

**Agenda item:** 5.1.5.1  
**Source:** National Taiwan University  
**Title:** Evaluation on double QC-LDPC codes with degree-3 for URLLC  
**Document for:** Discussion

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## 1 Introduction

This contribution follows the RAN1-#87[1] conclusion and agreement and further study should be prioritized for LDPC codes.

### Conclusion:

- One aspect that should be considered further for the NR LDPC design is the lifting size definition:
  - E.g. the lifting size  $z = c \cdot 2^d$ , where  $c$  is chosen from a set  $C$  of positive integers and, for each value of  $c$ ,  $d$  is taken from the set  $D = [0, 1, 2, 3, 4, 5, 6, 7, 8]$ 
    - FFS the set  $C$

### Agreement:

- Code extension of a parity-check matrix is used for IR HARQ/rate-matching support
  - Use lower-triangular extension, which includes diagonal-extension as a special case
- For the QC-LDPC design, the non-zero sub-blocks have circulant weight  $\leq 2$ 
  - Circulant weight is the number of superimposed circularly shifted  $Z \times Z$  identity matrices
- In parity check matrix design, the highest code rate ( $R_{\max,j}$ ) to design  $j$ -th H matrix for is
  - $R_{\max,j} \leq 8/9$
  - $R_{\max,j}$  is the code rate of the  $j$ -th H matrix before code extension is applied ( $0 \leq j < J$ )
  - $R_{\max,j}$  is the code rate after accounting for the built-in puncturing, if this is applied in H matrix design
  - Rate matching to support transmission code rate higher than  $R_{\max,j}$  is not precluded

We present and summarize double quasi-cyclic low-density parity check (DQC-LDPC) code with degree-3 following [2]. These codes are designed as  $z = 64$  or  $16$ . With the constraint of the same circulant size, the design could achieve BLER down to  $10^{-5}$  from short to long block length with IR-HARQ support and low decoding complexity.

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## 2 Design Result and Summary

Coding scheme selection of 5G new radio for eMBB, URLLC and mMTC consider the metrics of performance, implementation complexity, latency (decoding/encoding), flexibility (e.g., variable code length, code rate, HARQ (as applicable for particular scenario(s))). The LDPC code has been selected as the promising candidate according to the system simulation.

The DQC-LDPC is a special form of the classical QC-LDPC codes in the parity check matrix. The DQC code features double layers of circulant matrices (or circulant of circulants). Fig. 1 shows an example of double circulant in the parity matrix on the left submatrix of  $\mathbf{H}$ . For convenience, “\*” is used to denote the null matrix of size  $Q \times Q$ . As in QC-LDPC code, each small square blocks (or submatrices) of size  $Q \times Q$  are the null matrix (denoted by “\*”) or circulant permutation (right-shifted identity) matrices. A square collection of the small square blocks forms the outer layer of circulant matrix in  $\mathbf{H}$ . The unique feature helps to encode the parity with double shift register array.

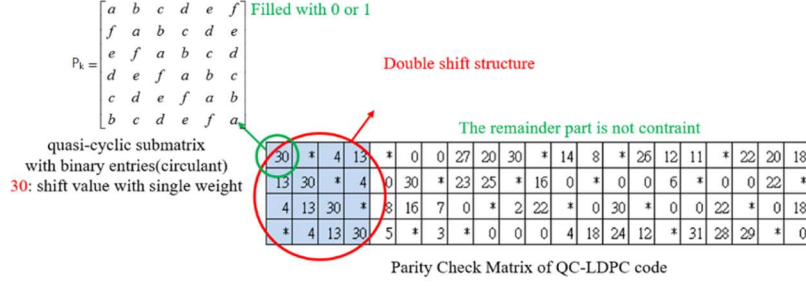


Fig. 1. Example of DQC-LDPC where the parity matrix is shaded on the left and the information is on the right.

The double QC-LDPC codes with degree-3 features superior support on these metrics.

1. **Correction:** The following figures show the typical BLER the code family. For each code , the BLER down to  $10^{-5}$  can be achieved.
2. **Implementation Complexity:** The number of degrees of the whole PCM indicates the codec complexity. The code family we propose has the fixed low degrees = 3 (column-degree)\*N(codeword length).
3. **Latency and throughput:** The code family we propose has the regular structure in the parity check matrix. If processing unit allow  $2^p$ -dim parallel processing, the decoder has predictable latency:  $N(\text{codeword length})/2^p(\text{parallel processing}) * (\text{average iteration})$  cycles. The final throughput then scales with the working clock frequency of decoder.
4. **Flexibility:** The following figures also show the flexibility of the code family with the code rate from 1/4 to 8/9, and information length is from 100 bits to 8000 bits.
5. **IR-HARQ support:** IR-HARQ scheme is also supported by code extension.

The following figures are the family of DQC-LDPC codes with circulant size  $z=16$  with PCM degree-3:

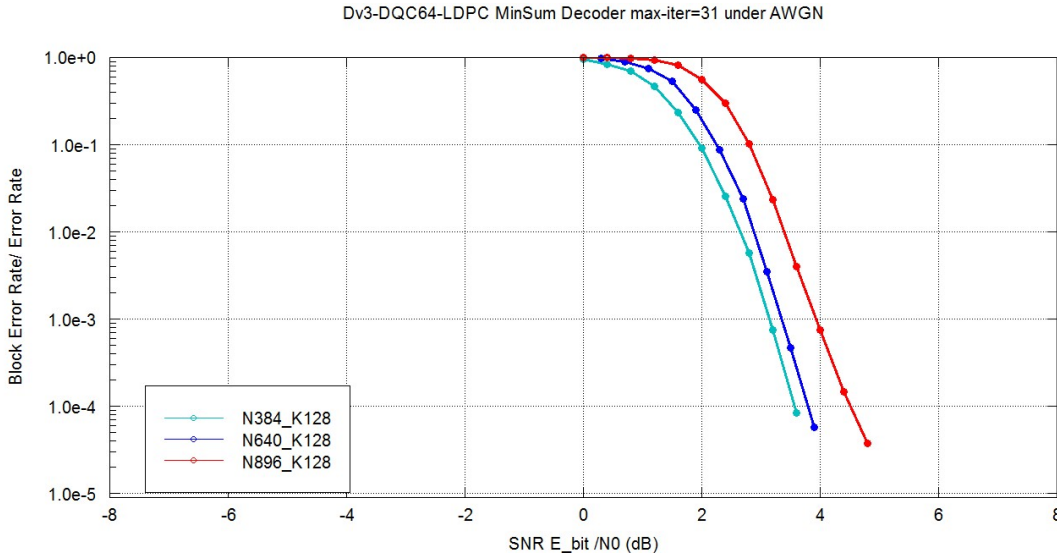


Fig. 1. DQC-LDPC codes with circulant size  $z=16$  with PCM degree-3 and  $K=128$ .

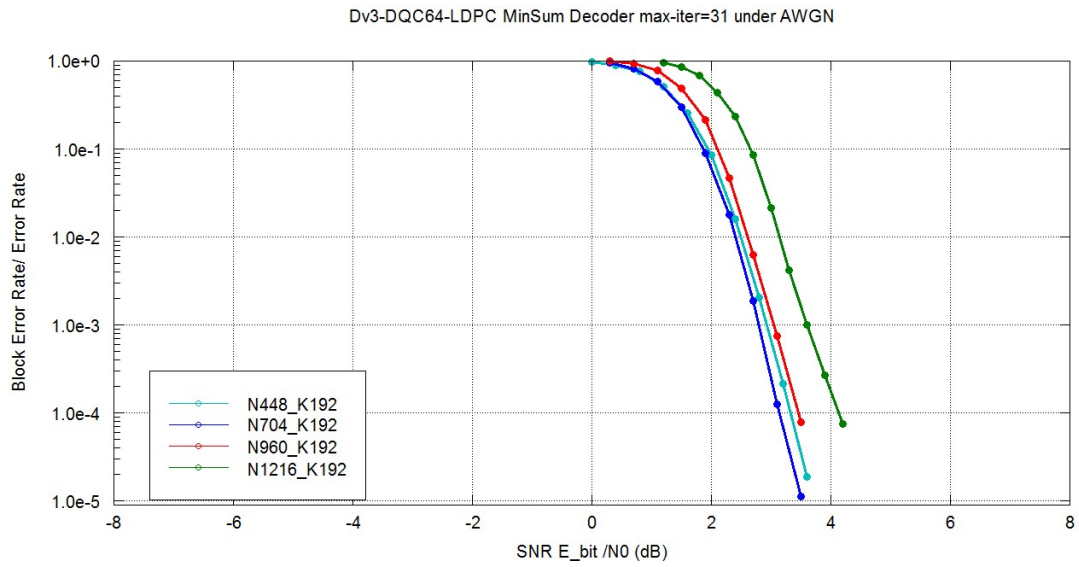


Fig. 2. DQC-LDPC codes with circulant size  $z=16$  with PCM degree-3 and  $K=192$ .

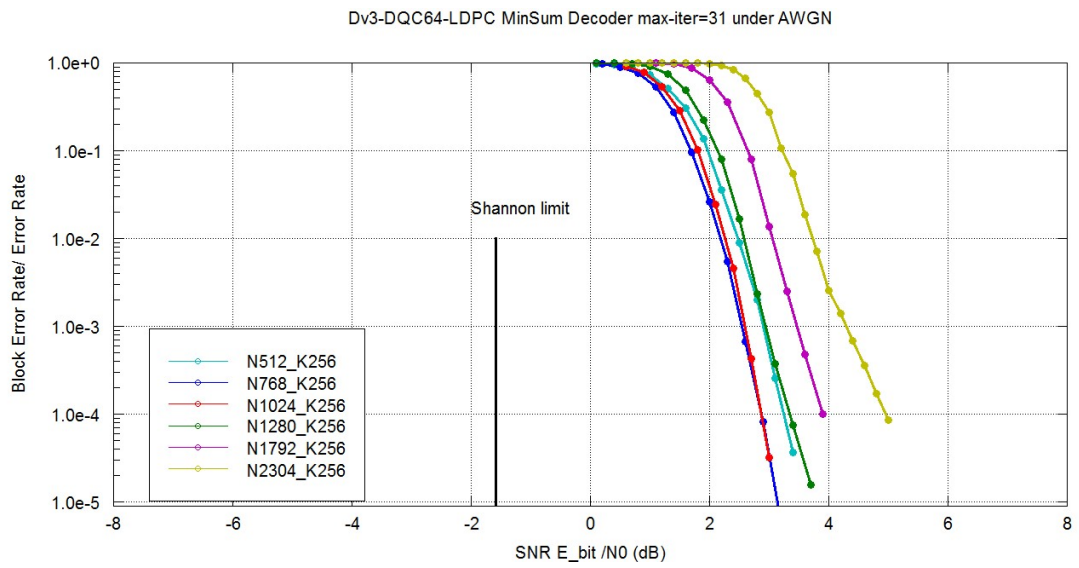


Fig. 3. DQC-LDPC codes with circulant size  $z=16$  with PCM degree-3 and  $K=256$ .

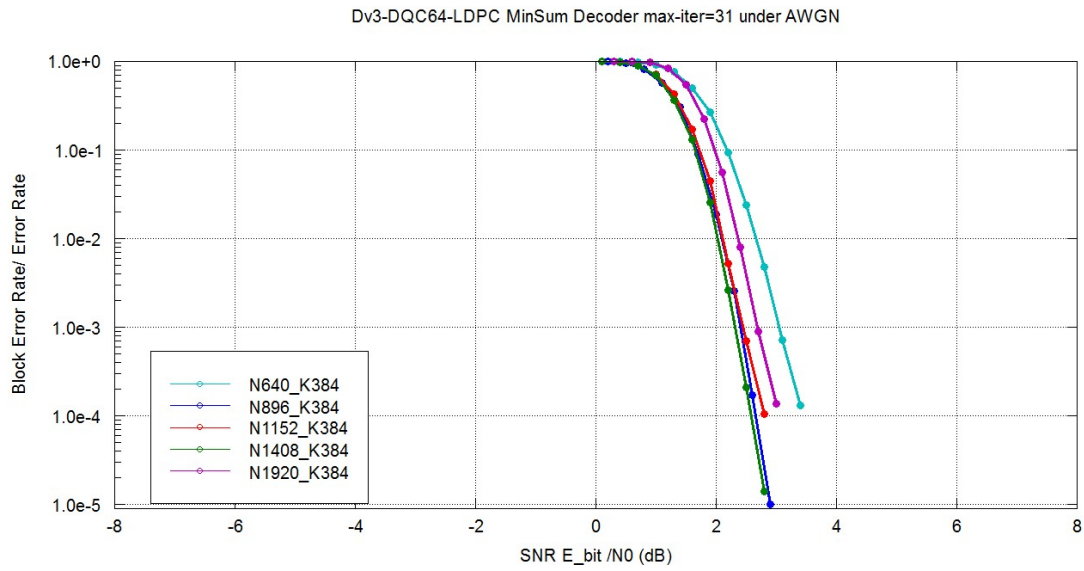


Fig. 4. DQC-LDPC codes with circulant size  $z=16$  with PCM degree-3 and  $K = 384$ .

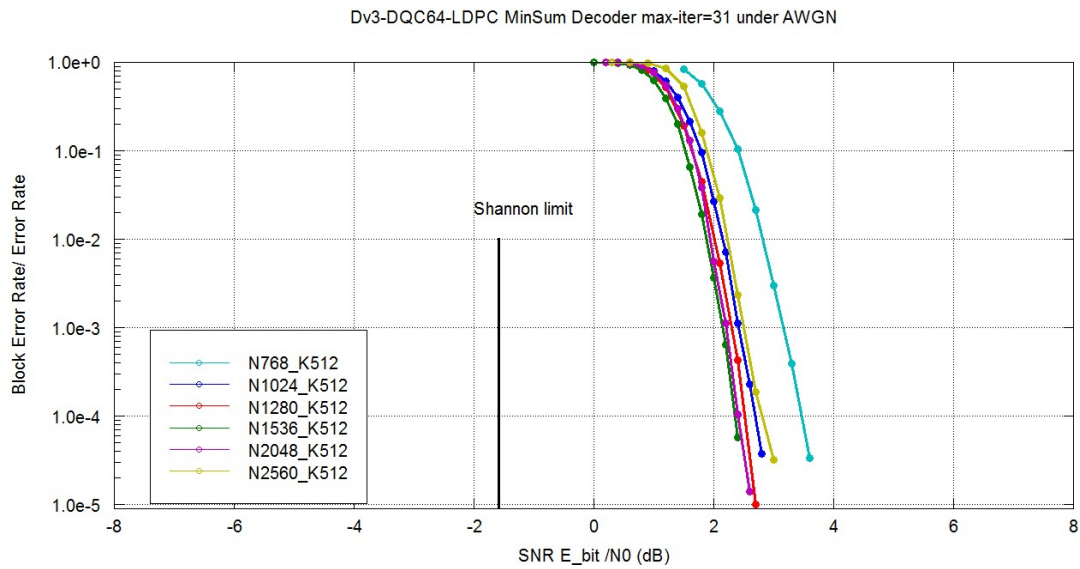


Fig. 5. DQC-LDPC codes with circulant size  $z=16$  with PCM degree-3 and  $K = 512$ .

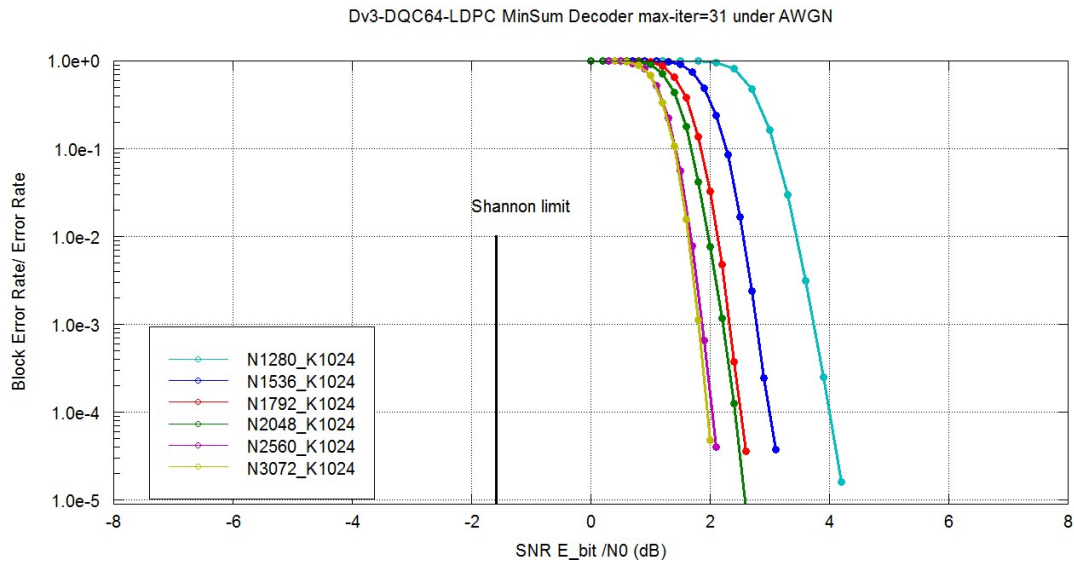


Fig. 6. DQC-LDPC codes with circulant size  $z=16$  with PCM degree-3 and  $K=1024$ .

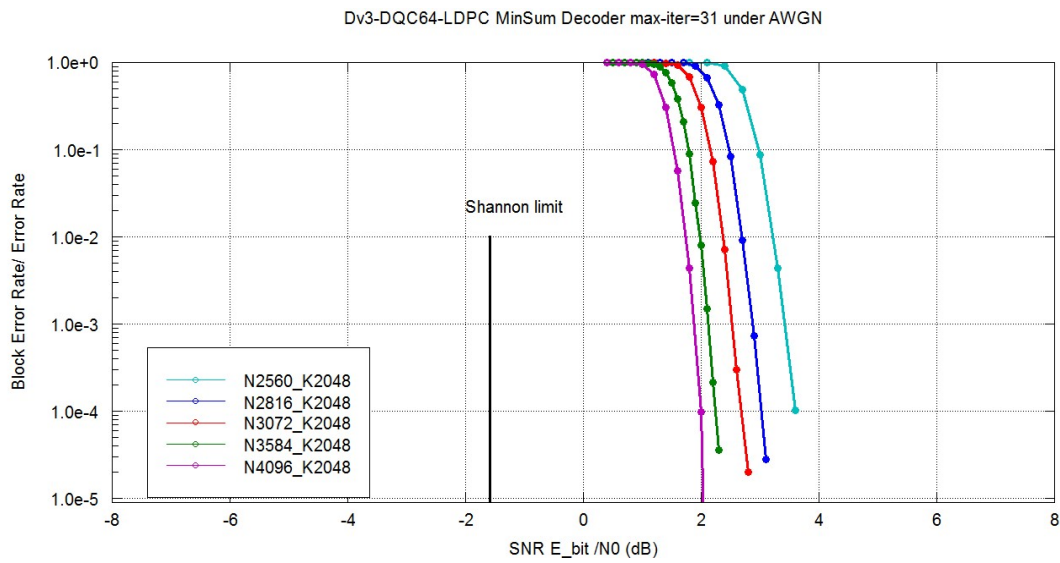


Fig. 7. DQC-LDPC codes with circulant size  $z=16$  with PCM degree-3 and  $K=2048$ .

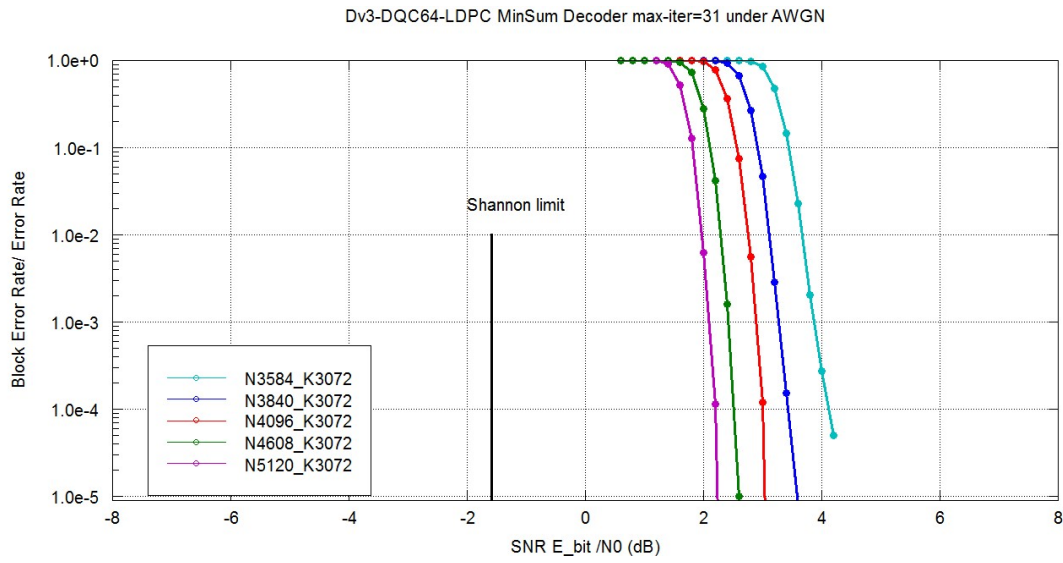


Fig. 8. DQC-LDPC codes with circulant size  $z=16$  with PCM degree-3 and  $K=3072$ .

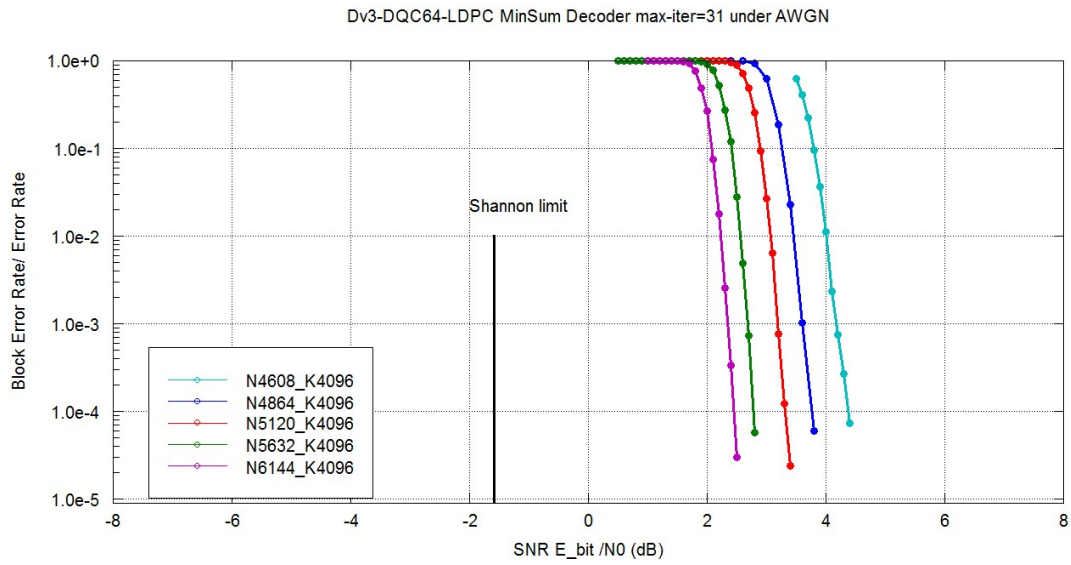


Fig. 9. DQC-LDPC codes with circulant size  $z=16$  with PCM degree-3 and  $K=4096$ .

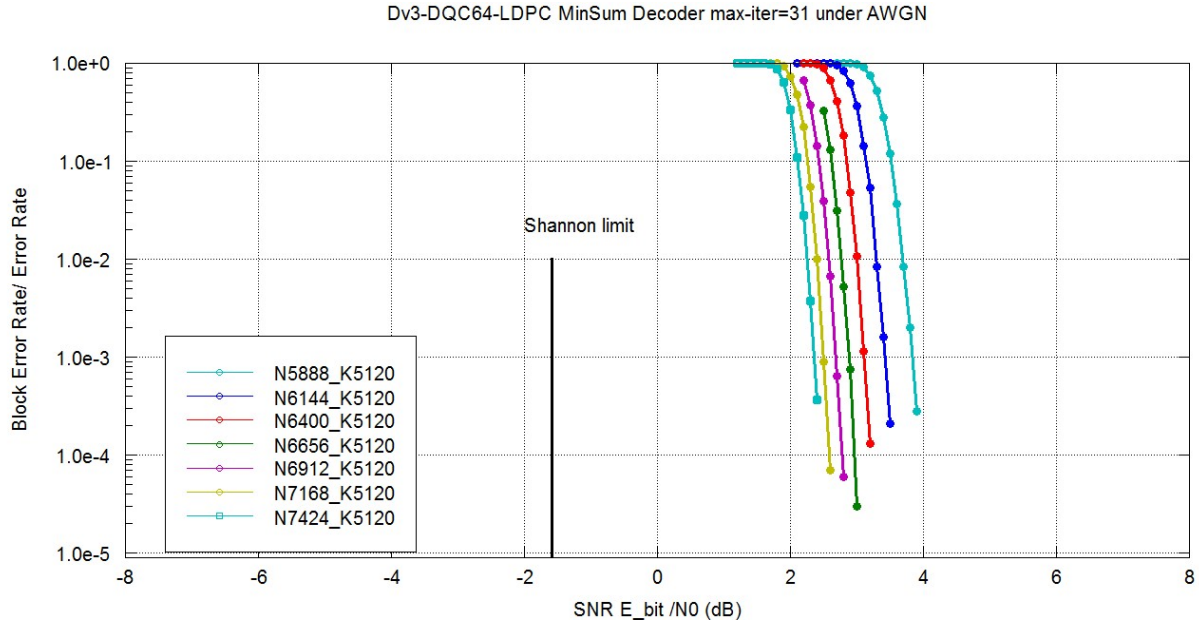


Fig. 10. DQC-LDPC codes with circulant size  $z=16$  with PCM degree-3 and  $K = 5120$ .

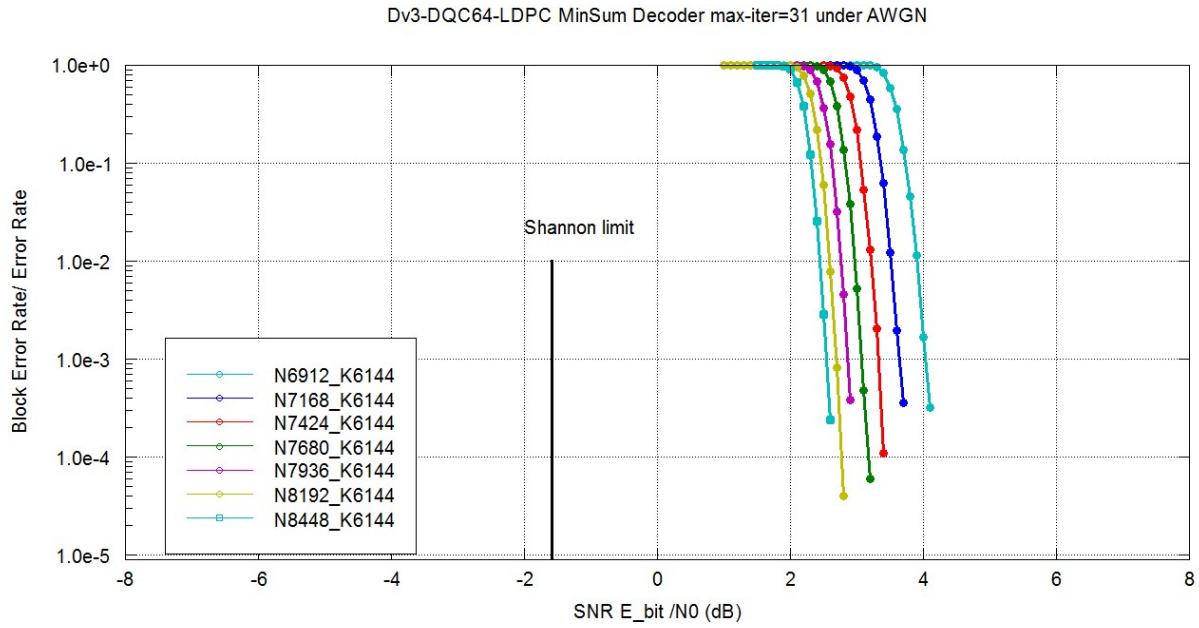


Fig. 11. DQC-LDPC codes with circulant size  $z=16$  with PCM degree-3 and  $K = 6144$ .

### 3 Conclusions

**Proposal 1:** Double QC-LDPC codes with degree-3 can provide achieve BLER down to  $10^{-5}$  from short to long block length with IR-HARQ support and low decoding complexity.

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## 4 References

- [1] R1-1613710, Chairman's Notes of AI 7.1.5 on channel coding and modulation for NR, 3GPP RAN1 meeting #87, Reno, Nevada, Nov. 2016.
- [2] R1-1609708, "Discussion of QC-LDPC code design with regular degree-3 for NR", National Taiwan University, 3GPP RAN1 meeting #86bis, Lisbon, Portugal.