1 Input files

1.1 INCAR

in INCAR file you specify the EMRI system by several parameters as listed in Table 1.1. Currently "METRIC" can be chosen from "Kerr", "KRZ" and "Custom".

Table 1: Explanation for each input value

name	explanation	unit	format
MASS	mass of central Black Hole (BH)	Solar mass	double
MASS_RATIO	mass ratio	1	double
METRIC	type of metric describing BH	/	string
DEFPAR_VAL	value of deformation parameters for non-Kerr metric	1	array
SPIN	spin parameter of central BH (J/M^2)	MASS^2	double
ECC	orbital eccentricity	1	double
P	orbital semi-latus rectum in geometrized unit	MASS	double
IOTA	orbital inclination angle	rad	double
TOTAL_TIME	upper limit for the time of evolution	second	double
THETA	observation latitudinal angle	rad	double
PHI	observation azimuthal angle	rad	double
R_PC	distance from source to observer	parsec	double

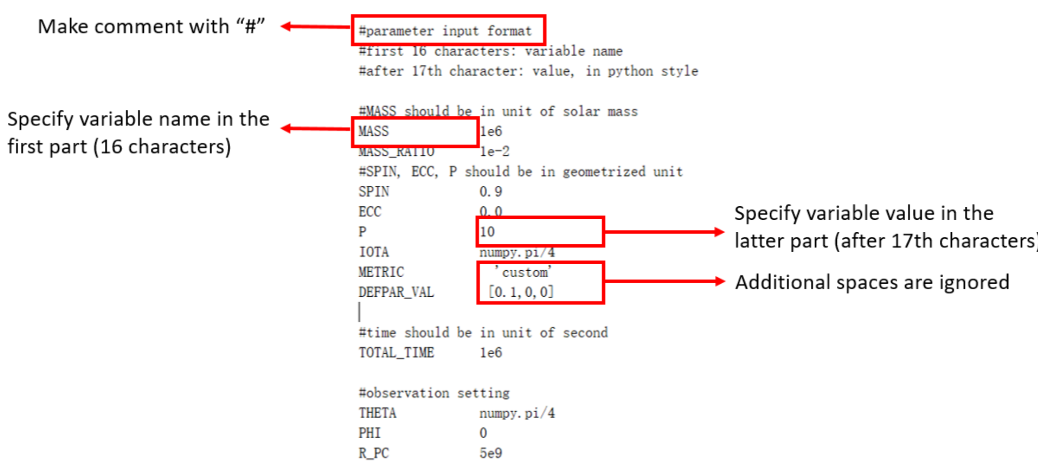

Figure 1: An Example of INCAR file.

Format of INCAR file (illustrated in Fig. 1):

- Comment with "#". Every line starting with "#" will be ignored.

- Each line that's not ignored specifies the variable name and value

    The first 16 characters specify the variable name

    The content after 17th character specifies the value. write it in python style.

- All strings are case-insensitive, i.e. "kerr" is equivalent to "Kerr" or "KERR".

1.2 metric.cpp

If your "METRIC" value is "custom", a cpp file is needed where you describe your metric. You need to write 4 functions in the file: metric(), metric_rderivatives(), metric_thderivatives() and metric_inverse(), which are used to calculate metric components, their derivatives with respect to r, their derivatives with respect to θ and the inverted metric in Boyer-Lindquist coordinates. Take care of the function names. See below for the details.

```

void metric(double spin /* the black hole spin */,
        double defpar[]/* deformation parameter in the custom metric */,
        double z1/* the r component of Boyer Lindquist coordinates */,
        double z2/* the theta component of Boyer Lindquist coordinates */,
        double g[][4]/* the metric elements */) {
...

g[0][0]=...;//write the expression for each metric element
...
return;
}

void metric_rderivatives(double spin /* the black hole spin */,
        double defpar[]/* deformation parameter in the custom metric */,
        double z1/* the r component of Boyer Lindquist coordinates */,
        double z2/* the theta component of Boyer Lindquist coordinates */,
        double rderg[][4]/* metric derivatives with respect to r */) {
...
rderg[0][0]=...;//write the expression for derivatives with respect to r
...
return;
}

void metric_thderivatives(double spin /* the black hole spin */,
        double defpar[]/* deformation parameter in the custom metric */,
        double z1/* the r component of Boyer Lindquist coordinates */,
        double z2/* the theta component of Boyer Lindquist coordinates */,
        double thderg[][4]/* metric derivatives with respect to theta */) {
...
thderg[0][0]=...;//write the expression for derivatives with respect to theta
...
return;
}

void metric_inverse(double spin /* the black hole spin */,
```

```
double defpar[]/* deformation parameter in the custom metric */,
double z1/* the r component of Boyer Lindquist coordinates */,
double z2/* the theta component of Boyer Lindquist coordinates */,
double invg[][4]/* inverted metric */) {
...
invg[0][0]=...;//write the expression for inverted metric components
...
return;
}
```

Note the relation between index numbers of g[ ][ ] (or rderg[ ][ ], thderg[ ][ ], invg[ ][ ]) and Boyer-Lindquist coordinates: $0 = t$; $1 = r$; $2 = \theta$; $3 = \phi$

2 Output files

2.1 OUTCAR

The OUTCAR file contains the log of the computing process and is self explained.

2.2 ORBCAR

The ORBCAR file contains the proper time, 4-location, 4-velocity and 4-acceleration of the orbiting object in each time step. It has 15 columns, as listed in Table. 2.2

Table 2: Explanation for each input value

column	explanation	unit	format
1	number of time steps	1	integer
2	coordinate time	second	10 digit decimal
3	proper time	geometric unit	10 digit decimal
4	coordinate time	geometric unit	10 digit decimal
5	r component in Boyer-Lindquist coordinates	geometric unit	10 digit E-notation
6	θ component in Boyer-Lindquist coordinates	rad	10 digit E-notation
7	ϕ component in Boyer-Lindquist coordinates	rad	10 digit E-notation
8	t component of 4-velocity	geometric unit	10 digit E-notation
9	r component of 4-velocity	geometric unit	10 digit E-notation
10	θ component of 4-velocity	rad	10 digit E-notation
11	ϕ component of 4-velocity	rad	10 digit E-notation
12	t component of 4-acceleration	geometric unit	10 digit E-notation
13	r component of 4-acceleration	geometric unit	10 digit E-notation
14	θ component of 4-acceleration	rad	10 digit E-notation
15	ϕ component of 4-acceleration	rad	10 digit E-notation

2.3 WAVECAR

The WAVECAR file contains the Gravitational wave emitted by the orbiting object evaluated by quadrupole formula. It has 3 columns, which are retarded time in seconds, "plus" components of the waveform and "cross" components of the waveform.

3 How radiation reaction is evaluated

The code solves the 2-order dynamical equations with radiative force F^μ (u is 4-velocity and Γ is Christoffel connection):

$$\frac{du^\mu}{d\tau} = -\Gamma^\mu_{\rho\sigma} u^\rho u^\sigma + \mathcal{F}^\mu, \quad u^\mu = \frac{dx^\mu}{d\tau} \tag{1}$$

($\mathcal{F}^\mu$ can be recovered from fluxes)

$$\begin{aligned}
&E = -u_t = -g_{tt}u^t - g_{t\phi}u^\phi \\
&L_z = u_\phi = g_{t\phi}u^t + g_{\phi\phi}u^\phi \\
&Q = (g_{\theta\theta}u^\theta)^2 + \cos^2\theta(a^2(\eta^2 - E^2) + (\frac{L_z}{\sin\theta})^2) \\
&g_{\mu\nu}u^\mu u^\nu = -1
\end{aligned} \tag{2}$$

$$\begin{aligned}
&\dot{E}u^t = -g_{tt}\mathcal{F}^t - g_{t\phi}\mathcal{F}^\phi \\
&\dot{L}_z u^t = g_{t\phi}\mathcal{F}^t + g_{\phi\phi}\mathcal{F}^\phi \\
&\dot{Q}u^t = 2g^2_{\theta\theta}u^\theta\mathcal{F}^\theta + 2\cos^2\theta a^2 E\dot{E} + 2\cos^2\theta\frac{L_z\dot{L}_z}{\sin^2\theta} \\
&g_{\mu\nu}u^\mu\mathcal{F}^\nu = 0
\end{aligned} \tag{3}$$

Currently we are using the fluxes from 2PN approximation with corrections explained in [1][2].

References

[1] Stanislav Babak, Hua Fang, Jonathan R. Gair, Kostas Glampedakis, and Scott A. Hughes. "kludge" gravitational waveforms for a test-body orbiting a kerr black hole. *Phys. Rev. D*, 75:024005, Jan 2007.

[2] Jonathan R Gair and Kostas Glampedakis. Improved approximate inspirals of test bodies into kerr black holes. *Phys. Rev. D*, 73:064037, Mar 2006.