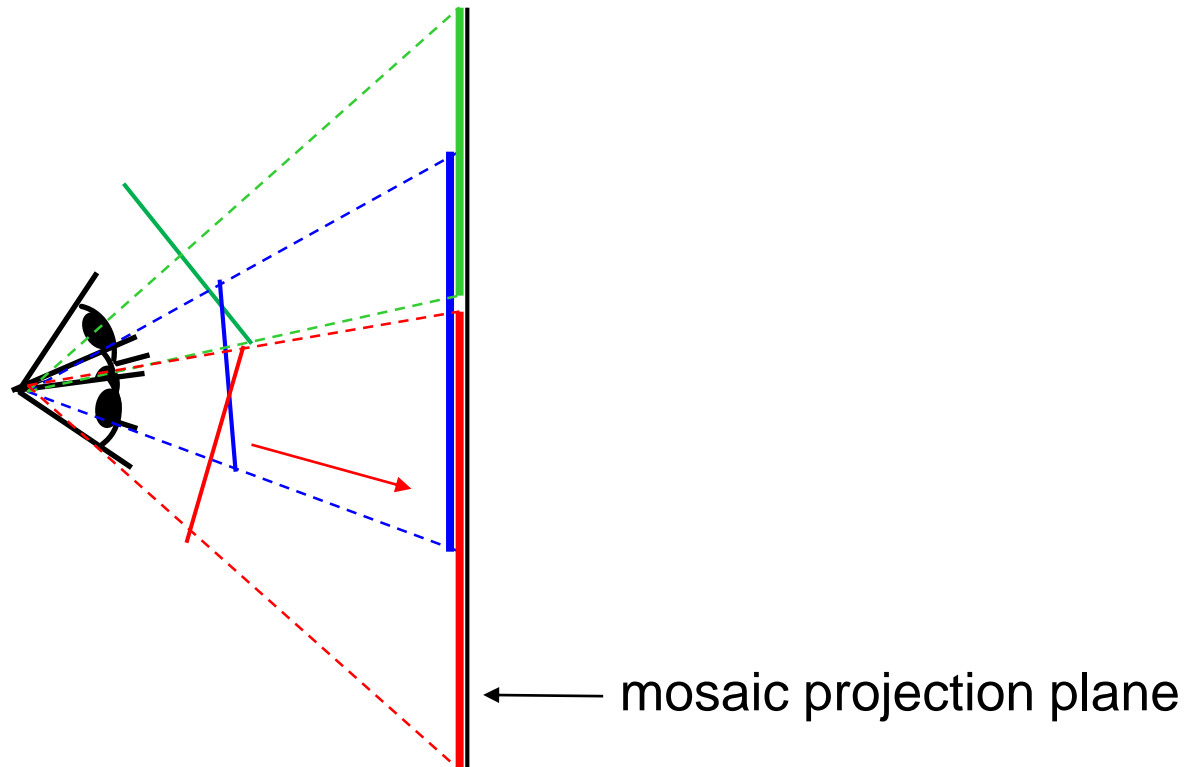
- A pencil of rays contains all views
  - Can generate any synthetic camera view as long as it has the *same center of projection*



# Stitch Together a Panorama

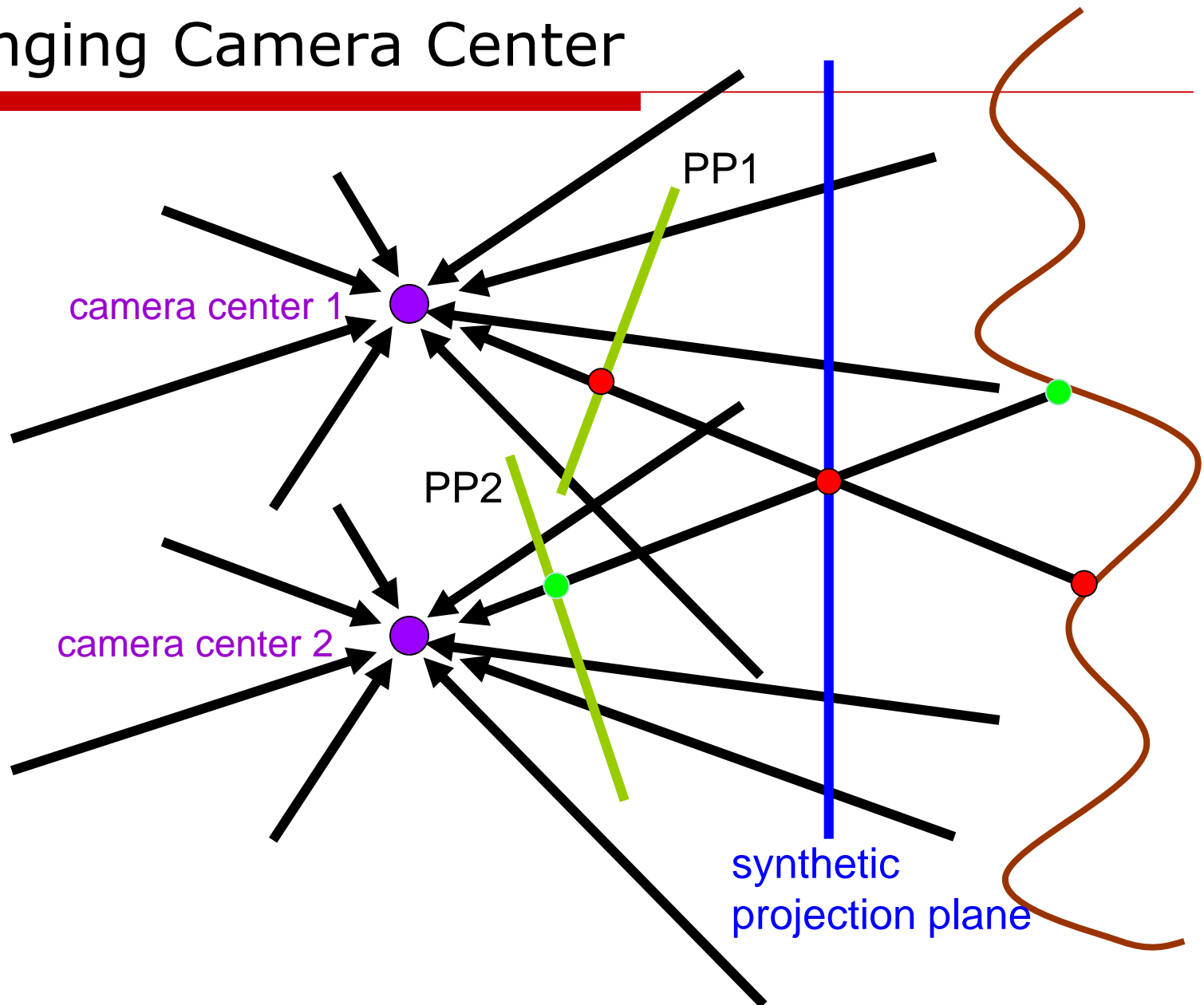
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- Mosaic as an image reprojection
  - The images are reprojected onto a common plane
  - The mosaic is formed on this plane
  - Mosaic is a synthetic wide-angle camera





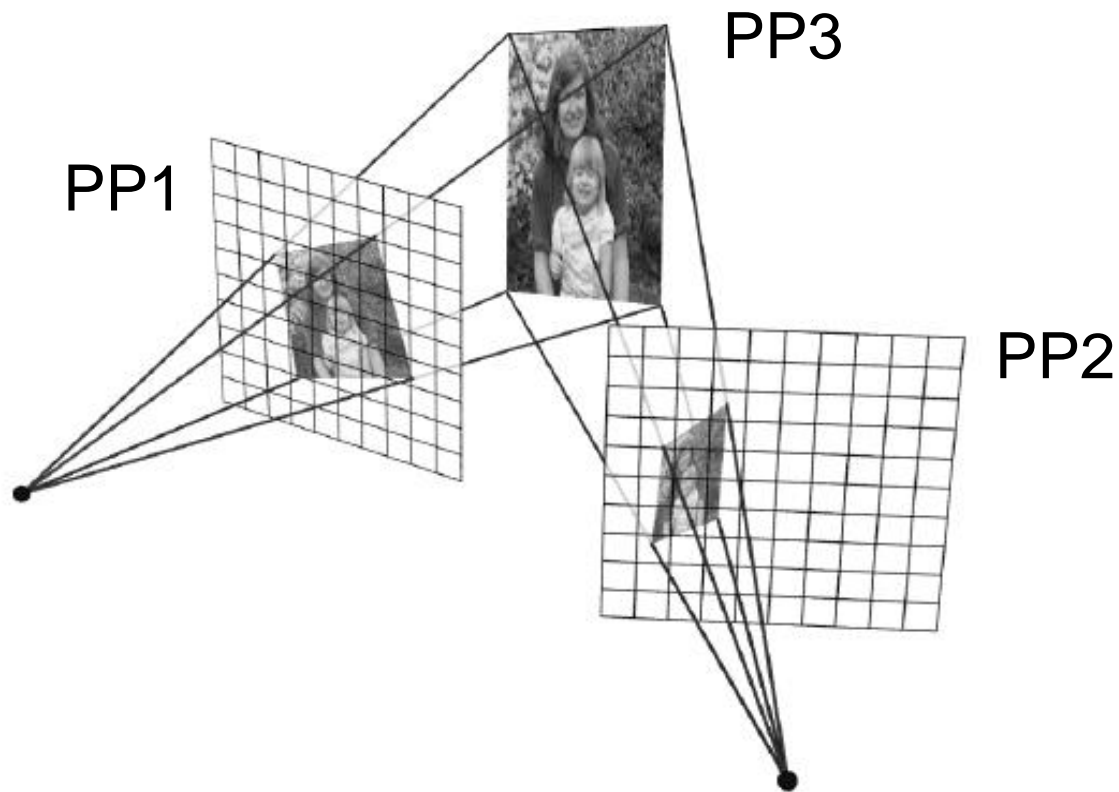
# Changing Camera Center



# Planar Scene (or Far Away)

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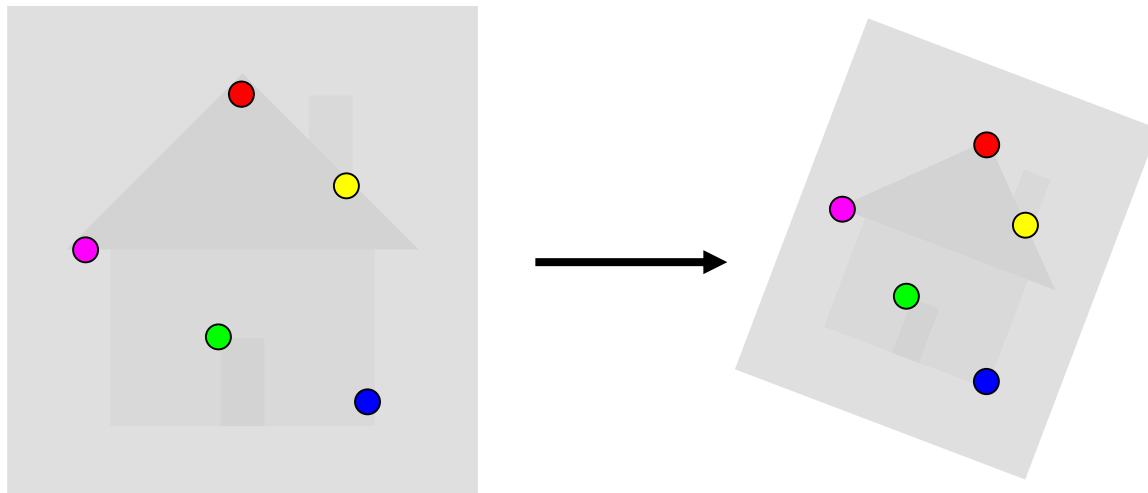
- ❑ PP3 is a projection plane of both centers of projection, so we are OK!
- ❑ This is how big aerial photographs are made



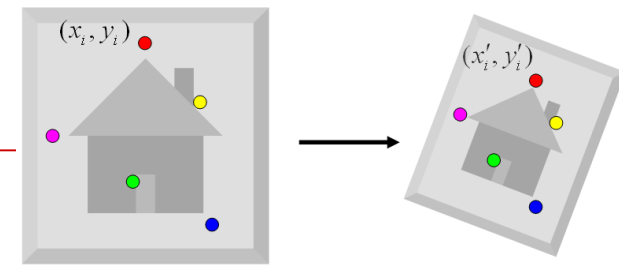
# Image Alignment

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- ❑ Two broad approaches:
  - Direct (pixel-based) alignment
    - ❑ Search for alignment where most pixels agree
  - Feature-based alignment
    - ❑ Search for alignment where *extracted features* agree
    - ❑ Can be verified using pixel-based alignment



# Image Alignment



## □ Fitting an affine transformation

- Assuming we know the correspondences, how do we get the transformation?

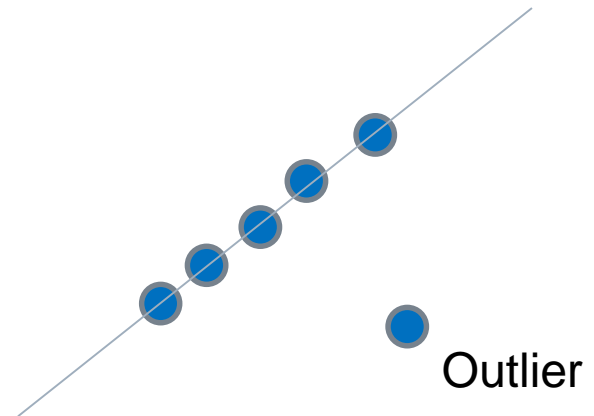
$$\begin{bmatrix} x'_i \\ y'_i \end{bmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} \Rightarrow \begin{bmatrix} \dots & \dots & \dots & \dots & \dots & \dots \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \dots \\ x'_i \\ y'_i \\ \dots \end{bmatrix} \quad \text{Ax=b}$$

- How many matches (correspondence pairs) do we need to solve for the transformation parameters?
  - (Translation+Rotation) 6 parameters  $\Rightarrow$  3 points
- Once we have solved for the parameters, we can compute the coordinates of the corresponding point for  $(x_{new}, y_{new})$

# Outliers

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- ❑ Outliers can hurt the quality of the parameter estimates, e.g.,
  - an erroneous pair of matching points from two images  $\Rightarrow$  wrong transformation matrix
  - an edge point that is noise, or doesn't belong to the line we are fitting.

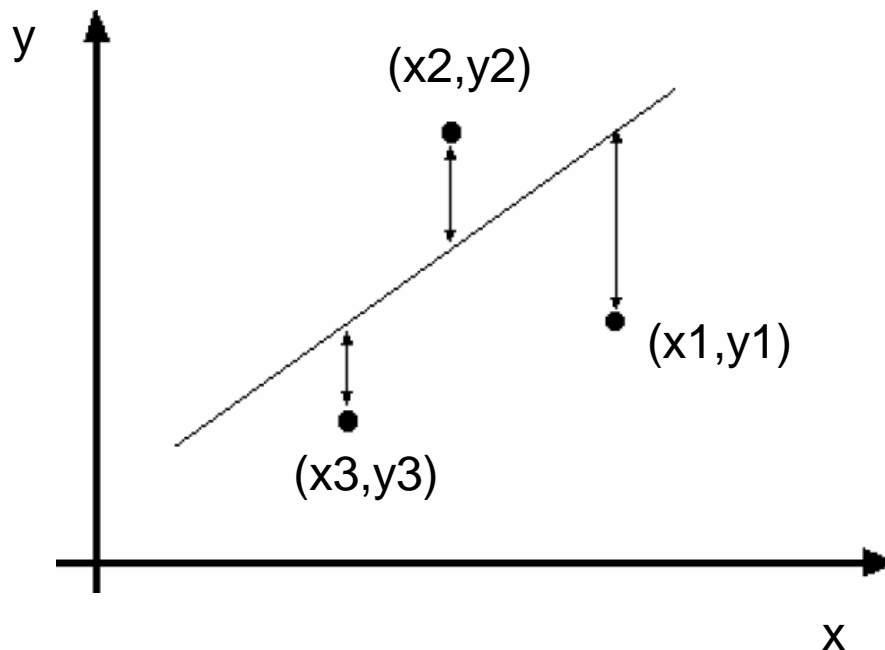




# Example: least squares line fitting

- Assuming all the points that belong to a particular line are known

$$f(a, b) = a + b x,$$



$$R^2(a, b) \equiv \sum_{i=1}^n [y_i - (a + b x_i)]^2$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ x_3 & 1 \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix}$$

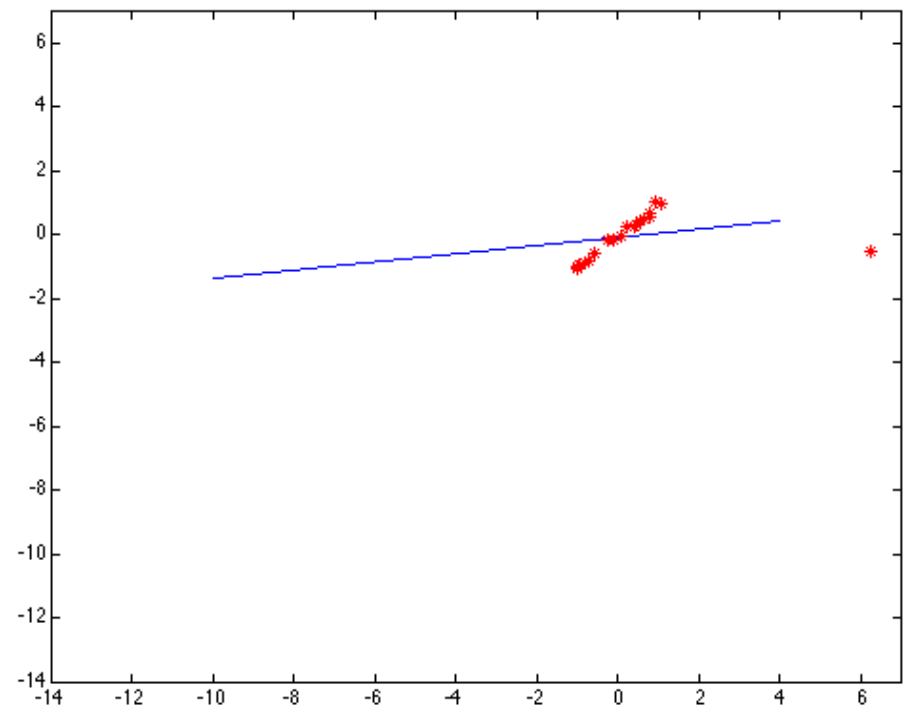
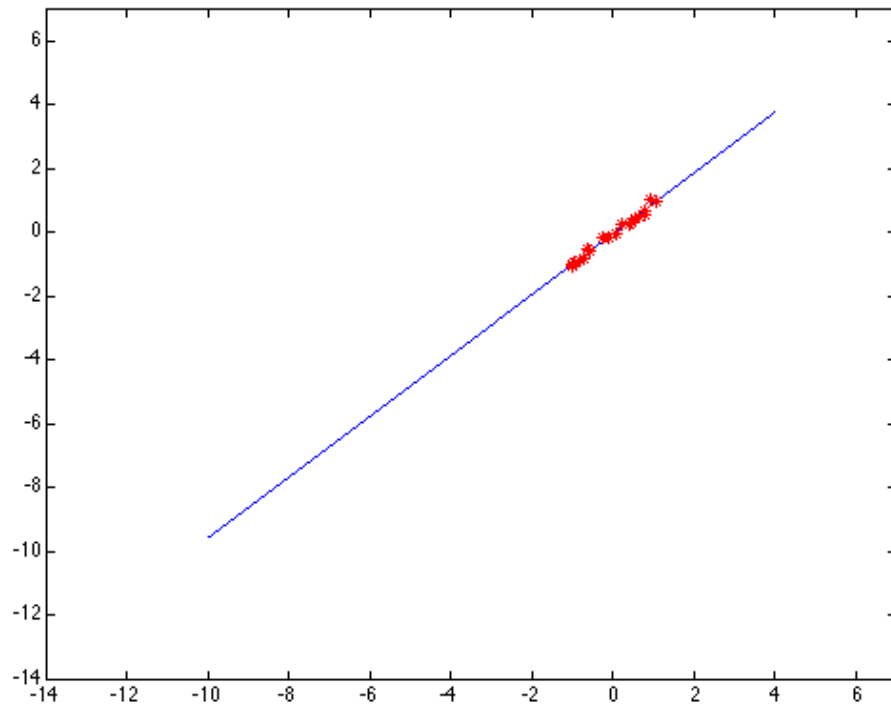
$$b = A \mathbf{x}$$

# Outliers affect least squares fit

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## □ 2D space

■ Given point data  $\Rightarrow$  obtain line parameters



# RANSAC

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## □ RANSAC : Random Sample Consensus

- an algorithm for robust fitting of models in the presence of many data outliers
- Given  $N$  data points  $x_i$ , assume that majority of them are generated from a model with parameters  $\Theta$ , try to recover  $\Theta$ .

## □ Algorithm

Run  $k$  times, in each time:

- (1) draw  $n$  samples randomly
- (2) fit parameters  $\Theta$  with these  $n$  samples
- (3) for each of other  $(N - n)$  points,  
calculate its distance to the fitted model,  
count the number of inlier points,  $c$

Output  $\Theta$  with the largest  $c$

How many times?

How big?  
Smaller is better

How to define?  
Depends on the problem.

# RANSAC Example: Line Fitting

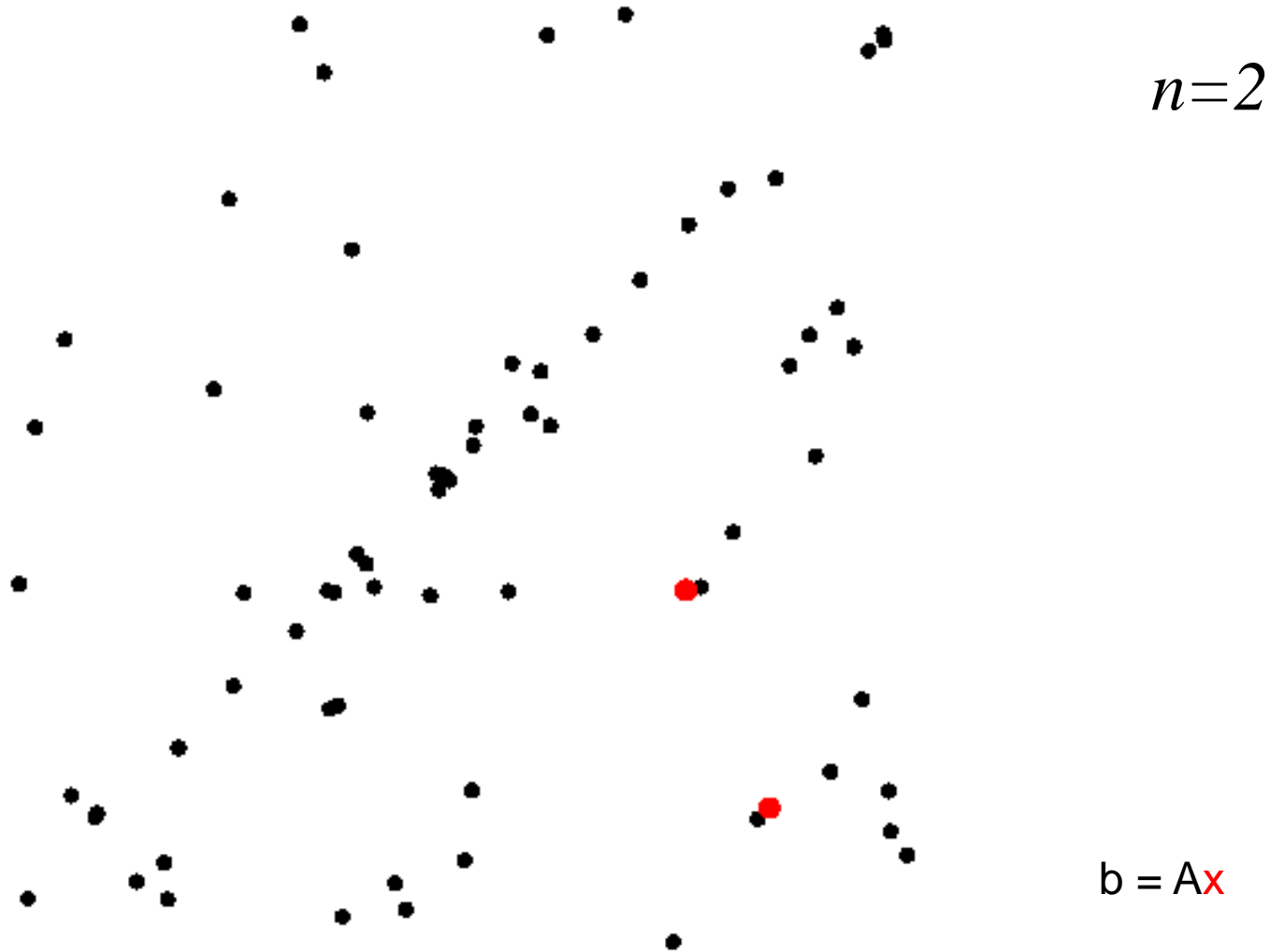
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$$b = Ax$$

# RANSAC Example: Line Fitting (1)

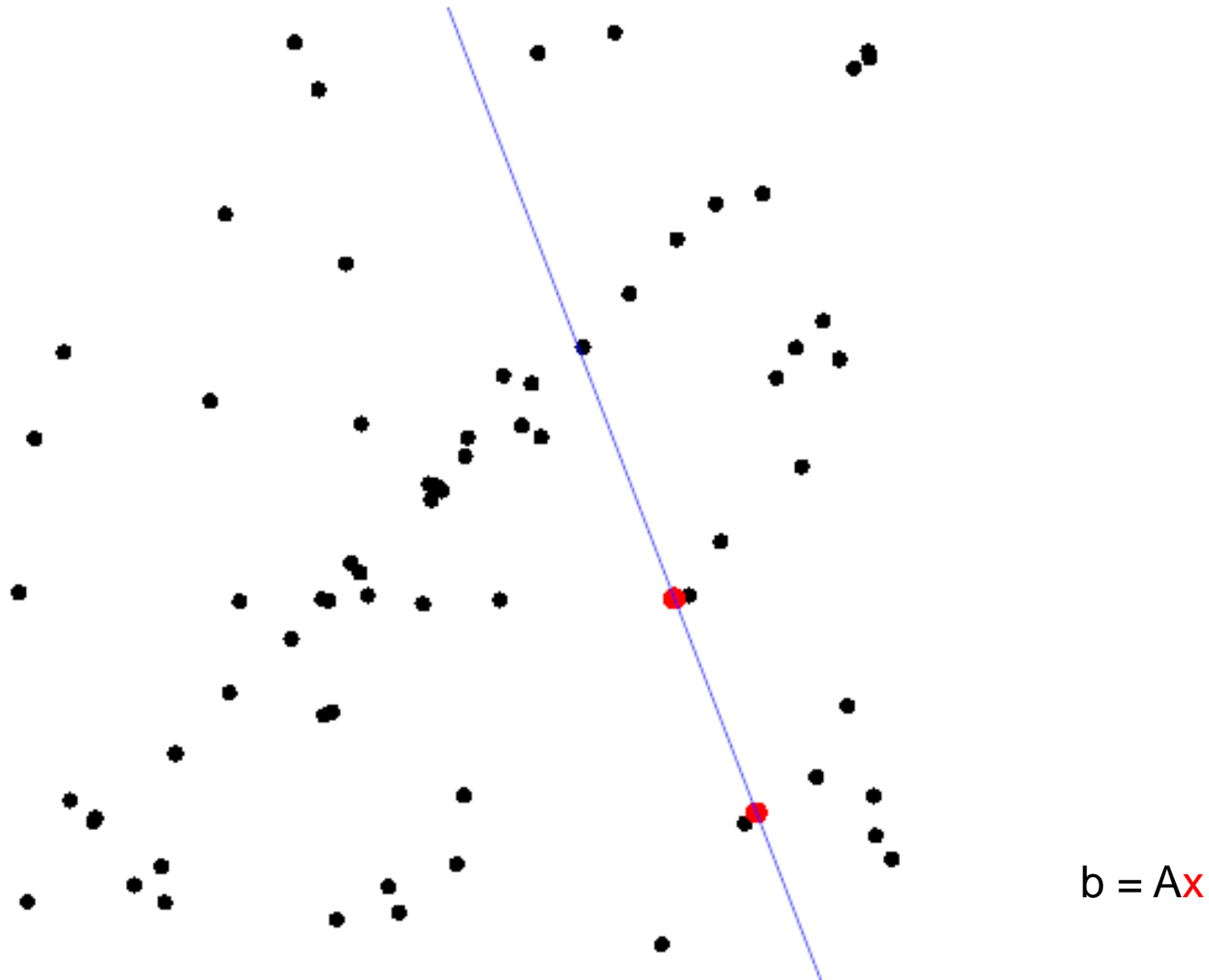
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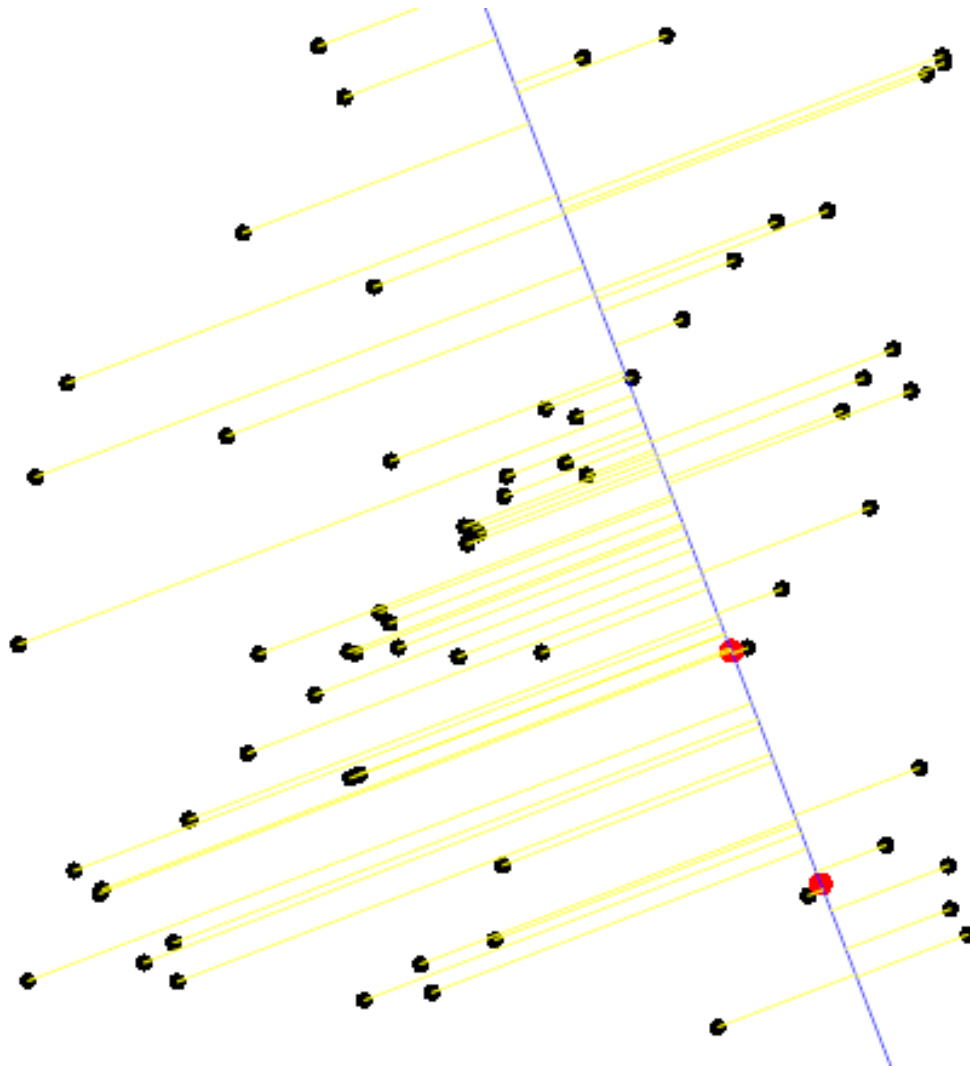
# RANSAC Example: Line Fitting (2)

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# RANSAC Example: Line Fitting (3)

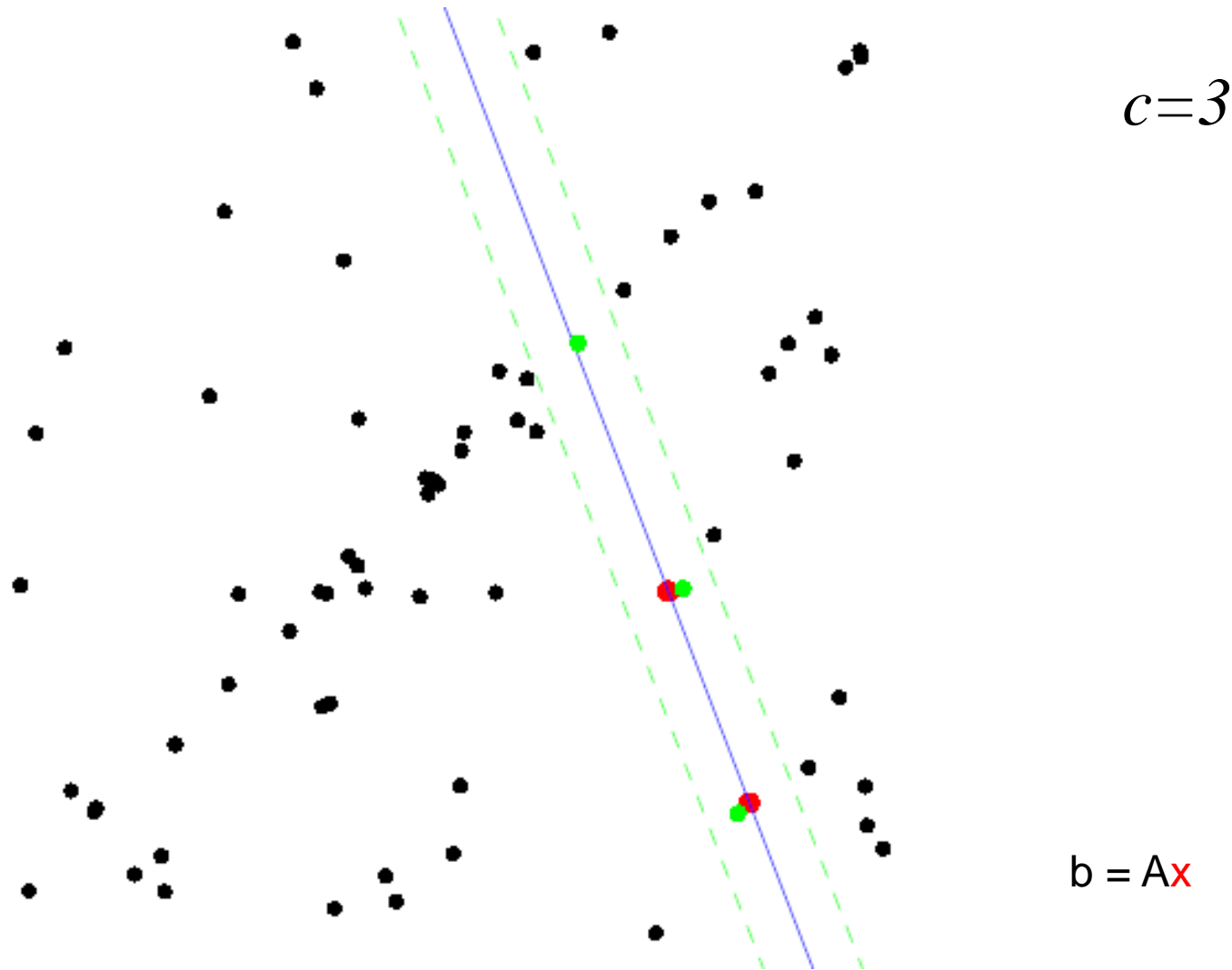
□ Measure distances



$$b = Ax$$

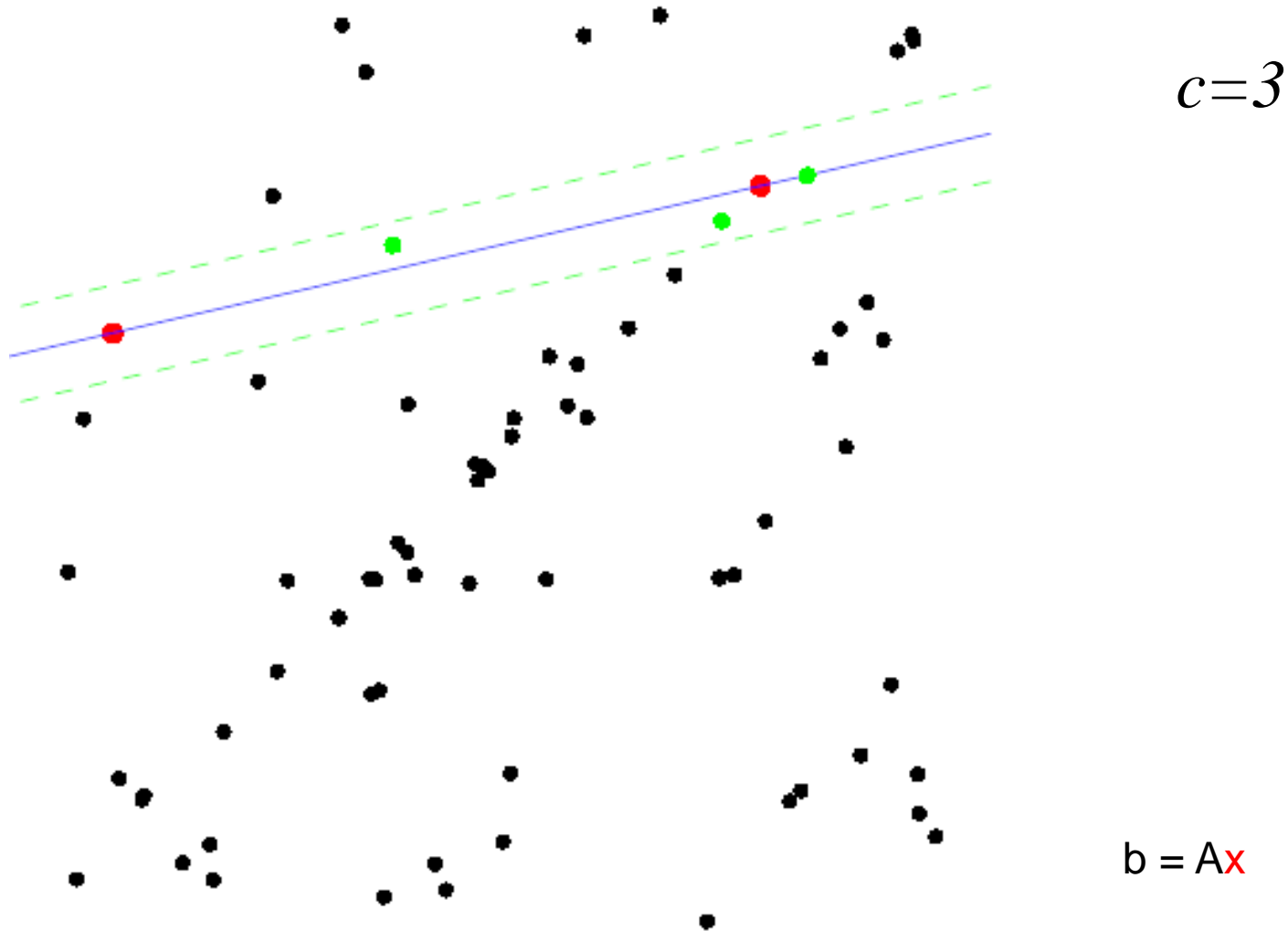
# RANSAC Example: Line Fitting (4)

□ Count inliers



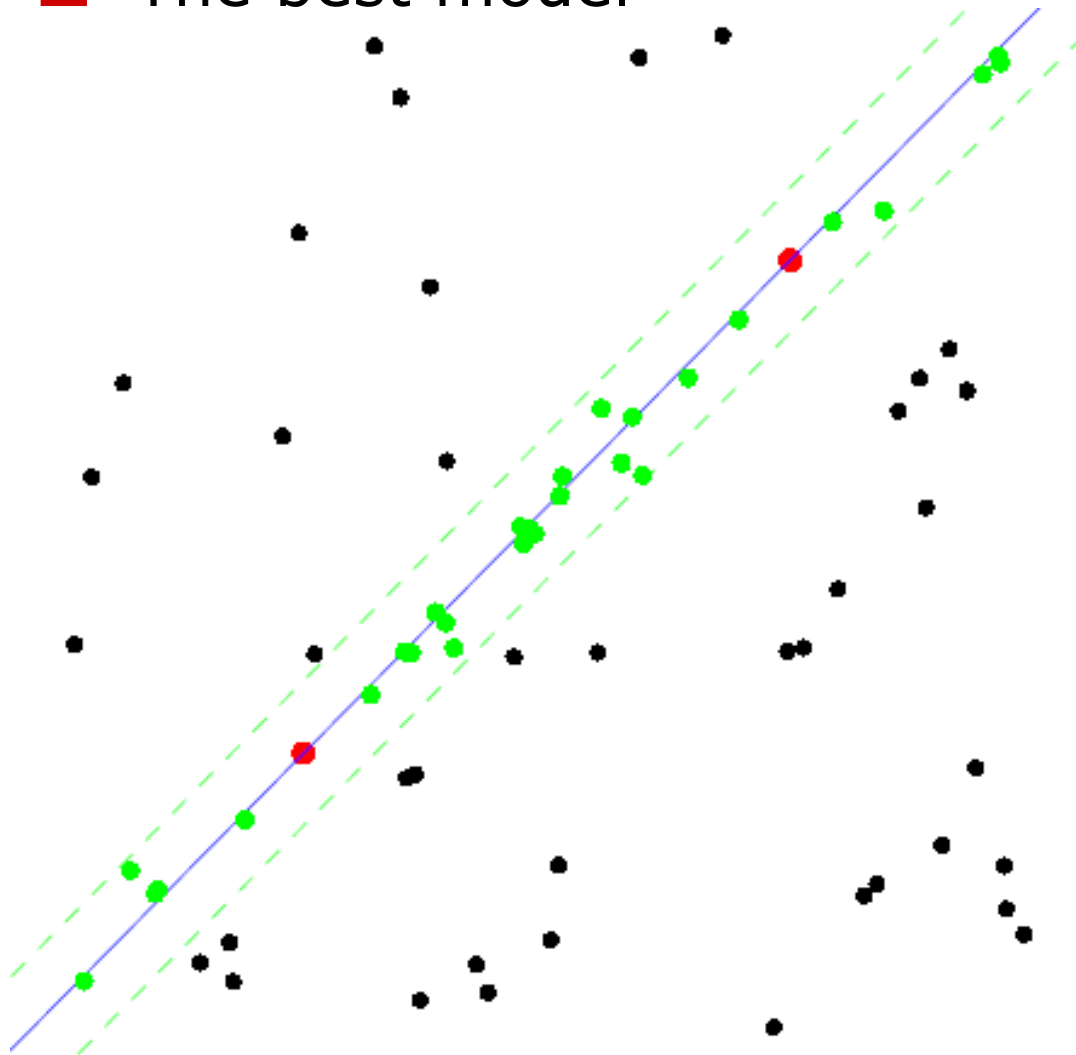
# RANSAC Example: Line Fitting (5)

□ Another trial



# RANSAC Example: Line Fitting (6)

□ The best model





# RANSAC

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## □ How to determine $k$

$p$ : probability of real inliers

$P$ : probability of success after  $k$  trials

$$P = 1 - \underbrace{(1 - p^n)}_{\text{n samples are all inliers}}^k$$

a failure

failure after  $k$  trials

$$k = \frac{\log(1 - P)}{\log(1 - p^n)}$$

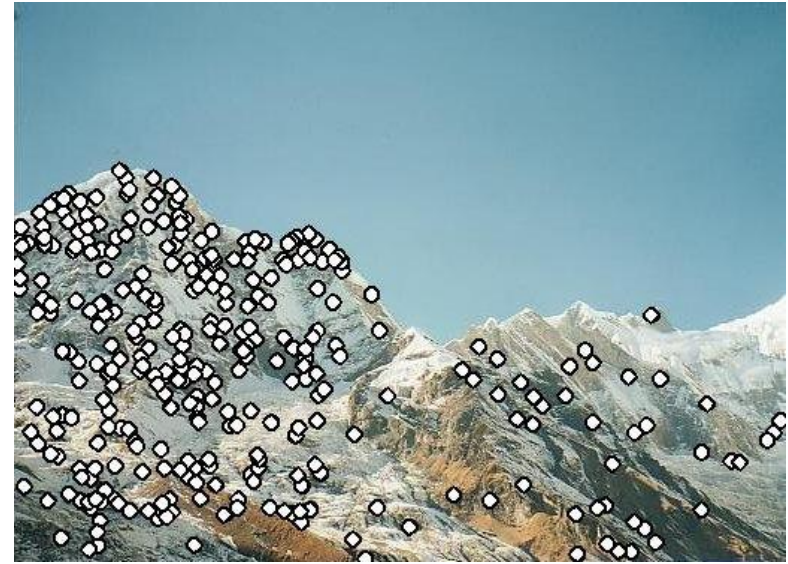
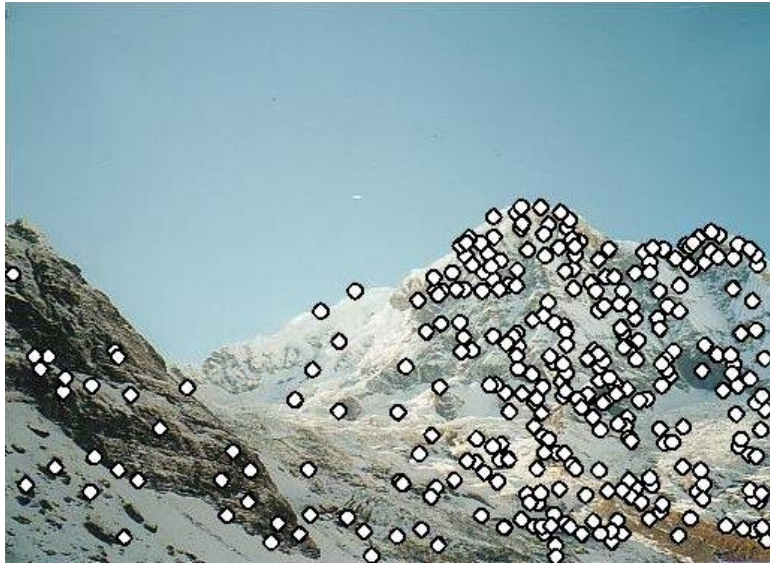
Define goal:  
given  $n, p, P$

for  $P=0.99$

$n$	$p$	$k$
3	0.5	35
6	0.6	97
6	0.5	293

# RANSAC for Homography

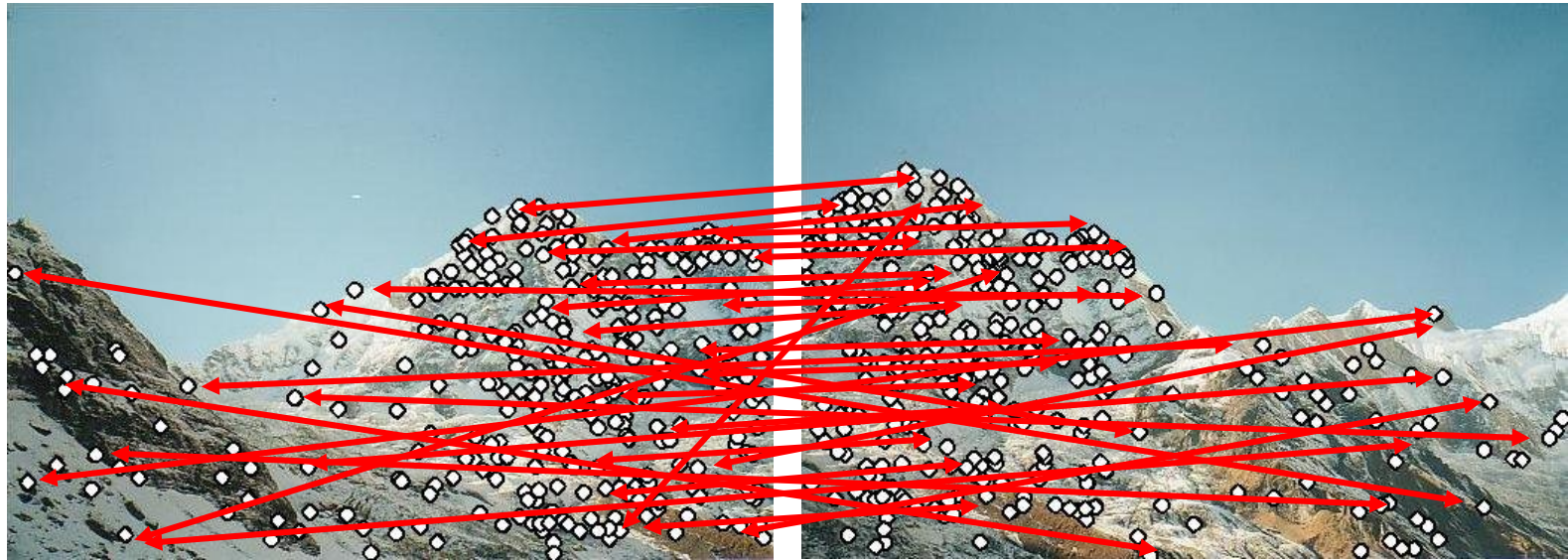
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□ Extract features

# RANSAC for Homography

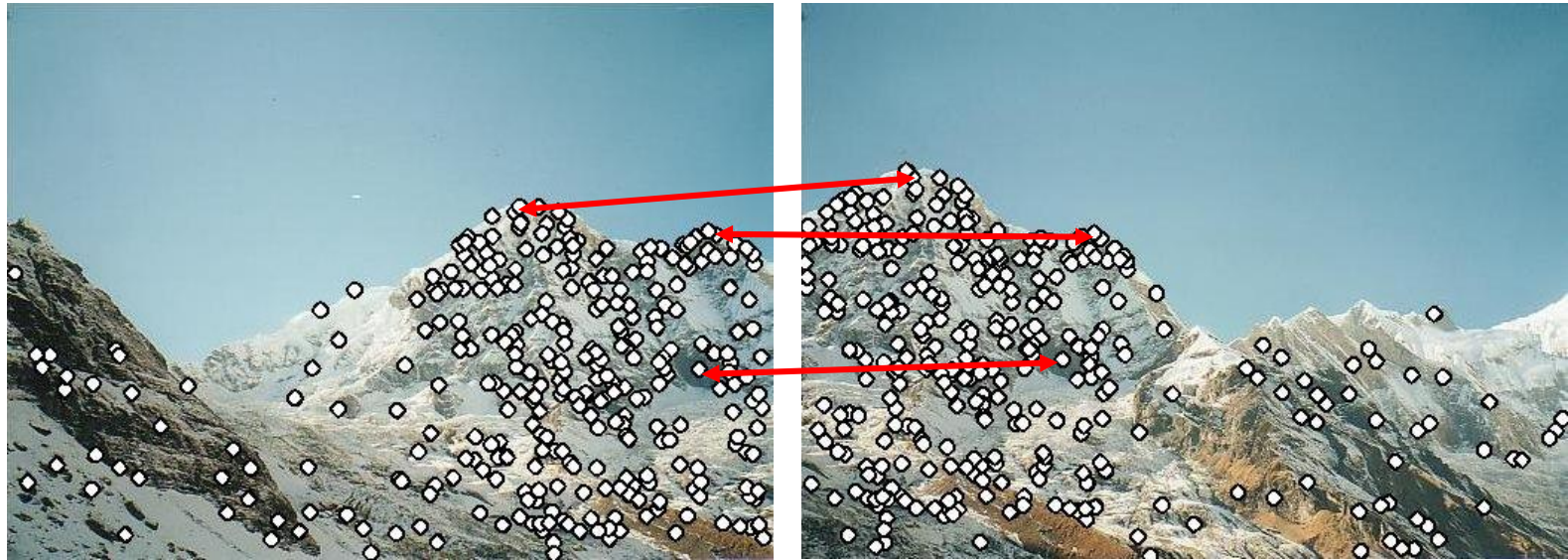
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- ❑ Extract features
- ❑ Compute *putative matches*

# RANSAC for Homography

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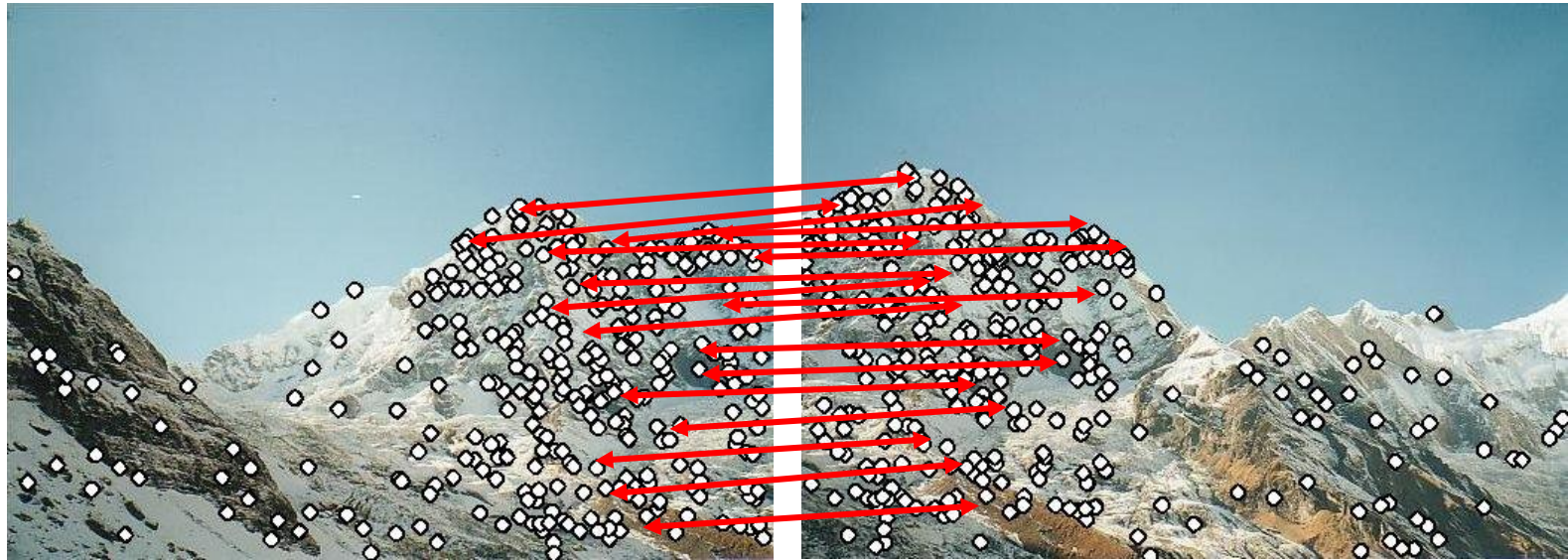


- ❑ Extract features
- ❑ Compute *putative matches*
- ❑ Loop:
  - *Hypothesize* transformation  $T$  (small group of putative matches that are related by  $T$ )



# RANSAC for Homography

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- ❑ Extract features
- ❑ Compute *putative matches*
- ❑ Loop:
  - *Hypothesize* transformation  $T$  (small group of putative matches that are related by  $T$ )
  - *Verify* transformation (search for other matches consistent with  $T$ )



# RANSAC for Homography

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- ❑ Extract features
- ❑ Compute *putative matches*
- ❑ Loop:

$b = Ax$

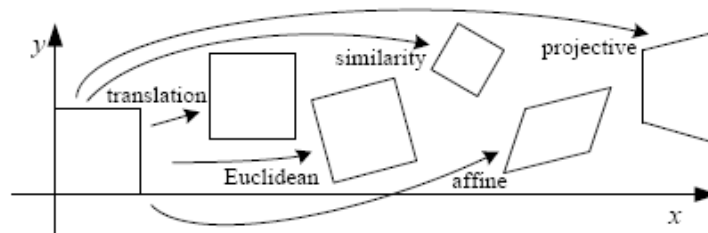
  - *Hypothesize* transformation (small group of putative matches that are related by  $x$ )
  - *Verify* transformation (search for other matches consistent with  $x$ )

# Large-Scale Mosaics

Perspective projection



## □ Motion models

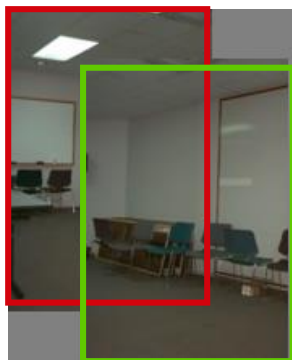


Translation

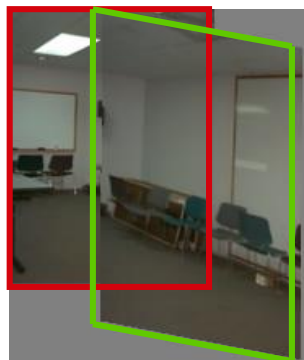
Affine

Perspective

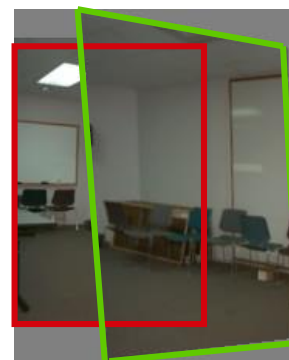
3D rotation



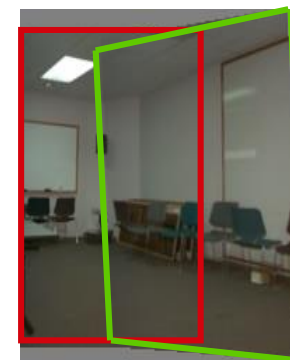
2 unknowns



6 unknowns



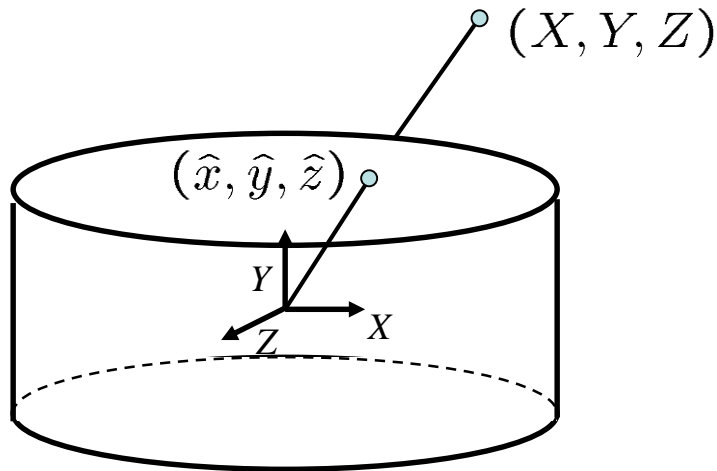
8 unknowns



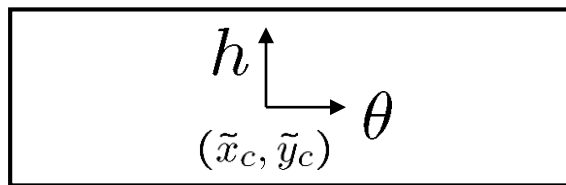
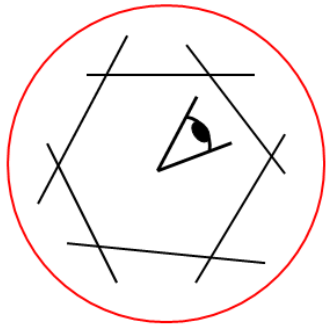
3 unknowns

# Cylindrical Projection for Panorama Stitching

Perspective projection



unit cylinder



unwrapped cylinder

- Map 3D point  $(X, Y, Z)$  onto cylinder

$$(\hat{x}, \hat{y}, \hat{z}) = \frac{1}{\sqrt{X^2 + Z^2}}(X, Y, Z)$$

- Convert to cylindrical coordinates

$$(\sin\theta, h, \cos\theta) = (\hat{x}, \hat{y}, \hat{z})$$

- Convert to cylindrical image coordinates

$$(\tilde{x}, \tilde{y}) = (f\theta, fh) + (\tilde{x}_c, \tilde{y}_c)$$



cylindrical image

# Cylindrical Reprojection

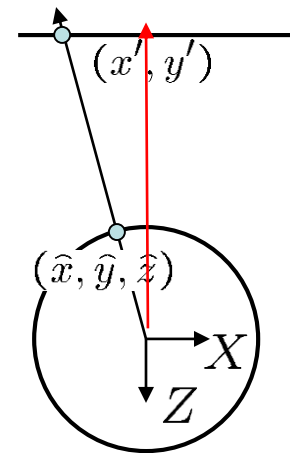
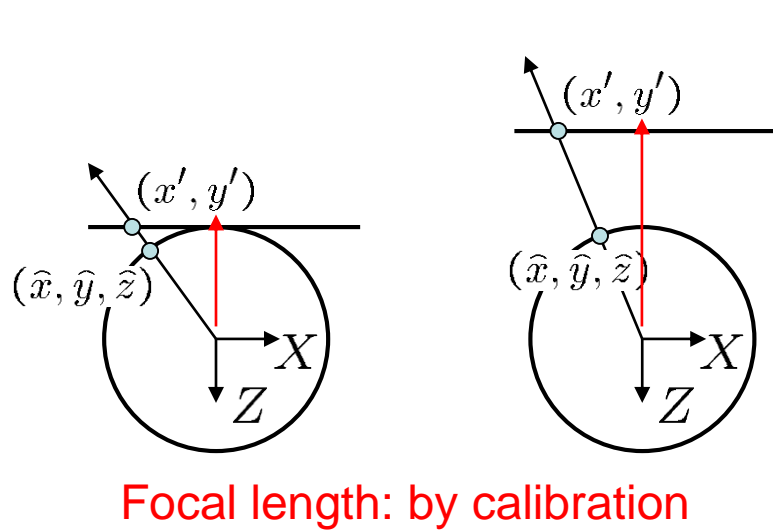
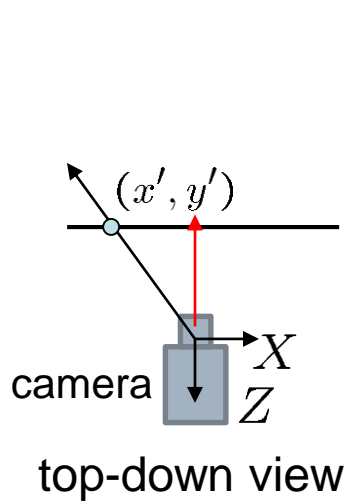


Image 384x300



$f = 180$  (pixels)



$f = 280$



$f = 380$

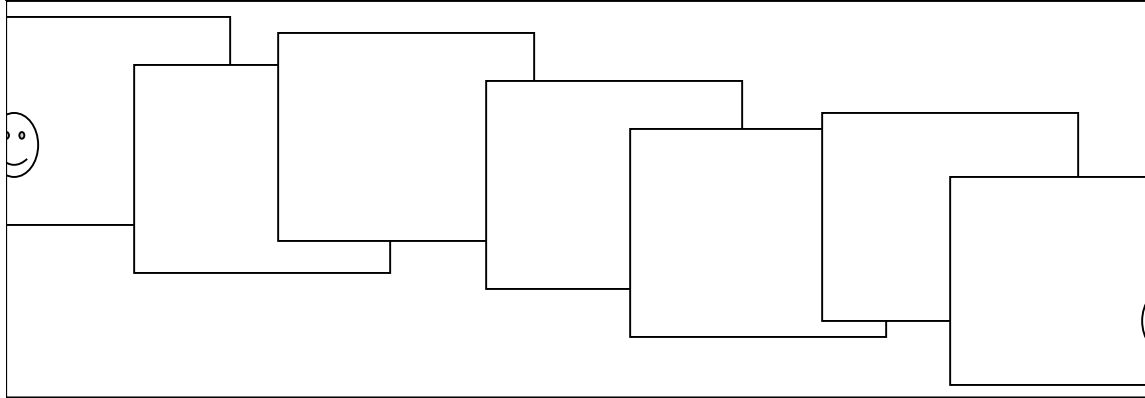


# Drift Problem

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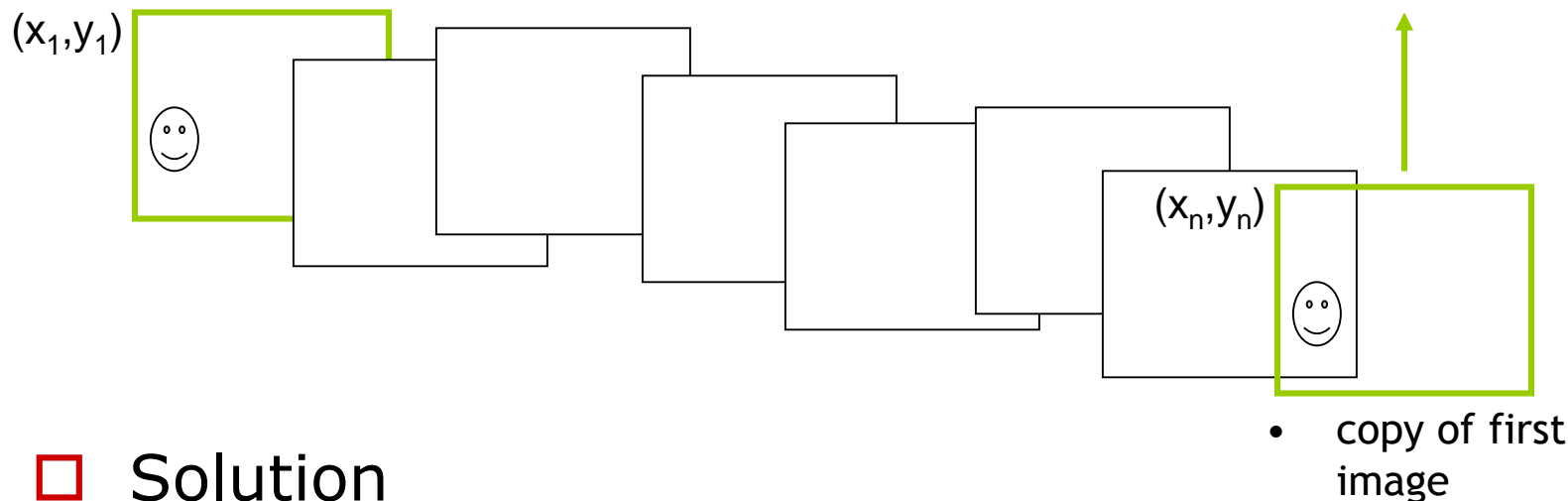
## ❑ Error accumulation

- small errors accumulate over time



# Drift Problem

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## □ Solution

- add another copy of first image at the end
- there are a bunch of ways to solve this problem
  - add displacement of  $(y_1 - y_n)/(n - 1)$  to each image after the first
  - compute a global warp:  $y' = y + ax$
  - run a big optimization problem, incorporating this constraint
    - best solution, but more complicated
    - known as “bundle adjustment”

# Blending

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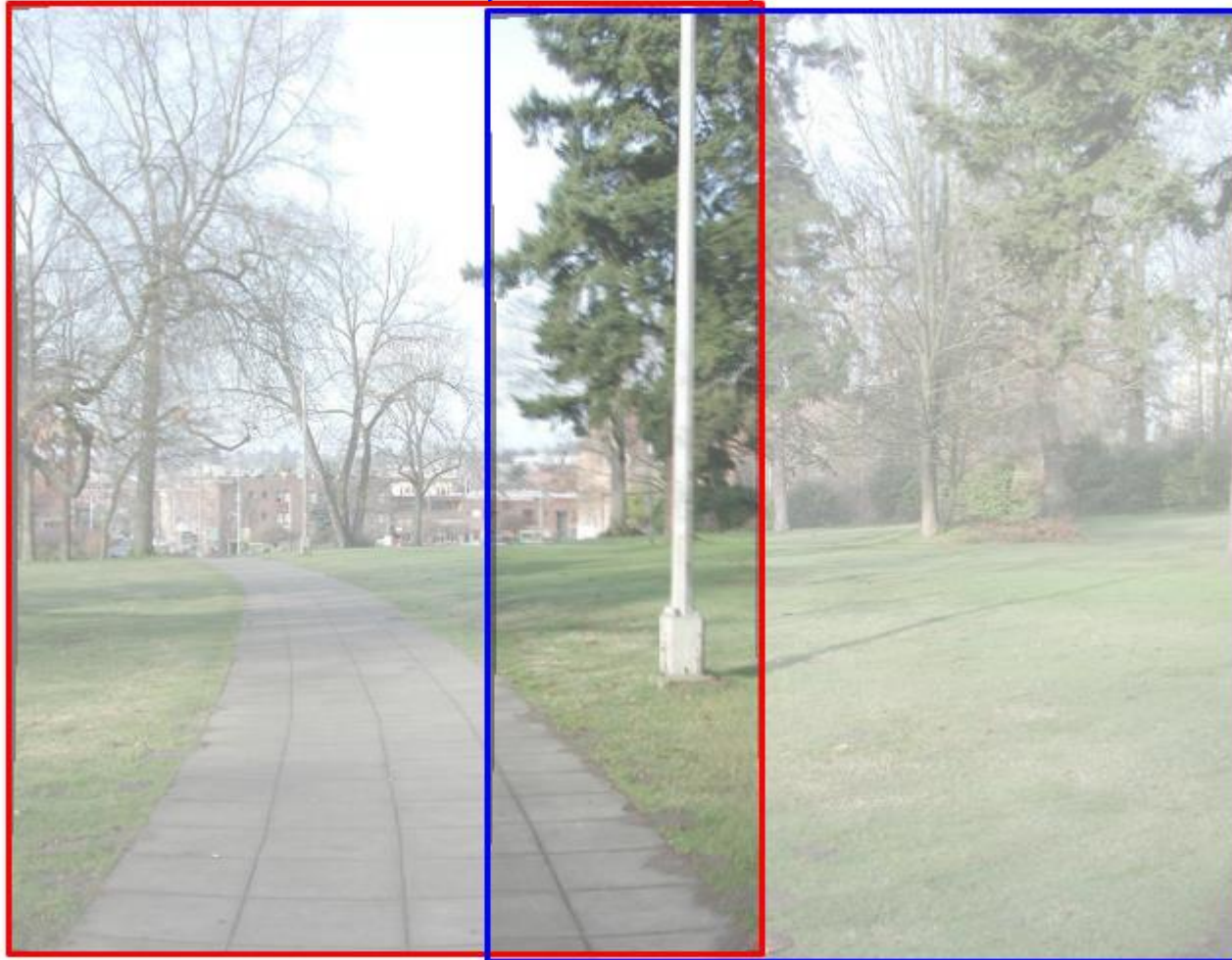
- Why blending: parallax, lens distortion, scene motion, exposure difference





# Linear Blending

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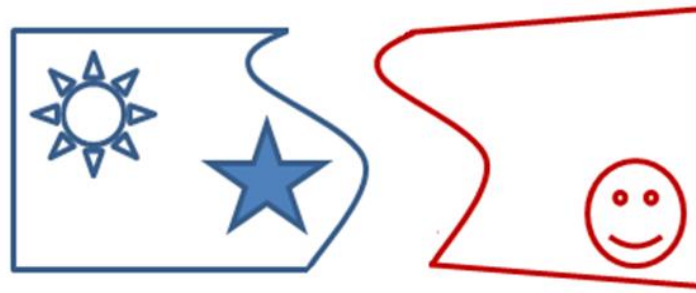
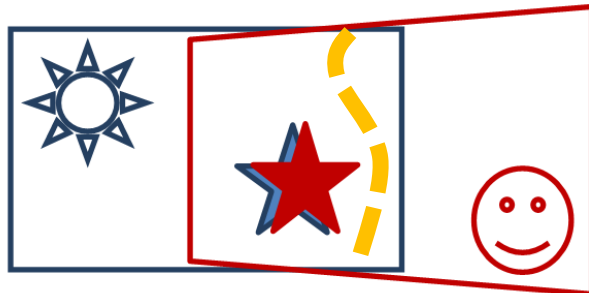
# Motion(Visual) Parallax, Ghosting Effect

- ❑ Large inconsistency in overlap region, after blending ...



- ❑ Solution

- Find a seam path avoid inconsistency in overlap region
- Cut off the overlap region of each image



# Face Morphing

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- ❑ Face recognition
- ❑ Feature point detection
- ❑ Image warping
- ❑ Color blending



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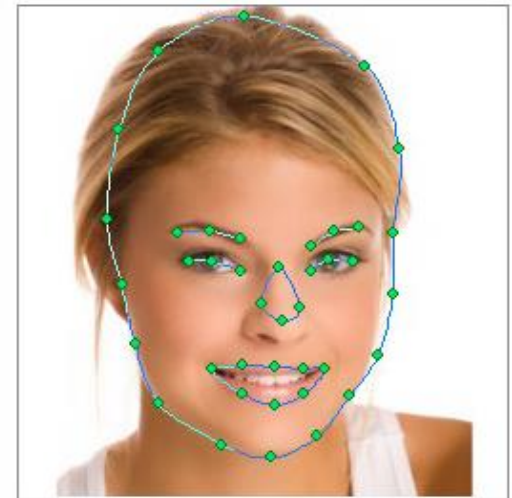
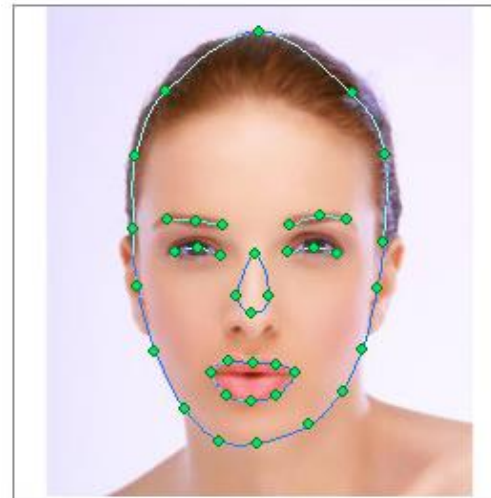


[Angelina Jolie](#)

[Jennifer Aniston](#)



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