```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from scipy import sparse
from scipy.stats import norm
from scipy.optimize import brentq, fsolve, minimize_scalar
```

Bachelier (1990), Black (1976), QNM (2023) classes

Bachelier (1990)

$$dF_t = \sigma_a dW_t$$

$$C_{BC}(F, K, \sigma_a, r, \tau) = e^{-r\tau}\left[(F - K)N(m_a) + \sigma_a\sqrt{\tau}n(m_a)\right]$$
$$P_{BC}(F, K, \sigma_a, r, \tau) = e^{-r\tau}\left[(K - F)(1 - N(m_a)) + \sigma_a\sqrt{\tau}n(m_a)\right]$$

where

$$m_a = \frac{F - K}{\sigma_a\sqrt{\tau}}$$

Black (1976)

$$dF_t = \sigma_G F_t dW_t$$

$$C_{BL}(F, K, \sigma_G, r, \tau) = e^{-r\tau}\left[FN(m_G + \frac{\sigma_G\sqrt{\tau}}{2}) - KN(m_G - \frac{\sigma_G\sqrt{\tau}}{2})\right]$$
$$P_{BL}(F, K, \sigma_G, r, \tau) = e^{-r\tau}\left[K(1 - N(m_G - \frac{\sigma_G\sqrt{\tau}}{2})) - F(1 - N(m_G + \frac{\sigma_G\sqrt{\tau}}{2}))\right]$$

where

$$m_G = \frac{ln(F - K)}{\sigma_G\sqrt{\tau}}$$

Quadratic Normal Model (Bouchouev, 2023)

$$dF_t = \sigma(F_t)dW_t = (\sigma_{ATM} + a + bF_t + cF_t^2)dW_t$$

$$C(F, K, a, b, c, r, \tau) = C_{BC}(F, K, \sigma_a = \sigma_{ATM}, r, \tau) + e^{-r\tau}U$$
$$P(F, K, a, b, c, r, \tau) = P_{BC}(F, K, \sigma_a = \sigma_{ATM}, r, \tau) + e^{-r\tau}U$$

where

$$U = \sqrt{\tau}n(\frac{F - K}{\sigma_{ATM}\sqrt{\tau}})[a + \frac{b}{2}(F + K) + \frac{c}{3}(F^2 + FK + K^2 + \frac{\sigma_{ATM}^2\tau}{2})]$$

```
# Bachelier (1900)
class Bachelier:
    def __init__(self, F, vol, r, tau):
      self.F = F
      self.vol = vol
      self.r = r
      self.tau = tau


    def option_pricer(self, K, vol = None, option_type = 'call'):
      '''
      Bachelier formula
      return call/put option price
      '''
      # default parameter (to compute implied vol)
      if vol == None:
        vol = self.vol

      m = (self.F - K) / (vol * self.tau**0.5)
      if option_type == 'call':
        return np.exp(-self.r * self.tau) * ((self.F - K) * norm.cdf(m) + vol * self.tau**0.5 * norm.pdf(m))
      elif option_type == 'put':
        return np.exp(-self.r * self.tau) * ((K - self.F) * (1 - norm.cdf(m)) + vol * self.tau**0.5 * norm.pdf(m))


    def implicit_FD(self, K, option_type = 'call'):
      '''
      The implicit finite difference
      return call/put option price
      '''
      # PDE: V_t + aV_ff + bV_f + cV = 0
      a = lambda f : 0.5 * self.vol**2
      b = lambda f : 0
      c = lambda f : -self.r
```

```python
        # Grid Parameters
        f_max = self.F + 100                                                        # space upper boundary
        f_min = self.F - 100                                                        # space lower boundary
        M = int(8*(f_max-f_min)) + 1                                                # the number of space steps
        N = int(100*252*self.tau)                                                   # the number of time steps

        # Grid Construction
        arr_f, df = np.linspace(f_min, f_max, M, retstep=True)                      # space discretization
        arr_t, dt = np.linspace(0, self.tau, N, retstep=True)                       # time discretization
        V = np.zeros((M, N))                                                        # grid initialization
        if option_type == 'call':
            V[:,0] = np.maximum(0, arr_f - K)                                       # European call option payoff (terminal condition)
            V[0,:] = 0                                                              # boundary condition
            V[-1,:] = (f_max - K) * np.exp(-self.r*arr_t)                           # boundary condition
        elif option_type == 'put':
            V[:,0] = np.maximum(0, K - arr_f)                                       # European put option payoff (terminal condition)
            V[0,:] = (K - f_min)* np.exp(-self.r*arr_t)                                     # boundary condition
            V[-1,:] = 0                                                             # boundary condition

        # Tri-diagonal Matrix Construction
        alpha = lambda f : -a(f)*dt/df**2 + b(f)*dt/2/df
        beta = lambda f : 1 + 2*a(f)*dt/df**2 - c(f)*dt
        gamma = lambda f : -a(f)*dt/df**2 - b(f)*dt/2/df

        D = np.zeros(shape = (M-2,M-2))
        D[0,0], D[0,1], D[-1,-2], D[-1,-1] = beta(arr_f[1]), gamma(arr_f[1]), alpha(arr_f[-2]), beta(arr_f[-2])
        for m in range(1, M-3):
            D[m,m-1], D[m,m], D[m,m+1] = alpha(arr_f[m+1]), beta(arr_f[m+1]), gamma(arr_f[m+1])
        D = sparse.csr_matrix(D)

        # Grid Computation
        rem = np.zeros(shape = (M-2,))
        for n in range(1,N):
            rem[0], rem[-1] = alpha(arr_f[1])*V[0,n-1], gamma(arr_f[-2])*V[-1,n-1]
            V[1:-1,n] =  sparse.linalg.spsolve(D, V[1:-1, n-1] - rem)

        return V[ round( (self.F - f_min)/df), -1]



# Black (1976)
class Black:
    def __init__(self, F, vol, r, tau):
      self.F = F
      self.vol = vol
      self.r = r
      self.tau = tau


    def option_pricer(self, K, vol = None, option_type = 'call'):
      '''
      Black formula
      return call/put option price
      '''
      # default parameter (to compute implied vol)
      if vol == None:
        vol = self.vol

      m = np.log(self.F / K) / (vol * self.tau**0.5)
      if option_type == 'call':
        return np.exp(-self.r * self.tau) * ( self.F * norm.cdf(m + 0.5*vol*self.tau**0.5) -
                                              K * norm.cdf(m - 0.5*vol*self.tau**0.5))
      elif option_type == 'put':
        return np.exp(-self.r * self.tau) * ( K * (1 - norm.cdf(m - 0.5*vol*self.tau**0.5)) -
                                              self.F * (1 - norm.cdf(m + 0.5*vol*self.tau**0.5)))



# Quadratic Normal Model (2023)
class QNM:
    def __init__(self, F, sig_atm, a, b, c, r, tau):
      self.F = F
      self.sig_atm = sig_atm
      self.a = a
      self.b = b
      self.c = c
      self.r = r
      self.tau = tau


    def option_pricer(self, K, option_type = 'call'):
      '''
      The method of linearization
      return call/put option price
      '''
      m = (self.F - K)/(self.sig_atm * self.tau**0.5)
      C_BC = np.exp(-self.r * self.tau) * ((self.F - K)*norm.cdf(m) + self.sig_atm * self.tau**0.5 * norm.pdf(m))
      P_BC = np.exp(-self.r * self.tau) * ((K - self.F)*(1-norm.cdf(m)) + self.sig_atm * self.tau**0.5 * norm.pdf(m))
      U = self.tau**0.5 * norm.pdf(m) * (self.a + self.b*(self.F + K)/2 +
```

```
                         self.c*(self.F**2 + self.F*K + K**2 + 0.5*self.sig_atm**2*self.tau)/3)

      if option_type == 'call':
        return C_BC + U*np.exp(-self.r * self.tau)
      elif option_type == 'put':
        return P_BC + U*np.exp(-self.r * self.tau)



    def implicit_FD(self, K, option_type = 'call'):
      '''
      The implicit finite difference
      return call/put option price
      '''
      # PDE: V_t + aV_ff + bV_f + cV = 0
      a = lambda f : 0.5 * (self.sig_atm + self.a + self.b*f + self.c*f**2)**2
      b = lambda f : 0
      c = lambda f : -self.r

      # Grid Parameters
      f_max = self.F + 100                                                      # space upper boundary
      f_min = self.F - 100                                                      # space lower boundary
      M = int(8*(f_max-f_min)) + 1                                              # the number of space steps
      N = int(100*252*self.tau)                                                 # the number of time steps

      # Grid Construction
      arr_f, df = np.linspace(f_min, f_max, M, retstep=True)                   # space discretization
      arr_t, dt = np.linspace(0, self.tau, N, retstep=True)                    # time discretization
      V = np.zeros((M, N))                                                      # grid initialization
      if option_type == 'call':
          V[:,0] = np.maximum(0, arr_f - K)                                     # European call option payoff (terminal condition)
          V[0,:] = 0                                                            # boundary condition
          V[-1,:] = (f_max - K) * np.exp(-self.r*arr_t)                         # boundary condition
      elif option_type == 'put':
          V[:,0] = np.maximum(0, K - arr_f)                                     # European put option payoff (terminal condition)
          V[0,:] = (K - f_min)* np.exp(-self.r*arr_t)                                   # boundary condition
          V[-1,:] = 0                                                           # boundary condition

      # Tri-diagonal Matrix Construction
      alpha = lambda f : -a(f)*dt/df**2 + b(f)*dt/2/df
      beta = lambda f : 1 + 2*a(f)*dt/df**2 - c(f)*dt
      gamma = lambda f : -a(f)*dt/df**2 - b(f)*dt/2/df

      D = np.zeros(shape = (M-2,M-2))
      D[0,0], D[0,1], D[-1,-2], D[-1,-1] = beta(arr_f[1]), gamma(arr_f[1]), alpha(arr_f[-2]), beta(arr_f[-2])
      for m in range(1, M-3):
          D[m,m-1], D[m,m], D[m,m+1] = alpha(arr_f[m+1]), beta(arr_f[m+1]), gamma(arr_f[m+1])
      D = sparse.csr_matrix(D)

      # Grid Computation
      rem = np.zeros(shape = (M-2,))
      for n in range(1,N):
          rem[0], rem[-1] = alpha(arr_f[1])*V[0,n-1], gamma(arr_f[-2])*V[-1,n-1]
          V[1:-1,n] =  sparse.linalg.spsolve(D, V[1:-1, n-1] - rem)

      return V[ round( (self.F - f_min)/df), -1]
```

```
# Check finite difference implementation
futures_price = 60.0
volaility_bachelier = 25
risk_free_rate = 0.005
time_to_maturity = 2.0
strike_price = 70

bachelier = Bachelier(futures_price, volaility_bachelier, risk_free_rate, time_to_maturity)
print(f'call formula = {bachelier.option_pricer(K = strike_price, option_type = "call"):.4f}')
print(f'call finite difference = {bachelier.implicit_FD(K = strike_price, option_type = "call"):.4f}\n')
print(f'put formula = {bachelier.option_pricer(K = strike_price, option_type = "put"):.4f}')
print(f'put finite difference = {bachelier.implicit_FD(K = strike_price, option_type = "put"):.4f}')
```

```
call formula = 9.5690
call finite difference = 9.5690

put formula = 19.4695
put finite difference = 19.4695
```

IBV and INV function

```
# IBV and INV
def implied_volatility(option_price, F, K, r, tau, option_type = 'call', model = 'black', method='brent', disp=True):
    """
        Return Implied volatility
        model: black (default), bachelier
        methods: brent (default), fsolve, minimization
    """
    # model
```

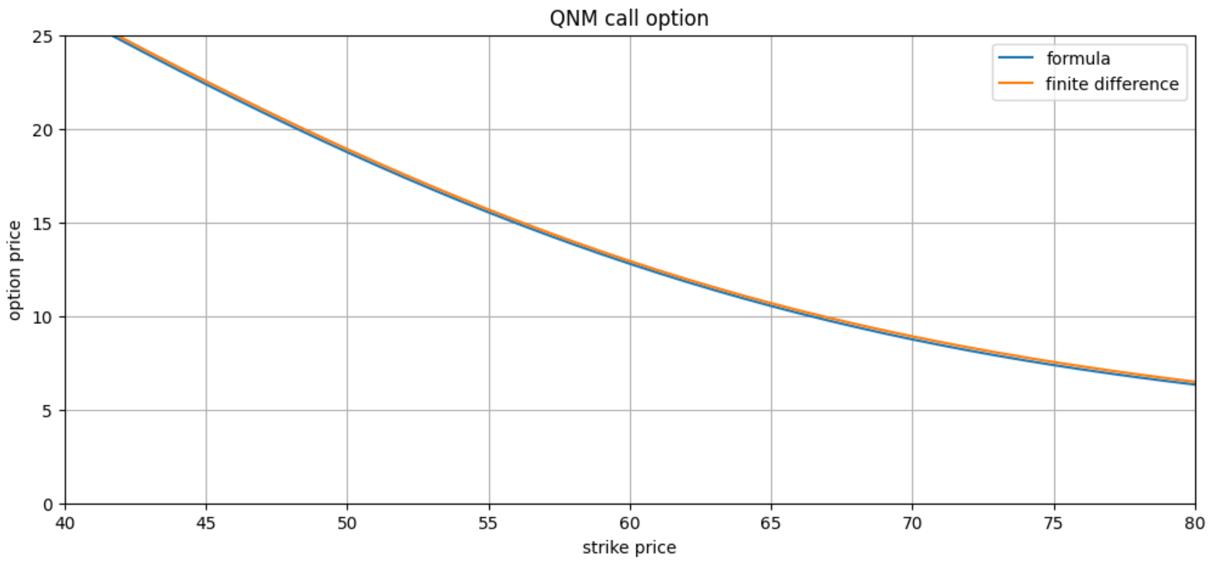
```
    if model == 'bachelier':
      bachelier_ = Bachelier(F, 30, r, tau)
      obj_fun = lambda vol : option_price - bachelier_.option_pricer(K = K, vol = vol, option_type = option_type)
    else: # model == 'black'
      black_ = Black(F, 0.1, r, tau)
      obj_fun = lambda vol : option_price - black_.option_pricer(K = K, vol = vol, option_type = option_type)

    # numerical method
    if method == 'minimization':
      obj_square = lambda vol : obj_fun(vol)**2
      res = minimize_scalar( obj_square, bounds=(1e-15, 8), method='bounded')
      if res.success == True:
        return res.x

    elif method == 'fsolve':
        X0 = [0.1, 0.5, 1, 3]   # set of initial guess points
        for x0 in X0:
            x, _, solved, _ = fsolve(obj_fun, x0, full_output=True, xtol=1e-8)
            if solved == 1:
              return x[0]

    else:
        x, r = brentq( obj_fun, a = 1e-15, b = 500, full_output = True)
        if r.converged == True:
          return x

    # display strikes with failed convergence
    if disp == True:
      print(method, K)
    return -1
```

Derman's approximation

$$
\begin{aligned}
INV(K,F) &\approx \frac{1}{K-F}\int_F^K \sigma(x)dx \\
&= \frac{1}{K-F}\int_F^K (\sigma_{ATM} + a + bx + cx^2)dx \\
&= \sigma_{ATM} + a + \frac{b}{2}(K+F) + \frac{c}{3}(K^2 + KF + F^2)
\end{aligned}
$$

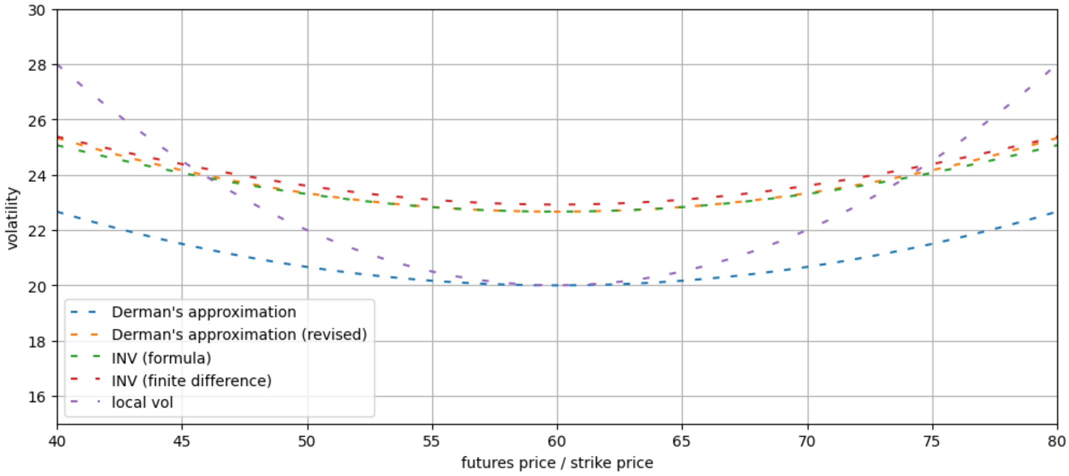
```
# QNM parameters
sig_atm, a, b, c = 20, 72, -2.4 ,0.02                 # 20 + 0.02*(F-60)^2
futures_price = 60.0
risk_free_rate = 0.0
time_to_maturity = 2.0


# local vol
sig_qnm = lambda f : sig_atm + a + b*f + c*f**2

# Derman's approximation
derman = lambda k : sig_atm + a + b/2*(k + futures_price) + c/3*(k**2 + k*futures_price + futures_price**2)

# Derman's approximation (revised)
derman_revised = lambda k : sig_atm + a + b/2*(k + futures_price) +\
                               c/3*(k**2 + k*futures_price + futures_price**2 + time_to_maturity*sig_atm**2/2)


# plot the local vol
step_size_f = 1.0
arr_f = np.arange(40, 80 + step_size_f/2, step_size_f)

plt.figure(figsize=(12,5))
plt.plot(arr_f, sig_qnm(arr_f))

plt.title('local vol')
plt.xlabel('futures price')
plt.ylabel('volatility')
plt.xlim(40,80)
plt.ylim(10,35)
plt.grid()

plt.show()
```


```
# compute call prices across strikes
qnm = QNM(futures_price, sig_atm, a, b, c, risk_free_rate, time_to_maturity)
arr_call_formula = qnm.option_pricer(K = arr_f, option_type = 'call')
list_call_fd = []
for k_ in arr_f:
    list_call_fd.append(qnm.implicit_FD(k_, option_type = 'call'))
arr_call_fd = np.array(list_call_fd)


# plot the call prices
plt.figure(figsize=(12,5))
plt.plot(arr_f, arr_call_formula, label = 'formula')
plt.plot(arr_f, list_call_fd, label = 'finite difference')

plt.title('QNM call option')
plt.xlabel('strike price')
plt.ylabel('option price')
plt.xlim(40,80)
plt.ylim(0,25)
plt.grid()
plt.legend()

plt.show()
```

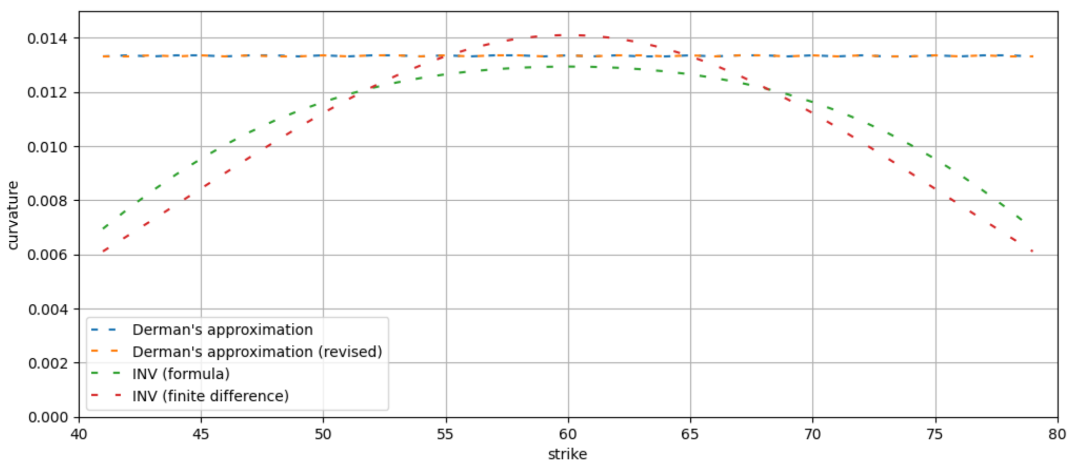



```
# compute the corresponding INVs across strikes
list_inv_formula = []
list_inv_fd = []
for stirke_, call_formula_, call_fd_ in zip(arr_f, arr_call_formula, arr_call_fd):
    list_inv_formula.append(implied_volatility(call_formula_, futures_price, stirke_, risk_free_rate, time_to_maturity,
                                                                option_type = 'call', model = 'bachelier', method='brent', disp=True))
    list_inv_fd.append(implied_volatility(call_fd_, futures_price, stirke_, risk_free_rate, time_to_maturity,
                                                                option_type = 'call', model = 'bachelier', method='brent', disp=True))
arr_inv_formula = np.array(list_inv_formula)
arr_inv_fd = np.array(list_inv_fd)


# plot the local vol, INVs, and Derman's approximations
arr_derman = derman(arr_f)
arr_derman_revised = derman_revised(arr_f)
plt.figure(figsize=(12,5))

plt.plot(arr_f, arr_derman, label = 'Derman\'s approximation', linestyle = (0,(3,5)))
plt.plot(arr_f, arr_derman_revised, label = 'Derman\'s approximation (revised)', linestyle = (0,(3,6)))
plt.plot(arr_f, arr_inv_formula, label = 'INV (formula)', linestyle = (0,(3,7)))
plt.plot(arr_f, arr_inv_fd, label = 'INV (finite difference)', linestyle = (0,(3,8)))
plt.plot(arr_f, sig_qnm(arr_f), label = 'local vol', linestyle = (0,(3,9)))

plt.xlabel('futures price / strike price')
plt.ylabel('volatility')
plt.xlim(40,80)
```

```
plt.ylim(15,30)
plt.grid()
plt.legend()

plt.show()
```




```
# compute the corresponding curvatures across strikes for INVs and Derman's approximations
arr_curve_formula = (arr_inv_formula[2:] + arr_inv_formula[:-2] - 2*arr_inv_formula[1:-1])/step_size_f**2
arr_curve_fd = (arr_inv_fd[2:] + arr_inv_fd[:-2] - 2*arr_inv_fd[1:-1])/step_size_f**2
arr_curve_derman = (arr_derman[2:] + arr_derman[:-2] - 2*arr_derman[1:-1])/step_size_f**2
arr_curve_derman_revised = (arr_derman_revised[2:] + arr_derman_revised[:-2] - 2*arr_derman_revised[1:-1])/step_size_f**2

# plot the curvatures for INVs and Derman's approximations
plt.figure(figsize=(12,5))

plt.plot(arr_f[1:-1], arr_curve_derman, label = 'Derman\'s approximation', linestyle = (0,(3,5)))
plt.plot(arr_f[1:-1], arr_curve_derman_revised, label = 'Derman\'s approximation (revised)', linestyle = (0,(3,6)))
plt.plot(arr_f[1:-1], arr_curve_formula, label = 'INV (formula)', linestyle = (0,(3,7)))
plt.plot(arr_f[1:-1], arr_curve_fd, label = 'INV (finite difference)', linestyle = (0,(3,8)))

plt.xlabel('strike')
plt.ylabel('curvature')
plt.xlim(40,80)
plt.ylim(0,0.015)
plt.grid()
plt.legend()

plt.show()
```

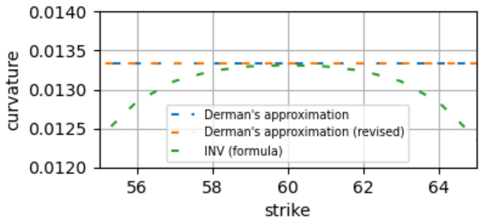


Derman's approximation demonstrates enhanced performance with shorter time to maturity, whereas a more refined strike grid has little impact on the situation.

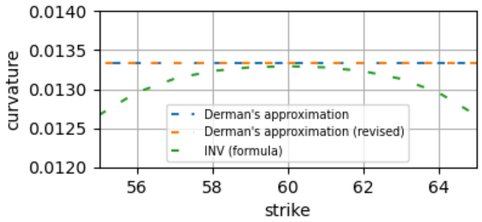
```
list_time_to_maturity = [1/12, 2/12, 4/12, 8/12]

for time_to_maturity_ in list_time_to_maturity:

    # compute call prices across strikes
    qnm = QNM(futures_price, sig_atm, a, b, c, risk_free_rate, time_to_maturity_)
```

```
    arr_qnm_call = qnm.option_pricer(K = arr_f, option_type = 'call')

    # compute the corresponding INVs and Derman's approximations across strikes
    list_inv_qnm = []
    for stirke_, qnm_ in zip(arr_f, arr_qnm_call):
        list_inv_qnm.append(implied_volatility(qnm_, futures_price, stirke_, risk_free_rate, time_to_maturity_,
                                               option_type = 'call', model = 'bachelier', method='brent', disp=True))
    arr_inv_qnm = np.array(list_inv_qnm)
    arr_derman = derman(arr_f)
    arr_derman_revised = derman_revised(arr_f)

    # compute the corresponding curvatures across strikes for INV and Derman's approximation
    arr_curve_inv = (arr_inv_qnm[2:] + arr_inv_qnm[:-2] - 2*arr_inv_qnm[1:-1])/step_size_f**2
    arr_curve_derman = (arr_derman[2:] + arr_derman[:-2] - 2*arr_derman[1:-1])/step_size_f**2
    arr_curve_derman_revised = (arr_derman_revised[2:] + arr_derman_revised[:-2] - 2*arr_derman_revised[1:-1])/step_size_f**2

    # plot the curvatures for INV and Derman's approximation
    print(f'\n time_to_maturity = {time_to_maturity_}')
    plt.figure(figsize=(12/3, 5/3))

    plt.plot(arr_f[1:-1], arr_curve_derman, label = 'Derman\'s approximation', linestyle = (0,(3,5)))
    plt.plot(arr_f[1:-1], arr_curve_derman_revised, label = 'Derman\'s approximation (revised)', linestyle = (0,(3,6)))
    plt.plot(arr_f[1:-1], arr_curve_inv, label = 'INV (formula)', linestyle = (0,(3,7)))

    plt.xlabel('strike')
    plt.ylabel('curvature')
    plt.xlim(55,65)
    plt.ylim(0.012,0.014)
    plt.grid()
    plt.legend(fontsize = 'x-small')

    plt.show()
```

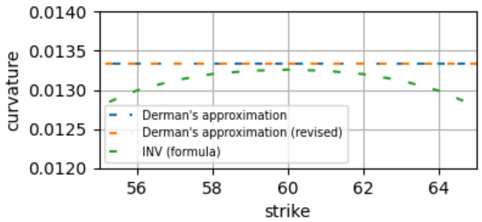
time_to_maturity = 0.08333333333333333



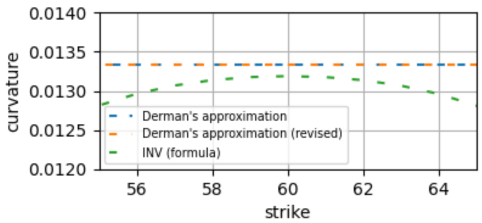
time_to_maturity = 0.16666666666666666



time_to_maturity = 0.3333333333333333



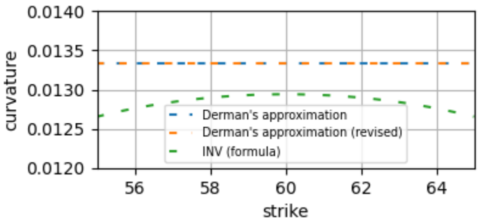
time_to_maturity = 0.6666666666666666



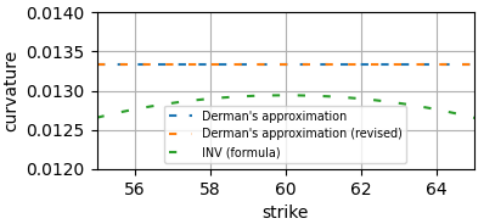

```
list_step_size_f = [0.1/8, 0.1/4, 0.1/2, 0.1]

for step_size_f_ in list_step_size_f:
    arr_f = np.arange(40, 80 + step_size_f_/2, step_size_f_)

    # compute call prices across strikes
    qnm = QNM(futures_price, sig_atm, a, b, c, risk_free_rate, time_to_maturity)
```

```
    arr_qnm_call = qnm.option_pricer(K = arr_f, option_type = 'call')

    # compute the corresponding INVs and Derman's approximations across strikes
    list_inv_qnm = []
    for stirke_, qnm_ in zip(arr_f, arr_qnm_call):
        list_inv_qnm.append(implied_volatility(qnm_, futures_price, stirke_, risk_free_rate, time_to_maturity,
                                                 option_type = 'call', model = 'bachelier', method='brent', disp=True))
    arr_inv_qnm = np.array(list_inv_qnm)
    arr_derman = derman(arr_f)
    arr_derman_revised = derman_revised(arr_f)

    # compute the corresponding curvatures across strikes for INV and Derman's approximations
    arr_curve_inv = (arr_inv_qnm[2:] + arr_inv_qnm[:-2] - 2*arr_inv_qnm[1:-1])/step_size_f_**2
    arr_curve_derman = (arr_derman[2:] + arr_derman[:-2] - 2*arr_derman[1:-1])/step_size_f_**2
    arr_curve_derman_revised = (arr_derman_revised[2:] + arr_derman_revised[:-2] - 2*arr_derman_revised[1:-1])/step_size_f_**2

    # plot the curvatures for INV and Derman's approximation
    print(f'\n step_f = {step_size_f_}')
    plt.figure(figsize=(12/3, 5/3))

    plt.plot(arr_f[1:-1], arr_curve_derman, label = 'Derman\'s approximation', linestyle = (0,(3,5)))
    plt.plot(arr_f[1:-1], arr_curve_derman_revised, label = 'Derman\'s approximation (revised)', linestyle = (0,(3,6)))
    plt.plot(arr_f[1:-1], arr_curve_inv, label = 'INV (formula)', linestyle = (0,(3,7)))

    plt.xlabel('strike')
    plt.ylabel('curvature')
    plt.xlim(55,65)
    plt.ylim(0.012,0.014)
    plt.grid()
    plt.legend(fontsize = 'x-small')

    plt.show()
```

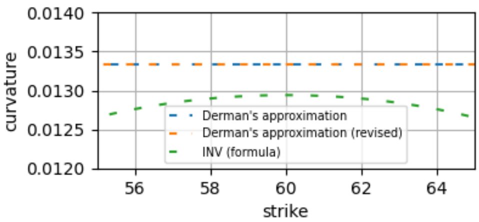
```
step_f = 0.0125
```



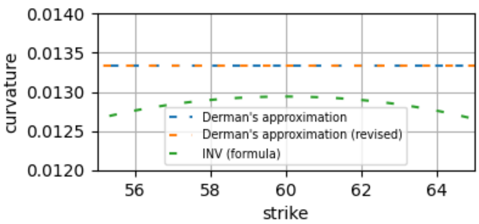

```
step_f = 0.025
```




```
step_f = 0.05
```




```
step_f = 0.1
```



The revised Derman's approximation performs effectively when the time to maturity is sufficiently long. For a one-month call option, it displays a maximum volatility error of only 1.09 across a range of strike prices.

```
'''# plot the maximum vol error as a function of time to matuirity
# i.e. ( max_k( | revised Derman's approximation - INV | ) )(tau)
```

```
arr_time_to_maturity = np.arange(1/12, 5.001, 1/12) #[1/12, 2/12, ...., 5]
list_max_vol_error = []

for time_to_maturity_ in arr_time_to_maturity:

    # compute call prices across strikes
    qnm = QNM(futures_price, sig_atm, a, b, c, risk_free_rate, time_to_maturity_)
    arr_qnm_call = qnm.option_pricer(K = arr_f, option_type = 'call')

    # compute the corresponding INVs and revised Derman's approximation across strikes
    list_inv_qnm = []
    for stirke_, qnm_ in zip(arr_f, arr_qnm_call):
        list_inv_qnm.append(implied_volatility(qnm_, futures_price, stirke_, risk_free_rate, time_to_maturity_,
                                               option_type = 'call', model = 'bachelier', method='brent', disp=True))
    arr_inv_qnm = np.array(list_inv_qnm)
    arr_derman_revised = derman_revised(arr_f)

    # compute the maximum difference between INV and revised Derman's approximation
    list_max_vol_error.append( (np.abs(arr_derman_revised - arr_inv_qnm)).max() )


# plot the maximum vol error(tau)
plt.figure(figsize=(12,5))

plt.plot(arr_time_to_maturity, list_max_vol_error, label = 'max_k( | Derman - INV | )')

plt.xlabel('time to maturity')
plt.ylabel('maximum error')
plt.grid()
plt.legend()

plt.show()'''
```

✓ 15 秒 完成時間：下午6:28