1 General comments

Throughout the code for both RJMCMC and SMC the prior parameter asssumed for the number and location of the changepoints is a Poisson process, so that if k is the number of changepoints over $[0, T]$ and $\tau_{1:k} = (\tau_1, \ldots, \tau_k)$ then

$$p(\tau_{1:k}, k) = \nu^k \mathrm{e}^{-\nu T} \mathbb{I}_{\mathbb{T}_k}(\tau_1, \ldots, \tau_k).$$

where $\mathbb{T}_k = \{\tau_{1:k} : 0 < \tau_1 < \ldots < \tau_k < T\}$.

2 Input options

- For a Poisson process modelprior1= α and modelprior2= β, see Section 3.
- For SNCP modelprior1= α and modelprior2= κ, see Section 4.
- To get results from Turcotte and Heard (2015), use the settings specified in the examples when using the -h option with the compiled code.

3 Poisson process

The data generating process $y(t)$ is assumed to be the increments of an inhomogeneous Poisson process with piecewise constant intensity $\lambda(t)$. So the stochastic process $y(t) = 0$ almost everywhere, otherwise $y(t) = 1$ at finitely many t, which will be referred to as the event times of the process.

For a data process observed over the interval $[0, T]$ the jumps in the intensity will correspond to the vector of changepoints $\tau_{1:k}$. Define the parameter vector $\lambda_{0:k} = (\lambda_0, \ldots, \lambda_k)$, such that $\lambda(t) = \sum_{i=0}^{k} \lambda_i \mathbb{I}_{(\tau_i, \tau_{i+1}]}(t)$, where $\lambda_i \in \mathbb{R}^+$ is the intensity of the process between τ_i and τ_{i+1}. The likelihood of the observed process data is

$$\mathcal{L}(y([0, T])|\tau_{1:k}, k, \lambda_{0:k}) = \prod_{i=0}^{k} \lambda_i^{r_i} \mathrm{e}^{-\lambda_i(\tau_{i+1} - \tau_i)},$$

where $r_i = \int_{\tau_i}^{\tau_{i+1}} y(t)\mathrm{d}t$ is the number of events between τ_i and τ_{i+1}.

The $(k + 1)$ intensities will be assumed to follow the independent conjugate priors, $\lambda_i \sim \text{Gamma}(\alpha, \beta)$.

For posterior inference the intensities $\lambda_{0:k}$ can be integrated out to give the posterior distribution for the changepoints, which is known only up to proportionality through

$$\gamma(\tau_{1:k}) = \nu^k \mathrm{e}^{-\nu T} \prod_{i=0}^{k} \frac{\beta^\alpha}{\Gamma(\alpha)} \frac{\Gamma(\alpha + r_i)}{(\beta + \tau_{i+1} - \tau_i)^{\alpha + r_i}},$$

since Z does not have an analytical solution.

And it follows that, conditional on the changepoints $\tau_{1:k}$, $\{\lambda_i\}$, for $i = 0 \ldots, k$, have the independent posterior distributions

$$[\lambda_i|\tau_{1:k}, k, y([0, T])] \equiv \text{Gamma}(\alpha + r_i, \beta + \tau_{i+1} - \tau_i).$$

4 Shot noise Cox process

SNCP. See Turcotte and Heard (2015).

5 RJMCMC

For a conjugate model the RJMCMC moves employed are similar to those of Green (1995), who also applies them to a Poisson process, and Denison et al. (2002) where the transitions are: (a) 'birth' of a new changepoint uniformly chosen over the time period; (b) 'death' of a randomly chosen changepoint; and (c) 'move' a randomly chosen changepoint uniformly over some interval around the changepoint.

For the non-conjugate model another transition is added (d) 'move' the parameter, see Turcotte and Heard (2015) for a desccription of the proposal distributions for the transitions.

6 SMC

Refer to Turcotte and Heard (2015) for a detailed description of the algorithm.

References

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