**2D TEz MOM Analysis**

*Authors: Adedayo Lawal and Blake Levy*

**Abstract – A two dimensional method of moments (MOM) simulation is presented. A normally incident plane impinges upon an infinitely long perfect electric conducting cylinder.**

1. **INTRODUCTION**
2. **FORMULATION**
   1. **Discretization**

Unlike other methods in computational electromagnetics, the geometrical mesh does not include the free space environment. The discretization occurs on the boundaries of the objects of interest. Figure XX shows an example of a cylinder divided into 8 segments with their midpoints represented by circles.

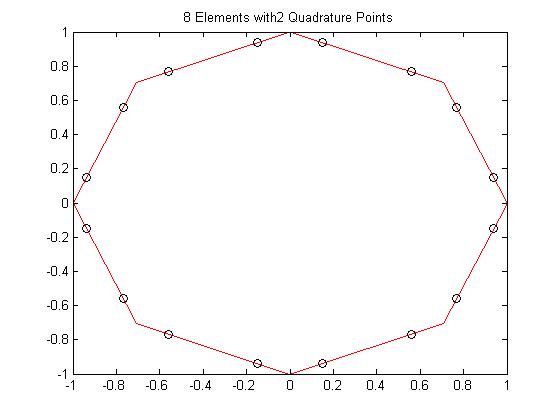


Figure 1 Circular Mesh of 8 segments with midpoints

* 1. **Boundary Conditions (D- EFIE, B-MFIE)**

For a TEz incident plane wave, the boundary conditions on the surface of the cylinder are determined by equations (2.2.1) and (2.2.2). The fields outside of the cylinder are represented with a subscript of 2 whereas the fields inside the cylinder are represented with a subscript of 1. The problem calls for a perfectly conducting cylinder which makes the fields inside the cylinder identically equal to zero.

* 1. **Fundamental Equations (D-EFIE,B-MFIE)**
     1. **MFIE**

The magnetic field integral equation begins with the boundary condition shown in equation (2.2.1). The exterior field is comprised of both the incident and scattered magnetic field. The tangential component of the exterior field is equal to the surface current at the boundary of the PEC cylinder as seen from equation (2.3.1).

The use of the magnetic vector potential, A, is used to determine the scattered magnetic field. The magnetic vector potential in Equation (2.3.2) uses an outward propagating Green’s function equal to a first order Hankel function of the second kind.

The scattered field is now evaluated from equation (2.3.3). A singularity occurs if the Green’s function is equal to zero. To account for the singularity, a principle value integral is used and the subsequent incident magnetic field integral equation is found by equation (2.3.4).

Source coordinates are differentiated from the observer coordinates with the use of a prime. The use of the letter, , represented the parameterized tangential components on the surface of the PEC cylinder.

* + 1. **EFIE**

EFIE solution for scattering on a PEC cylinder starts with a different set of boundary conditions (2.2.2). However, both the electric and magnetic fields are inside zero inside a PEC cylinder. The tangential component of the incident field is therefore related to the scattered field (2.3.2).

Using (2.3.2-2.3.4) we can then formulate the final TEz EFIE equation.

* + 1. **Point Matching**

MFIE has the advantage that point matching can be employed. To point matching uses a delta function to represent for the test function. These test function evaluate to the integrated on the nodes of the domain. This test function however cannot be employed for EFIE, due to the additional differentiability requirement on the test function.

* + 1. **N-Point Quadrature**

Quadrature was the chosen method to compute the terms in the system matrices for both EFIE and MFIE. The number of quadrature points used for MFIE and EFIE were different. 1-point Gaussian-Legendre quadrature rule was implemented for MFIE while a 3-pont Gaussian-Legendre quadrature was used for EFIE. Higher-order quadrature rule will

* 1. **Matrix Formulation (D-EFIE, B-MFIE)**
     1. **D-EFIE**

Formulating the EFIE matrix equation from (2.3.5) requires several considerations to be made in regards to the basis and testing functions (2.4.1-2.4.3).

The higher order derivatives on the scalar potential in (2.3.5) will increase the differentiability requirements on the basis functions for the surface current. The testing functions can also no longer be delta functions as a result of the differentiability requirement from 2.4.3 moving onto the testing functions through integration by parts. Unlike in MFIE, where pulse functions were used to represent the surface currents, those same basis functions would result in artificial charge accumulation on the nodes. To resolve this situation, roof-top basis functions were adopted instead. These basis functions when formulated as an EFIE equation result in the final form for the EFIE system matrix. The test functions are pulses with a span of one domain length. These pulses satisfy the differentiability requirements given by (2.4.3) on the test functions. In formulating the system matrix, several approximations can be made [2]. When these approximations are combined with the roof-top basis functions for the surface currents on the cylinder, the final form of the system matrix Z is given in (2.4.1-2.4.3).

The input vector V give given in (2.4.6)

With the system matrix formulated, an N-point quadrature rule can be employed to compute the integrals in (2.4.4).

* + 1. **Singularity Extraction**
       1. **D-EFIE**

In computing the integrals in (2.4.4) care must be taken to integrate the singularity when the observer to source distance in the Henkel function becomes zero or nearly zero. To integrate the singularity a Gaussian Lin-Log rule is employed to resolve the singularity more accurately. Careful computation of the singularity is critical in creating a well-conditioned matrix for the iterative solver, especially since a large number of these singular terms occur on or near the diagonals.

1. **RESULTS**
   1. **Error**

Analytic solutions for both the surface current and bistatic echo width are shown in equations 3.1.1 and 3.1.2 respectively. The simulation results determine the coefficients for the surface current on each element.

* 1. **Convergence**

1. **Future Work**

Figure 2 Top: Ez Reflection Coefficient Bottom: Hz Reflection Coefficient Comparison to [1]

* 1. **CFIE**
     1. **Internal Resonance**
  2. **TMz**

1. **CONCLUSION**
2. **REFERENCES**
3. J. Jin, *The Finite Element Method in Electromagnetics*, 2nd edition, Wiley, 2002.
4. A. W. Glisson & D.R. Wilton, IEEE T. Ant. Prop., Sept. 1980