LONG SQUEEZE IN COMMODITY FUTURES MARKET

THESIS

Submitted in Partial Fulfillment of

the Requirements for

the Degree of

MASTER OF SCIENCE
(Financial Engineering)

at the

NEW YORK UNIVERSITY
TANDON SCHOOL OF ENGINEERING

by

Wu-Yen Sun

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Vita

Wu-Yen Sun was born in Taipei, Taiwan, on June 2, 1993. He obtained his Bachelor of Business Administration in Finance from National Taiwan University in June 2016. Driven by his academic and professional aspirations, he then pursued a Master of Science in Financial Engineering from New York University, where he worked on this thesis between Dec 2022 and May 2023. He is set to graduate from the program in May 2023.

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ABSTRACT

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by

Wu-Yen Sun

Advisor: Prof. David Shimko, Ph.D.

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This paper investigates the futures market manipulation in a two-period batch-trading model with asymmetric information. Building upon the model of Kyle (1984), we incorporate logistics and storage to accommodate occasional futures price crashes observed in commodity markets. Our model shows that a manipulator who privately knows uninformed trading activity can profit by manipulating the futures market and stockpiling a significant amount of the commodity at the delivery location to create a long squeeze. We find a symmetric pattern between the long squeeze in our study and the short squeeze discussed in Kyle (1984); in both works, an increase in the range of uninformed trading activity leads to a shift in equilibrium from separating to pooling. Additionally, we examine the local relationship between the random storage supply and the pooling equilibrium. Overall, this study contributes to a better understanding of the dynamics of commodity markets and offers potential solutions to prevent long squeezes.

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1 Introduction

Why do negative prices occur in commodity markets? For example, natural gas in Permian Basin, West Texas in 2019 and wholesale electricity in Ontario in 2009 and in Germany in 2017. More recently, on April 20, 2020, the May 2020 West Texas Intermediate (WTI) crude oil futures contract settled at an unprecedented negative price of -$37.63 per barrel, with an intraday trading low of -$40.32 per barrel. It is worth noting that the corresponding Brent Contract and all other WTI expirations settled at positive prices on that day. April 20 was the penultimate day of the May contract, which meant that traders who failed to close out their positions by settlement on the following day would be forced to take physical delivery at Cushing, a landlocked trading hub with limited storage and pipeline capacity.

The CFTC Interim Staff Report, released on November 23, 2020, revealed that the working storage available at the Cushing facility was near capacity, and non-reportable traders had contributed to the unusual open interest in the May contract at the start of the April 20 trading session. According to Fernandez-Perez et al. (2023), the negative prices can be explained by cash-and-carry activities and a lack of storage capacity at Cushing. The COVID-19 lockdowns and geopolitical tensions aggravated the already oversupplied crude oil market, resulting in a super contango that encouraged cash-and-carry traders to long the May contract, book up the storage at Cushing, and simultaneously short more distant contracts. The authors posit that the price crash was caused by the desperate reversing trades of the long traders who had not pre-booked storage at Cushing. Alternatively, Bouchouev (2020) attributes the collapse of oil prices and the unusual open interest to the illiquidity of the market on a penultimate day and the popularity of the oil-linked retail products, such as YuanYouBao sponsored by Bank of China, which held large amounts of the May contracts on behalf of its retail investors at the start of the April 20 and liquidated the positions during the trading hours. Bouchouev (2021) estimates that the constraint on storage can account for approximately a $5 drop per barrel, suggesting that other factors are responsible for the remaining $55 drop per barrel. Despite this, the author acknowledges that storage tightness and fear of a long squeeze were indeed the catalysts leading to the event. Most price crashes in commodities share a similar pattern: limited storage availability at the delivery point and logistical capacity constraints give rise to concerns regarding the high costs of owning the commodity following physical settlement. By providing extra time to a long, storage can mitigate the logistics bottleneck or involuntary disposal, both determining the theoretical lower bound of the commodity price. If storage is scarce, a long without storage capacity is forced to dispose of or transport the commodity away from the settlement location within a short period, the simultaneous rush of outflow transportation driving up the logistic cost so high that negative prices are possible.

Commodities are characterized by storage and logistics. Both time (storage) and space (logistics) dimensions are highly relevant in the functioning and economics of commodity markets. Furthermore, the two are not independent of but interrelated with each other. This study examines how storage and logistics interact under the context of long squeezes. Specifically, we focus on how the supply of working storage affects the manipulator's decision on the transportation of the commodity to the settlement location.

While a short squeeze demands excessive deliveries to manipulate the market, a long squeeze involves making excessive deliveries. The literature has given less attention to long squeezes, which are less common than short squeezes throughout history. An example of long squeeze is the 1956 onion market in Chicago – Sam Siegel and Vincent Kosuga bought a significant quantity of onions and flooded the market to profit from a short position in the onion futures market. Both long and short squeezes result in a glut at the delivery location after the physical settlement and are almost mirror images of each other, except for a few differences. First, there is an inherent asymmetry due to futures contract specifications. For instance, a short has the delivery option, which hurts a short but benefits a long squeezer. Second, the glut harms a short squeezer, also known as the bury-the-corpse effect, but favors a long squeezer. Third, the market characteristics that make a market vulnerable to one kind of squeeze are usually different from those that make it vulnerable to the other kind: Pirrong (1993) demonstrates that a long squeezer must face a downward-sloping demand curve to profitably manipulate the market, which the upward-sloping supply curves of storage and outflow transportation can achieve. Consequently, commodities with inelastic storage and outflow logistics, such as electricity, crude oil, and perishable products, are more susceptible to long squeezes. Nonetheless, these characteristics impede short squeezes by exacerbating the bury-the-corpus effect.

But how does the manipulator acquire market power in the first place? In Kyle (1984), Pirrong (1995), and our work, the manipulator, through security trading, disguises his intent of market manipulation and accumulates a large position in the futures market. The uncertainty of uninformed trading activity offers camouflage for this abnormal order. Once the manipulator has established a significant position, he can exercise his market power as described in Fackler (1993). The manipulator in our study seeks to establish a substantial short position in the commodity futures market, then move significant amounts of the underlying asset to the delivery point for the coming physical settlement and profit from the resulting depressed price due to a long squeeze. To make the market squeeze profitable, the manipulator cannot reveal his intentions in the security trading stage. Otherwise, market participants would have anticipated the long squeeze and set the futures price reasonably low to reflect the coming price crash. Our model is a variation of Kyle (1984). In both works, the manipulator privately knows the realization of the uninformed trading activity, and, most importantly, the equilibrium has a closed-form expression. In contrast, all market participants in Pirrong (1995) have identical information, but the model can only be implemented numerically. In the following part, we will revisit the literature on theoretical market manipulation and devote more space to Kyle (1984) and Pirrong (1995).

Allen and Gale (1992) identifies three categories of manipulation techniques. The first category, information-based manipulation, involves the release of false or misleading information that is intended to inflate or depress the price of the asset. The second category, trade-based manipulation, involves the use of trading activities to influence the price of the asset. The third category, action-based manipulation, occurs when a manipulator takes actions or misuses assets to affect the value of those assets. These categories are not always mutually exclusive and may overlap in some cases. Our work falls into the intersection of second and third categories as the manipulator in our work attempts to acquire a large short position in the security trading and transport a large amount of the commodity to the settlement location to squeeze the market.

The theoretical market manipulation literature is founded on imperfect competition assumption rather than the Walrasian paradigm. Both Putniņš (2012) and Pirrong (2017) provide comprehensive surveys of the literature on market manipulation, the latter specifically focusing on the commodity market. For information-based and action-based manipulation, Vila (1989) presents a simple model in which a manipulator short-sells a stock, disseminates damaging rumors about the company, and then liquidates his short position at a deflated price. Bagnoli and Lipman (1996) examines stock price manipulation in the context of takeover bids and finds that banning such manipulation could have undesirable consequences. In their model, manipulation involves a bidder announcing a takeover bid, selling his holdings at an inflated stock price, and subsequently dropping the bid. There are two competing effects of manipulation on the equilibrium distribution of bidders: firstly, some serious bidders may become manipulators, and secondly, a decrease in prebid stock price may attract more serious bidders. The dominance of the effects depends on the level of prior takeover activity, ultimately determining whether a ban on manipulation would be socially efficient. The profitability of the manipulation in Vila (1989) and Bagnoli and Lipman (1996) depends crucially on the credibility of the messages released. In repeated games, the credibility can erode rapidly, rendering the manipulation scheme unprofitable. However, Benabou and Laroque (1992) demonstrates that noise in private information can make it difficult for the public to distinguish between an honest mistake and an intentional deception, allowing opportunistic individuals to repeatedly profitably manipulate the market by mixing truthful and deceptive announcements. In Van Bommel (2003), an informed investor with limited investment capacity, who has invested in the stock market, may opt to disseminate imprecise rumors to a group of followers. In equilibrium, these rumors are informative, and the followers are willing to trade on them. Subsequently, the informed investor can capitalize on any resulting stock price overshooting by reversing his position to earn extra profits. Introducing costly talk and bounded rational uninformed traders, Eren and Ozsoylev (2006) shows that the informed trader may engage in hype-and-dump manipulation if there is at least one naive trader in the market and the cost of spreading dishonest rumors is sufficiently high. After the informed trader spreads a dishonest rumor, both sophisticated and naive uninformed traders become more aggressive in the trading game, as they believe the other side is incorrect. This behavior provides camouflage for the informed trader to enjoy more informational rent. The cost of disseminating a dishonest rumor due to regulatory enforcement ensures that the informed trader sends a convincing, albeit deceptive, rumor in equilibrium.

Many theoretical studies on trade-based manipulation are conducted on variations of the seminal models of Kyle (1985) and Glosten and Milgrom (1985). Allen and Gorton (1991) argues that a stock purchase is more likely than a sale to be driven by an information motive. Hence, an uninformed speculator can long a position, mimicking the informed trader, to bid up the price and liquidate the position at a relatively high price with little price impact due to the asymmetry of price responses. In equilibrium, the specialist rationally expects the manipulation activity and revises the bid and ask prices such that the manipulation earns zero profit. The uninformed speculator in Allen and Gale (1992) can imitate the informed trader and earn a positive profit in the pooling equilibrium. The unique information structure and the existence of risk-averse agents are essential to their results. Aggarwal and Wu (2006) expands on the framework of Allen and Gale (1992) and shows that a rise in the number of information seekers causes increased competition and improves market liquidity, inducing entry of more uninformed manipulators and harming market efficiency. When the market is uncertain about the entry of an informed trader, Chakraborty and Yılmaz (2004a) and Chakraborty and Yılmaz (2004b) demonstrate that the informed trader may mimic the uninformed to convince market participants that the informed has not entered the market, resulting in short-term losses but enjoying more informational rent in the long run. Several studies have examined how certain events or market mechanisms can enable profitable trade-based manipulation. In Gerard and Nanda (1993), the informed trader shorts a stock just before a seasoned equity offering, pushing down the prices and covering the position in the offering at a discount price. Profitable manipulation is possible because of the difference between the price-setting mechanisms in the secondary market and the seasoned equity offering. Fishman and Hagerty (1995) and John and Narayanan (1997) examine the manipulation using the mandatory disclosure rules for large trades and insider trades, respectively. In their work, the manipulator, pooling with the informed trader, discloses large purchases to drive up prices and subsequently sells the position anonymously through small trades. Investigating "banging the close" in the cash-settled futures market, Kumar and Seppi (1992) finds that uninformed traders can earn positive expected profits by establishing a futures position and subsequently trading in the spot market in order to manipulate the spot price used to compute the cash settlement. The manipulation is profitable because of the unique information structure in the spot and futures markets.

Kyle (1984), Pirrong (1995), and Allen et al. (2006) are trade-based manipulation related to market squeezes. In Kyle (1984), the cheapest-to-delivery mechanism plays the central role, where a squeezer can take a sizeable long position to force the short to settle with the high-quality commodity. However, without camouflage, the manipulator's intention would be exposed due to the abnormal buy order sent to the futures market. Knowing the trades from hedgers privately, the manipulator can conceal his trades so that other market participants cannot tell whether there will be a squeeze in the subgame. Hence, the equilibrium futures price reflects the expectation of squeeze occurrence. The manipulator profits in two ways: if the trades from hedgers are high, he goes short at an inflated price; on the other hand, if the trades from hedgers are low, he goes long at a deflated price and forces the short to settle with the high-quality commodity.

Pirrong (1993) presents a structural model demonstrating that for commodities produced and consumed in multiple locations, marginally increasing transportation costs create a downward-sloping demand curve, forming the basis of Fackler (1993)'s reduced-form analysis of how a large long facing a downward sloping demand curve would exercise market power to squeeze the market. Pirrong (1995) shows how a manipulator acquires market power through security trading described in Kyle (1985). The manipulator will then exercise a market squeeze, as in Fackler (1993), in the subgame if and only if he owns a sufficiently large long position acquired from the previous security trading stage. In equilibrium, the manipulator randomizes between buying and selling the futures contract in the security trading stage, anticipating a strictly positive profit and hurting the liquidity traders. Some implications are as follows: (1) a short squeeze occurs with a positive probability if noise trades exist and the supply curve in the delivery market is upward sloping; (2) markets are vulnerable to a short squeeze if they have volatile order flows, inelastic delivery market supply curves, and/or elastic delivery market demand curves; (3) market manipulation reduces the market depth and the information efficiency of the futures price.

Allen et al. (2006) develop a variant of Grossman and Stiglitz (1980) and show that squeezes can arise from rational behavior. The informed manipulator is the only strategic trader in the model, whereas others are price takers but try to extract private information from the (subsequently) observed transaction price, just as the traders in rational expectations equilibrium by Radner (1979). In the partially-revealing equilibrium, when receiving bad news, the manipulator tries to buy up the random supply of the security to squeeze the informed trades who would be on the short side and will subsequently face difficulty covering their positions.

Our contribution to the literature is to develop a tractable model for long squeeze and incorporate logistics and storage into the analysis of market manipulation. This paper proceeds as follows. In Section 2, we present a model of futures market manipulation. In section 3, we extend the model by introducing a random supply of storage. Section 4 presents concluding remarks and regulatory implications.

2 Model of Long Squeeze

Consider a physically settled commodity futures market. The futures market opens twice, and the physical settlement occurs after the second trade. All agents in the model do not have a time preference.

The commodity outside the settlement location has a common value of $\tilde{v} \in L^1(\Omega, \mathcal{F}, \mathbb{P})$ to all agents; the random variable $\tilde{v}$ will be realized just before the second trade. Sufficient amounts of the commodity are stored at the location for the coming physical settlement. However, the excess supply of working storage at the location "right after the settlement" is $\tilde{z} - \tilde{s}$ whose realization depends on nature's choice $\tilde{z} \in L^1(\Omega, \mathcal{F}, \mathbb{P})$ and $\tilde{s}$ of the commodity agents collectively transport to the settlement location. $\tilde{v}$ and $\tilde{z}$ are independent. Both $\tilde{z}$ and $\tilde{s}$ are positive almost surely, and their realizations are common knowledge to all agents right before the second trade. Given a fixed number of contracts settled at the expiration, the excess supply of working storage is generally higher if the short delivers the commodity previously stored at the settlement location rather than the commodity elsewhere. When a market squeeze occurs, i.e., $\tilde{z} - \tilde{s} \leq 0$, the commodity at the settlement location has a common value of $\tilde{v} - l$. On the other hand, if $\tilde{z} - \tilde{s} > 0$, the commodity at the settlement location has a value of $\tilde{v}$ to all agents. Pirrong (1993) and Fackler (1993) explain how a large short exercises market power to drive down the price, justifying l in the setting.

This is a two-period order-driven batch trading model with three types of agents: exogenous traders, one manipulator, and competitive market makers. Exogenous traders include but are not limited to hedgers and index trackers in the financialization of commodities described in Basak and Pavlova (2016). We will reserve capital letters for the agents' strategies. First, exogenous traders collectively buy $\tilde{H}$ and then reverse their position by selling $\tilde{H}$ in the two trading periods. The exogenous traders aim to hedge the endowed risk associated with $\tilde{v}$, realized just before the second trade. The futures market allows exogenous traders to transfer the endowed risk to other agents. We implicitly assume that the spot market hinders risk-sharing activity among market participants due to a much higher transaction cost. Assume $h_1 > h_0$ and $\lambda \in (0,1)$. The random variable $\tilde{H}$, independent of $\tilde{v}$ and $\tilde{z}$, is defined as the following:

$$\tilde{H} = \begin{cases} h_1, & \text{with probability } \lambda \\ h_0, & \text{with probability } 1-\lambda \end{cases}$$

We can also interpret h_1 as unusual cash-and-carry activity without pre-booked storage capacity prepared for physical delivery.

Second, the strategy choice of the manipulator consists of three measurable functions, denoted $\tilde{X_1} : \mathbf{R} \to \mathbf{R}$, $\tilde{S} : \mathbf{R^2} \to \mathbf{R}$, and $\tilde{X_2} : \mathbf{R^5} \to \mathbf{R}$. The quantities chosen by the manipulator are given by:

$$\begin{aligned} \tilde{x_1} &= \tilde{X_1}(\tilde{h}) \\ \tilde{s} &= \tilde{S}(\tilde{h}, \tilde{x_1}) \\ \tilde{x_2} &= \tilde{X_2}(\tilde{h}, \tilde{x_1}, \tilde{s}, \tilde{z}, \tilde{v}) \end{aligned}$$

The manipulator privately knows the realization of $\tilde{H}$ and submits a market order $\tilde{x_1}$ in the first trade. Then, the manipulator can transport $\tilde{s}$ units of the commodity to the settlement location for the physical delivery or for filling up idle storage at the settlement location, the associated cost being c per unit commodity, including but not limited to expenses for the inflow transportation and for the corresponding storage till the physical settlement. The manipulator submits another market order $\tilde{x_2}$ to the market in the second trade. Notice that the arguments in the functions are consistent with the manipulator's information set. For example, when choosing $\tilde{x_1}$, the manipulator can use different strategies based on the realization of $\tilde{H}$, capturing the idea that he knows the realization of $\tilde{H}$ before his choice of $\tilde{x_1}$. We will refer the manipulator receiving $\tilde{H} = h_0$ and $\tilde{H} = h_1$ as h_0-type manipulator and as h_1-type manipulator, respectively.

Third, the strategy choice of the competitive market makers consists of two functions, denoted $P_1 : \mathbf{R} \to \mathbf{R}$ and $P_2 : \mathbf{R^5} \to \mathbf{R}$. The clearing futures prices for the two trades chosen by the market makers are given by:

$$\begin{aligned} p_1 &= P_1(\tilde{x_1} + \tilde{h}) \\ p_2 &= P_2(\tilde{x_1} + \tilde{h}, \tilde{s}, \tilde{z}, \tilde{v}, \tilde{x_2} - \tilde{h}) \end{aligned}$$

After seeing the order imbalance $\tilde{y_1} := \tilde{x_1} + \tilde{h}$ in the first trade, the market makers set the clearing futures price p_1 for the first trade. The market makers observe only the order imbalance $\tilde{y_1}$ rather than $\tilde{x_1}$ and $\tilde{h}$ directly in setting p_1. For the second trade, the market makers set another clearing futures price p_2 after knowing $\tilde{y_1}$, $\tilde{s}$, $\tilde{z}$, and $\tilde{v}$ and observing the next order imbalance $\tilde{y_2} := \tilde{x_2} - \tilde{h}$. Notice that $\tilde{s}$ and $\tilde{z}$ are arguments of p_2, indicating that the market makers know whether there will be a squeeze after the physical settlement before setting p_2. Perhaps, the market makers contact storage providers to update the information before assigning the clearing futures price for the second trade.

We assume the manipulator is endowed with a plentiful commodity outside the settlement location so that $\tilde{s}$ can be any non-negative real number. Let $\tilde{\pi}(\tilde{X_1}, P_1, \tilde{S}, \tilde{X_2}, P_2)$ denotes the utility of the manipulator given a strategy profile $(\tilde{X_1}, P_1, \tilde{S}, \tilde{X_2}, P_2)$:

$$\tilde{\pi}(\tilde{X_1}, P_1, \tilde{S}, \tilde{X_2}, P_2) := \\ \tilde{X_1}(\tilde{v} - P_1) + \tilde{X_2}(\tilde{v} - P_2) - c\tilde{S} - l(\tilde{X_1} + \tilde{X_2} + \tilde{S})\mathbb{1}_{\tilde{z} \leq \tilde{S}} \tag{1}$$

To successfully engineer a market squeeze, the manipulator must transport more than $\tilde{z}$ units of the commodity to the settlement location. The last term in (1) captures changes in the utility due to market squeeze. Specifically, the manipulator experiences an extra loss if he owns the commodity at the delivery point after the settlement and an additional gain if he is required to deliver more than he has transported because he can procure a cheaper commodity at the settlement location than outside.

An equilibrium is defined as a strategy profile $(\tilde{X_1}, P_1, \tilde{S}, \tilde{X_2}, P_2)$ that satisfies the below three conditions:

First, for all $(\tilde{X_1}', \tilde{S}', \tilde{X_2}')$,

$$\mathbb{E}[\tilde{\pi}(\tilde{X_1}, P_1, \tilde{S}, \tilde{X_2}, P_2)] \geq \mathbb{E}[\tilde{\pi}(\tilde{X_1}', P_1, \tilde{S}', \tilde{X_2}', P_2)] \tag{2}$$

Second,

$$P_1 = \mathbb{E}[P_2 \mid \tilde{X_1} + \tilde{H}] \tag{3}$$

Third,

$$P_2 = \begin{cases} \tilde{v} - l, & \text{if } \tilde{z} \leq \tilde{s} \\ \tilde{v}, & \text{if } \tilde{z} > \tilde{s} \end{cases} \tag{4}$$

Condition (2) ensures that the manipulator does not strictly prefer to modify his strategy choice unilaterally in equilibrium. Essentially, both h_0-type and h_1-type manipulators weakly prefer to stay with the equilibrium strategy. Condition (4) well-defines P_2 such that the commodity at the settlement location has a price of $\tilde{v} - l$ if a market squeeze will occur and a price of $\tilde{v}$ otherwise. Let's first simplify the problem through backward induction.

Lemma 2.1. *Let* $\tilde{X_2} = -\tilde{x_1} - \tilde{s}$ *and* P_2 *be defined as (4). For all tuple* $(\tilde{X_1}, P_1, \tilde{S}, \tilde{X_2}')$*,* $\tilde{\pi}(\tilde{X_1}, P_1, \tilde{S}, \tilde{X_2}, P_2) \geq \tilde{\pi}(\tilde{X_1}, P_1, \tilde{S}, \tilde{X_2}', P_2)$*.*

Proof. Suppose the statement is false, then there exists a realization such that the above inequality does not hold. A generic realization of $\tilde{\pi}(\tilde{X_1}, P_1, \tilde{S}, \tilde{X_2}, P_2) - \tilde{\pi}(\tilde{X_1}, P_1, \tilde{S}, \tilde{X_2}', P_2)$ is $(-\tilde{x_1} - \tilde{s} - x_2')(\tilde{v} - P_2) + l(\tilde{x_1} + x_2' + \tilde{s})\mathbb{1}_{\tilde{z} \leq \tilde{s}}$. If $\tilde{z} > \tilde{s}$, the realization is zero. If $\tilde{z} \leq \tilde{s}$, the realization becomes $-l(\tilde{x_1} + \tilde{s} + x_2') + l(\tilde{x_1} + x_2' + \tilde{s})$, which is also zero. We have exhausted all possibilities, but no violation has been found. □

The manipulator cannot mislead the market makers in the second trade because the relevant information now is symmetric, and the competition among market makers guarantees that the manipulator will be treated fairly in the second trade. Therefore, the manipulator is indifferent to all choices of $\tilde{x_2}$. One best response for the manipulator is to submit the order such that the manipulator would deliver exactly $\tilde{s}$ units of the commodity at the physical settlement. By Lemma 2.1, we can simplify the problem by setting:

$$\tilde{X_2} = -\tilde{x_1} - \tilde{s} \tag{5}$$

The utility of the manipulator (1) becomes:

$$\tilde{\pi}(\tilde{X_1}, P_1, \tilde{S}) := \tilde{X_1}(\tilde{v} - P_1) - l(\tilde{X_1} + \tilde{S})\mathbb{1}_{\tilde{z} \leq \tilde{S}} - c\tilde{S} \tag{6}$$

We will discuss $\tilde{z}$ as a constant in this section and will consider $\tilde{z}$ as a continuous random variable in the next section. In the former, the math is less involved, allowing the readers to grasp the gist of the thesis and see the symmetric counterpart of the short squeeze described in Kyle (1984); in the latter, we can analyze the relationship between random storage supply and the equilibrium behavior.

Lemma 2.2. *Suppose $\tilde{z} = z$ almost surely and let $\tilde{S}$ be defined as the below (7), then for all tuple $(\tilde{X}_1, \tilde{S}', P_1)$, $\tilde{\pi}(\tilde{X}_1, \tilde{S}, P_1) \geq \tilde{\pi}(\tilde{X}_1, \tilde{S}', P_1)$ almost surely.*

$$\tilde{S} = \begin{cases} z, & \text{if } \tilde{x_1} \leq -\frac{z(c+l)}{l} \\ 0, & \text{if } \tilde{x_1} > -\frac{z(c+l)}{l} \end{cases} \tag{7}$$

Proof. A realization of (6) is $\tilde{x_1}(\tilde{v} - p_1) - l(\tilde{x_1} + \tilde{s})\mathbb{1}_{z \leq \tilde{s}} - c\tilde{s}$ almost surely, the partial derivative of which with respect to $\tilde{s}$ on the domain of $\mathbb{R}_+ \setminus \{z\}$ is strictly negative almost surely. Hence, in equilibrium $\tilde{s} \in \{0, z\}$ almost surely. The realization with $\tilde{s} = 0$ and with $\tilde{s} = z$ are respectively $\tilde{x_1}(\tilde{v} - p_1)$ and $\tilde{x_1}(\tilde{v} - p_1) - l(\tilde{x_1} + z) - cz$ almost surely. The difference between the two and the strictly negative partial derivative demonstrate that for all tuple $(\tilde{X}_1, \tilde{S}', P_1)$, $\tilde{\pi}(\tilde{X}_1, \tilde{S}, P_1) \geq \tilde{\pi}(\tilde{X}_1, \tilde{S}', P_1)$ almost surely. □

It is costly to move the commodity to the delivery location and store them till the physical settlement. The only purpose of choosing a non-zero $\tilde{s}$ is to squeeze the market. The optimal quantity to attain so is z. Any $\tilde{s} > z$ is an unnecessary waste to the manipulator. Furthermore, the above (7) suggests that a significant short position in the first trade induces the manipulator to squeeze the market in the subgame, transporting the underlying to the settlement location and dampening the futures price in the second trade. In terms of economic modeling, it is undesirable to see that the manipulator squeezes the market by transporting exact z units of the commodity. Such an issue disappears if $\tilde{z}$ is a continuous random variable, which will be discussed later in the next section.

We now show that market squeeze does not occur in equilibrium if the cost of engineering a market squeeze c is sufficiently high or the range of exogenous traders' trades $(h_1 - h_0)$ is sufficiently small, the latter associated with the revenue of deviation.

Theorem 2.3. *Suppose $\tilde{z} = z$ almost surely and $(h_1 - h_0)l - z(c + l) \leq 0$, then there exists an equilibrium such that no squeeze occurs and $p_1 = \mathbb{E}[\tilde{v}]$ almost surely.*

Proof. We claim that the following strategy profile (8), (9), (10), (11), and (12) constitute an equilibrium with the property claimed if $(h_1 - h_0)l - z(c + l) \leq 0$.

$$\tilde{X}_1 = 0 \tag{8}$$

$$P_1 = \begin{cases} \mathbb{E}[\tilde{v}] - l, & \text{if } \tilde{y_1} < h_0 \\ \mathbb{E}[\tilde{v}], & \text{if } \tilde{y_1} \geq h_0 \end{cases} \tag{9}$$

$$\tilde{S} = \begin{cases} z, & \text{if } \tilde{x_1} \leq -\frac{z(c+l)}{l} \\ 0, & \text{if } \tilde{x_1} > -\frac{z(c+l)}{l} \end{cases} \tag{10}$$

$$\tilde{X}_2 = -\tilde{x_1} - \tilde{s} \tag{11}$$

$$P_2 = \begin{cases} \tilde{v} - l, & \text{if } \tilde{z} \leq \tilde{s} \\ \tilde{v}, & \text{if } \tilde{z} > \tilde{s} \end{cases} \tag{12}$$

We verify that the strategy profile satisfies conditions (2), (3), and (4). Condition (4) is trivially true from (12). (8) and (10) imply that $\tilde{x_1} = 0$ almost surely and that $\tilde{y_1} \in \{h_0, h_1\}$ and $\tilde{s} = 0$ almost surely, so (9) fulfills condition (3). Lemma 2.1 and Lemma 2.2 simplifies condition (2): we only need to check whether (8) maximizes $\mathbb{E}[\tilde{\pi}(\tilde{X}_1, P_1, \tilde{S}, \tilde{X}_2, P_2)]$ given (9), (10), (11), and (12).

The h_0-type manipulator receives zero expected utility under the claimed equilibrium; for any $\tilde{x_1} > 0$, he would still receive zero expected utility; for any $\tilde{x_1} < 0$, he would receive negative expected utility due to the pricing scheme (9). On the other hand, the h_1-type manipulator receives zero expected utility under the claimed equilibrium; for any $\tilde{x_1} \geq -(h_1 - h_0)$, he would receive zero expected utility due to the assumption $(h_1 - h_0)l - z(c + l) \leq 0$; for any $\tilde{x_1} \leq -(h_1 - h_0)$, he would receive negative expected utility due to the pricing scheme (9). Hence, the strategy profile meets condition (2). Moreover, $p_1 = \mathbb{E}[\tilde{v}]$ almost surely due to (8) and (9), and no squeeze occurs almost surely because $\tilde{s} = 0$ almost surely. The assertion is proved. □

In the separating equilibrium described in Theorem 2.3, the equilibrium futures price $p_1 = \mathbb{E}[\tilde{v}]$ almost surely because market participants rationally expect that a market squeeze does not occur with probability one. To sustain the equilibrium, the h_1-type manipulator cannot strictly prefer to deviate and squeeze the market. (10) implies that the manipulator will squeeze the market in the subgame only if he has a sufficiently large short position from the first trade. The h_1-type manipulator can mimic h_0-type manipulator by submitting a market order $-(h_1 - h_0)$ to the trading floor, resulting in a large short position, nonetheless, not enough to induce h_1-type manipulator to squeeze the market under the assumption $(h_1 - h_0)l - z(c + l) \leq 0$. If h_1-type manipulator submits any market order less than $-(h_1 - h_0)$ to the trading floor, market makers will see $\tilde{y_1} < h_0$, believing that there will certainly be a market squeeze, and set a depressed futures price. Rationally expecting this, the manipulator has no taste in setting up a market squeeze in the first place. In equilibrium, both types of manipulators receive zero expected utility, and exogenous traders receive zero expected trading profits.

The sufficient condition $(h_1-h_0)l-z(c+l) \leq 0$ implies that the separating equilibrium can be sustained when z is high, c is high, or $(h_1 - h_0)$ is low. The intuition goes as follows. $\tilde{x_1}$ has to be low enough to induce a market squeeze. Still, a narrow $(h_1 - h_0)$ renders the market makers to observe $\tilde{y_1} < h_0$ and to set a depressed futures price if the manipulator indeed submits a low enough $\tilde{x_1}$ to induce himself to squeeze the market in the subgame. Rationally expecting this, the manipulator prefers to stick with the equilibrium strategy. On the other hand, high z or c make $\tilde{x_1}$ have to be even lower to induce a market squeeze in the subgame.

We now turn to the pooling equilibrium in which the market makers always see the same order imbalance in the first trade and can not acquire finer information on the realization of $\tilde{H}$ upon seeing the order imbalance $\tilde{y_1}$. In the pooling equilibrium, a market squeeze occurs with a probability λ: the h_1-type manipulator always squeezes the market while the h_0-type manipulator never squeezes the market. To sustain the pooling equilibrium, the cost of engineering a market squeeze, c and z, is sufficiently low, the range of exogenous traders' trades $(h_1 - h_0)$ is sufficiently high, or the prior probability of $\tilde{H} = h_1$ is sufficiently low, the latter two associated with the revenue of engineering a market squeeze. The challenge is that condition (3) is uniquely defined only up to events with strictly positive probability. Although we can assign values to P_1 for zero-probability events, the choice would affect (2). To sustain such an equilibrium, we need to prove the existence of P_1 that is compatible with conditions (2) and (3). It turns out that there is only one equilibrium order imbalance that can meet the conditions.

Theorem 2.4. *Suppose $\tilde{z} = z$ almost surely and $(1-\lambda)^2(h_1-h_0)l-z(c+l) \geq 0$, then there exists an equilibrium such that market squeeze occurs if and only if $\tilde{H} = h_1$ almost surely, $\tilde{y_1} = \lambda h_1 + (1 - \lambda)h_0$ almost surely, and $p_1 = \mathbb{E}[\tilde{v}] - \lambda l$ almost surely.*

Proof. Define $p_1^* := \mathbb{E}[\tilde{v}] - \lambda l$ and $y_1^* := \lambda h_1 + (1-\lambda)h_0$. We claim that the following strategy profile (13), (14), (15), (16), and (17) constitute an equilibrium with the property claimed if $(1-\lambda)^2(h_1-h_0)l - z(c+l) \geq 0$.

$$\tilde{X_1} = \begin{cases} (1-\lambda)(h_0-h_1), & \text{if } \tilde{H} = h_1 \\ \lambda(h_1-h_0), & \text{if } \tilde{H} = h_0 \end{cases} \tag{13}$$

$$P_1 = \begin{cases} \mathbb{E}[\tilde{v}], & \text{if } \tilde{y_1} > h_1 \\ \mathbb{E}[\tilde{v}] - l, & \text{if } \tilde{y_1} < h_0 \\ \min(\mathbb{E}[\tilde{v}], \max(\mathbb{E}[\tilde{v}] - l, p^* + \frac{l}{h_1-h_0}(y-y^*)), & \text{otherwise} \end{cases} \tag{14}$$

$$\tilde{S} = \begin{cases} z, & \text{if } \tilde{x_1} \leq -\frac{z(c+l)}{l} \\ 0, & \text{if } \tilde{x_1} > -\frac{z(c+l)}{l} \end{cases} \tag{15}$$

$$\tilde{X_2} = -\tilde{x_1} - \tilde{s} \tag{16}$$

$$P_2 = \begin{cases} \tilde{v} - l, & \text{if } \tilde{z} \leq \tilde{s} \\ \tilde{v}, & \text{if } \tilde{z} > \tilde{s} \end{cases} \tag{17}$$

We verify that the strategy profile satisfies conditions (2), (3), and (4). Condition (4) is trivially true from (17). (13), (15) and $(1-\lambda)^2(h_1-h_0)l - z(c+l) \geq 0$ imply that $\tilde{y_1} = y_1^*$ almost surely and $\tilde{s} \geq z$ if and only if $\tilde{H} = h_1$ almost surely, so (14) fulfills condition (3) as $P_1(\tilde{y_1} = y_1^*) = p_1^*$. Lemma 2.1 and Lemma 2.2 simplifies condition (2): we only need to check whether (13) maximizes $\mathbb{E}[\tilde{\pi}(\tilde{X_1}, P_1, \tilde{S}, \tilde{X_2}, P_2)]$ given (14), (15), (16), and (17). We further define the following to continue the verification process:

$$\begin{aligned}
&\pi_0(y,p) = (y-h_0)(\mathbb{E}[\tilde{v}] - p) \\
&\pi_0^* = \lambda^2(h_1-h_0)l \\
&I_0 = \{(y,p) \in (h_0,h_1) \times \mathbb{R} \mid \pi_0(y,p) > \pi_0^*\} \\
&\pi_1(y,p) = (y-h_1)(\mathbb{E}[\tilde{v}] - p) - [(y-h_1)l + z(c+l)]\mathbb{1}_{y \leq h_1 - \frac{z(c+l)}{l}} \\
&\pi_1^* = (1-\lambda)^2(h_1-h_0)l - z(c+l) \\
&I_1 = \{(y,p) \in (h_0,h_1) \times \mathbb{R} \mid \pi_1(y,p) > \pi_1^*\}
\end{aligned}$$

Notice that h_0-type and h_1-type manipulators weakly prefer to set $\tilde{x_1} \in [0, h_1-h_0]$ and $\tilde{x_1} \in [h_0-h_1, 0]$, respectively, due to the pricing scheme (14). Hence, we can contain our verification in the domain $\{(y,p) \mid (y,p) \in [h_0,h_1] \times \mathbb{R}\}$. $\pi_0(y,p)$ is the expected utility for h_0-type manipulator, π_0^* is the expected utility for the h_0-type manipulator at (y^*, p^*), and I_0 represents the collection of (y, p) pairs such that the h_0-type manipulator strictly prefers (y, p) to (y^*, p^*). On the other hand, $\pi_1(y, p)$ is the expected utility for h_1-type manipulator, π_1^* is the expected utility for the h_1-type manipulator at (y^*, p^*), and I_1 represent the collection of (y, p) pairs such that the h_1-type manipulator strictly prefers (y, p) to (y^*, p^*).

$\pi_0(y, p)$ is increasing in y and decreasing in p, while π_1(y,p) is decreasing in y and increasing in p. Hence, I_0 (I_1) is located at the lower-right (upper-left) corner on the (y, p)-plane. The slopes of the I_0's and I_1's boundaries, which are their indifference curves, in the (y, p)-plane are $\frac{\mathbb{E}[\tilde{v}]-p}{y-h_0}$ and $\frac{\mathbb{E}[\tilde{v}]-p-l\mathbb{1}_{y\leq h_1-\frac{z(c+l)}{l}}}{y-h_1}$, both evaluated at (y^*, p^*) being $\frac{l}{h_1-h_0}$. Moreover, the slope is decreasing for the I_0's boundary while increasing for the I_1's boundary except for the kink $y = h_1 - \frac{z(c+l)}{l}$. Notice that the boundary of I_1 becomes the horizontal line $p = \mathbb{E}[\tilde{v}] + k$ for some $k > 0$ when $y > h_1 - \frac{z(c+l)}{l}$. Hence, the kink is above $p = \mathbb{E}[\tilde{v}]$, and I_0 and I_1 do not intersect P_1 defined in (14). In other words, given the piece-wise linear pricing scheme (14), both types of manipulators weakly prefer their equilibrium strategy to all $\tilde{x_1} \in \mathbb{R}$. Hence, the strategy profile meets condition (2).

The order imbalance $\tilde{y_1} = y^*$ almost surely due to (13). From (13), (15), and $(1-\lambda)^2(h_1 - h_0)l - z(c + l) \geq 0$, a market squeeze occurs if and only if $\tilde{H} = h_1$ almost surely, and $p_1 = p_1^*$ almost surely due to (13) and (14). □

In the pooling equilibrium, the h_1-type manipulator sets $\tilde{x_1} = -(1 - \lambda)(h_1 - h_0) < 0$ and squeezes the market in the subgame, while h_0-type manipulator submits $\tilde{x_1} = \lambda(h_1 - h_0) > 0$ and does not squeeze the market. The order imbalance is always $\lambda h_1 + (1 - \lambda)h_0$ in equilibrium. The uncertainty of $\tilde{H}$ makes the pooling plausible. The market makers in the first trading period cannot distinguish the two types and set a futures price to reflect the uncertainty of a market squeeze in the subgame. Therefore, both types of manipulators earn a positive expected profit in the equilibrium. The exogenous traders receive zero expected trading profit due to the assumption of competitive market makers. Still, additional risk is created because of the market manipulation, discouraging optimal risk sharing among market participants.

The sufficient condition in Theorem 2.4 suggests that the pooling equilibrium exists if the event $\tilde{H} = h_1$ is an unexpected outlier, i.e., a small λ and a large $(h_1 - h_0)$, and if the cost to set up a market squeeze is mild, i.e. c and z are small. The sufficient condition is equivalent to the positive expected utility for the h_1-type manipulator in the pooling equilibrium.

There are uncountably many P_1 that can sustain the equilibrium, but all are tangent to both types' indifference curves at the equilibrium order imbalance and the equilibrium futures price. Less obviously, the equilibrium order imbalance must be $\lambda h_1 + (1-\lambda)h_0$; otherwise, there does not exist P_1 compatible with conditions (2) and (3).

In equilibrium, the futures price $p_1 = \mathbb{E}[\tilde{v}] - \lambda l$ almost surely, so both types of manipulator receive strictly positive expected utility. A greater $(h_1 - h_0)$ results in higher expected utility for both types, as both types hold a larger position in magnitude. However, an increase in λ boosts the expected utility for the h_0-type manipulator but deflates that for the h_1-type manipulator. This is because the equilibrium futures price becomes lower and the equilibrium order imbalance is closer to h_1. An increase in c and z lowers the expected utility for the h_1-type manipulator. After all, c and z raise the cost of engineering a market squeeze. Unconditional expected utility for the manipulator is:

$$\pi^* = \lambda[(1-\lambda)(h_1 - h_0)l - z(c+l)]$$

π^* is strictly positive because both types of manipulators receive strictly positive expected utility. It is easy to see that a greater $(h_1 - h_0)$, a smaller z, or a smaller c result in higher unconditional expected utility. Observe that λ is quadratic in π^*, and the λ that maximizes π^* is $\frac{(h_1-h_0)l-z(c+l)}{2(h_1-h_0)l}$, which is positive but less than $\frac{1}{2}$ if the sufficient condition in Theorem 2.4 holds. The intuition is that when λ is very small (large), there is a high probability of the $h_0(h_1)$-type manipulator appearing, but the equilibrium profit of that type is extremely low.

We have seen that a separating equilibrium prevails when $(h_1 - h_0)$ is sufficiently small and a pooling equilibrium takes over when $(h_1 - h_0)$ is sufficiently large. What happens when $(h_1 - h_0)$ is somewhere in between? The answer is partial pooling equilibrium in which the h_1-type manipulator randomizes between two market orders, one pooling with and the other separating from the h_0-type manipulator. Upon seeing the pooling order imbalance, the market makers are unsure whether there will be a market squeeze and set a futures price to reflect this uncertainty using an updated belief. In contrast, the market makers know $\tilde{H} = h_1$ when observing the separating order imbalance.

Theorem 2.5. *Suppose $\tilde{z} = z$ almost surely and $\frac{z(c+l)}{l} \leq h_1 - h_0 \leq \frac{z(c+l)}{l(1+\lambda)^2}$, then there exist an equilibrium and $\mu^* \in [0,1]$ such that market squeeze occurs with probability $\lambda\mu^*$, and it occurs only if $\tilde{H} = h_1$ almost surely.*

Proof. Define $\mu^* = \frac{1-\lambda}{\lambda}[\frac{(h_1-h_0)-z(c+l)}{(1-\lambda)(h_1-h_0)+z(c+l)}]$, $p_1^* := \mathbb{E}[\tilde{v}] - \frac{\lambda\mu^*}{(1-\lambda)+\lambda\mu^*}l$ and $y_1^* := \lambda\mu^* h_1 + (1-\lambda\mu^*)h_0$. Notice that $\mu^* \in [0,1]$ from the assumption $\frac{z(c+l)}{l} \leq h_1 - h_0 \leq \frac{z(c+l)}{l(1+\lambda)^2}$. We claim that the following strategy profile (18), (19), (20), (21), and (22) constitute an equilibrium with the property claimed if $\frac{z(c+l)}{l} \leq h_1 - h_0 \leq \frac{z(c+l)}{l(1+\lambda)^2}$.

$$\tilde{X}_1 = \begin{cases} \begin{cases} (1-\lambda\mu^*)(h_0-h_1), & \text{with probability } \mu^* \\ 0, & \text{with probability } 1-\mu^* \end{cases} & \text{if } \tilde{H} = h_1 \\ \lambda\mu^*(h_1-h_0), & \text{if } \tilde{H} = h_0 \end{cases} \tag{18}$$

$$P_1 = \begin{cases} \mathbb{E}[\tilde{v}], & \text{if } \tilde{y_1} > h_1 \\ \mathbb{E}[\tilde{v}] - l, & \text{if } \tilde{y_1} < h_0 \\ \min(\mathbb{E}[\tilde{v}], \max(\mathbb{E}[\tilde{v}] - l, p^* + \frac{l}{h_1-h_0}(y-y^*)), & \text{otherwise} \end{cases} \tag{19}$$

$$\tilde{S} = \begin{cases} z, & \text{if } \tilde{x_1} \leq -\frac{z(c+l)}{l} \\ 0, & \text{if } \tilde{x_1} > -\frac{z(c+l)}{l} \end{cases} \tag{20}$$

$$\tilde{X_2} = -\tilde{x_1} - \tilde{s} \tag{21}$$

$$P_2 = \begin{cases} \tilde{v} - l, & \text{if } \tilde{z} \leq \tilde{s} \\ \tilde{v}, & \text{if } \tilde{z} > \tilde{s} \end{cases} \tag{22}$$

We verify that the strategy profile satisfies conditions (2), (3), and (4). Condition (4) is trivially true from (22). (18), (20) and $\frac{z(c+l)}{l} \leq h_1 - h_0 \leq \frac{z(c+l)}{l(1+\lambda)^2}$ imply that $\tilde{s} \geq z$ if and only if $\tilde{y_1} = y^*$ and $\tilde{H} = h_1$ almost surely, so (19) fulfills condition (3) as $P_1(\tilde{y_1} = y_1^*) = p_1^*$. Lemma 2.1 and Lemma 2.2 simplifies condition (2): we only need to check whether (18) maximizes $\mathbb{E}[\tilde{\pi}(\tilde{X_1}, P_1, \tilde{S}, \tilde{X_2}, P_2)]$ given (19), (20), (21), and (22). We further define the following to continue the verification process:

$$\pi_0(y,p) = (y-h_0)(\mathbb{E}[\tilde{v}] - p)$$
$$\pi_0^* = \frac{(\lambda\mu^*)^2(h_1-h_0)l}{(1-\lambda)+\lambda\mu^*}$$
$$I_0 = \{(y,p) \in (h_0,h_1) \times \mathbb{R} \mid \pi_0(y,p) > \pi_0^*\}$$
$$\pi_1(y,p) = (y-h_1)(\mathbb{E}[\tilde{v}] - p) - [(y-h_1)l + z(c+l)]\mathbb{1}_{y \leq h_1 - \frac{z(c+l)}{l}}$$
$$\pi_1^* = 0$$
$$I_1 = \{(y,p) \in (h_0,h_1) \times \mathbb{R} \mid \pi_1(y,p) > \pi_1^*\}$$

Notice that h_0-type manipulator and h_1-type manipulator weakly prefer to set $\tilde{x_1} \in [0, h_1 - h_0]$ and $\tilde{x_1} \in [h_0 - h_1, 0]$, respectively, due to the pricing scheme (19). Hence, we can contain our verification in the domain $\{(y,p) \mid (y,p) \in [h_0,h_1]\times\mathbb{R}\}$. $\pi_0(y,p)$ is the expected utility for h_0-type manipulator, π_0^* is the expected utility for the h_0-type manipulator at (y^*,p^*), and I_0 represents the collection of (y,p) pairs such that the h_0-type manipulator strictly prefers (y,p) to (y^*,p^*). On the other hand, $\pi_1(y,p)$ is the expected utility for h_1-type manipulator, π_1^* is the expected utility for the h_1-type manipulator at (y^*,p^*),, and I_1 represent the collection of (y,p) pairs such that the h_1-type manipulator strictly prefers (y,p) to (y^*,p^*). Notice that $(h_1,\mathbb{E}[\tilde{v}])$ is on the I_1's boundary, consistent with the use of random strategy.

$\pi_0(y,p)$ is increasing in y and decreasing in p, while π_1(y,p) is decreasing in y and increasing in p. Hence, I_0 (I_1) is located at the lower-right (upper-left) corner on the (y,p)-plane. The slopes of the I_0's and I_1's boundaries in the (y,p)-plane are $\frac{\mathbb{E}[\tilde{v}]-p}{y-h_0}$ and $\frac{\mathbb{E}[\tilde{v}]-p-l\mathbb{1}_{y\leq h_1-\frac{z(c+l)}{l}}}{y-h_1}$, both evaluated at (y^*,p^*) being $\frac{l}{h_1-h_0}$. Moreover, the slope is decreasing for the I_0's boundary while increasing for the I_1's boundary except for the kink $y = h_1 - \frac{z(c+l)}{l}$. Notice that the boundary of I_1 becomes the horizontal line $p = \mathbb{E}[\tilde{v}]$ when $y > h_1 - \frac{z(c+l)}{l}$. Hence, the kink is on $p = \mathbb{E}[\tilde{v}]$, and both I_0 and I_1 do not intersect P_1 defined in (19). In other words, given the piece-wise linear pricing scheme (19), both types of manipulators weakly prefer their equilibrium strategy to all $\tilde{x_1} \in \mathbb{R}$. Hence, the strategy profile meets condition (2).

Given (18), (20), and $\frac{z(c+l)}{l} \leq h_1 - h_0 \leq \frac{z(c+l)}{l(1+\lambda)^2}$, market squeeze occurs with probability $\lambda\mu^*$, and it occurs only if $\tilde{H} = h_1$ almost surely. □

In the partial pooling equilibrium, the h_0-type manipulator always submits $\tilde{x_1} = \lambda\mu^*(h_1 - h_0)$ while the h_1-type randomizes between $\tilde{x_1} = 0$ and $\tilde{x_1} = -(1-\lambda\mu^*)(h_1-h_0)$ with the probability $1-\mu^*$ and μ^*, respectively. When the h_1-type manipulator submits $\tilde{x_1} = 0$, the market makers know that $\tilde{H} = h_1$ and that there will not be a market squeeze, setting the equilibrium price $p_1 = \mathbb{E}[v]$. On the other hand, when the market makers observe $\tilde{y_1} = \lambda\mu^* h_1 + (1-\lambda\mu^*)h_0$, they cannot tell whether this order imbalance is from h_1-type or from h_0-type manipulators and set the equilibrium price $p_1 = \mathbb{E}[v] - \frac{\lambda\mu^*}{(1-\lambda)+\lambda\mu^*}l$ to reflect the uncertainty using the updated belief. Hence, h_0-type manipulator expects a strictly positive utility. In contrast, the h_1-type manipulator expects zero utility because he must be indifferent between the two market orders. Indeed, μ^* is chosen so that h_1-type manipulator earns zero expected utility when setting $\tilde{x_1} = -(1-\lambda\mu^*)(h_1-h_0)$. As z increases, the probability of a market squeeze decreases because μ^* is decreasing in z. The exogenous traders receive zero expected trading profit due to the assumption of competitive market makers. Still, additional risk is created because of the market manipulation, discouraging the optimal risk sharing among market participants.

We can see a symmetric pattern between the long squeeze here and the short squeeze described in Kyle (1984). Type 1, type 2, and type 3 equilibrium in Kyle (1984) corresponds to Theorem 2.3, 2.5, and 2.4 here. Furthermore, as $(h_1 - h_0)$ increases, the equilibrium evolves from separating to pooling in both works.

3 Random Supply of Storage

In the previous section, we assume $\tilde{z}$ to be a constant, so the manipulator squeezes the market by transporting exactly z units of the commodity. This section considers $\tilde{z}$ as a continuous random variable. There will be two layers of uncertainty regarding a market squeeze: $\tilde{z}$ and $\tilde{s}$. The manipulator control only the latter and may be uncertain whether a market squeeze will occur in the subgame. Furthermore, $\tilde{z} \neq \tilde{s}$ almost surely, so a market squeeze always occurs with excess demand for the working storage at the settlement location. Similar results to the previous section hold, and moreover, the relationship is examined between equilibrium behavior and the location and scale of a continuous uniform random variable $\tilde{z}$.

We assume the manipulator strictly prefers not to trade when he expects to get zero utility. As discussed below, the assumption makes the conditions in Theorem 2.3 and Theorem 2.4 necessary and will be utilized in Theorem 3.3. Recall that in the partial pooling equilibrium, the h_1-type manipulator receives zero expected utility but submits non-zero market order occasionally. Hence, we will exclude partial pooling equilibrium and focus only on the separating and pooling equilibrium in this section. Let's first impose some regularity assumptions on the random variable $\tilde{z}$ for the sake of tractability. Denote the cumulative distribution function and probability density function of $\tilde{z}$ as $Pr[\tilde{z} \leq z]$ and $Pr[\tilde{z} = z]$, respectively.

Lemma 3.1. *Suppose random variable $\tilde{z}$ with a compact support $[z_d, z_u]$ has a differentiable, weakly decreasing probability density function on (z_d, z_u). Then there exist $x_1^* \leq x_1^{**}$ such that $\tilde{S}$ defined as the below (30) has the property that $\tilde{x_1} + s^* < 0$, $\tilde{S}$ is a function decreasing in $\tilde{x_1}$, and for all tuple $(\tilde{X_1}, \tilde{S}', P_1)$, $\mathbb{E}[\tilde{\pi}(\tilde{X_1}, \tilde{S}, P_1)] \geq \mathbb{E}[\tilde{\pi}(\tilde{X_1}, \tilde{S}', P_1)]$.*

$$\tilde{S} = \begin{cases} z_u, & \text{if } \tilde{x_1} \leq x_1^* \\ s^* \text{ uniquely defined by } Pr(\tilde{z} \leq s^*) + (\tilde{x_1} + s^*)Pr(\tilde{z} = s^*) + \frac{c}{l} = 0, & \text{if } x_1^* < \tilde{x_1} < x_1^{**} \\ 0, & \text{if } \tilde{x_1} \geq x_1^{**} \end{cases} \tag{23}$$

Proof. To maximize $\mathbb{E}[\tilde{\pi}(\tilde{X_1}, P_1, \tilde{S})] = \tilde{X_1}(\mathbb{E}[\tilde{v}] - P_1) - l(\tilde{X_1} + \tilde{S})\mathbb{P}[\tilde{z} \leq \tilde{S}] - c\tilde{S}$ with respect to $\tilde{S}$ given $\tilde{X_1}$ and P_1, we can assign an optimal positive real number to each realization $(\tilde{h}, \tilde{x_1}, p_1)$ since the measurable function $\tilde{S}$ is allowed to depend on $(\tilde{H}, \tilde{X_1}, P_1)$. That is, $\tilde{S}(\tilde{h}, \tilde{x_1}, p_1) \in argmax_{s \in \mathbb{R}^+} f(s) :=$

$\tilde{x_1}(\mathbb{E}[\tilde{v}] - \tilde{p_1}) - l(\tilde{x_1} + s)\mathbb{P}[\tilde{z} \leq s] - cs$. The hypothesis implies $f(s)$ is differentiable on $(0, \infty)$ and twice-differentiable on $(0, \infty) \setminus \{z_u, z_d\}$:

$$\frac{df(s)}{ds} = -l[Pr(\tilde{z} \leq s) + (\tilde{x_1} + s)Pr(\tilde{z} = s)] - c$$

$$\frac{d^2 f(s)}{ds^2} = -l[2Pr(\tilde{z} = s) + (\tilde{x_1} + s)\frac{dPr(\tilde{z} = s)}{ds}]$$

The first derivative implies that the optimal $\tilde{s} \notin (0, z_d) \cup (z_u, \infty)$ almost surely, and that if the optimal $\tilde{s} > 0$, then $\tilde{x_1} + \tilde{s} < 0$ almost surely. Hence, s^* satisfying the first-order condition must be in the support $[z_d, z_u]$. Notice that s^* is unique given $\tilde{x_1}$: suppose there are two critical points $s_2^* > s_1^* > 0$, then the following equation must hold since the first-order condition holds for both:

$$Pr(\tilde{z} \leq s_2^*) - Pr(\tilde{z} \leq s_1^*) = (\tilde{x_1} + s_1^*)Pr(\tilde{z} = s_1^*) - (\tilde{x_1} + s_2^*)Pr(\tilde{z} = s_2^*)$$

However, the left-hand side is positive, while the right-hand side is strictly negative, which is a contradiction.

Furthermore, the $s^* \in (z_d, z_u)$ is a local maximum because the second derivative is strictly negative. To find the global maximum, we only need to compare $s = s^*$ with $s = 0$ and $s = z_u$ since $s = z_d$ is always dominated by $s = 0$ because $c > 0$. Observe that s^* is decreasing in $\tilde{x_1}$: set the first derivative equal to zero and differentiate both sides of the equation with respect to $\tilde{x_1}$ by considering s^* as a function of $\tilde{x_1}$, arriving at the first derivative of s^* with respect to $\tilde{x_1}$:

$$\frac{ds^*(\tilde{x_1})}{d\tilde{x_1}} = -\frac{Pr(\tilde{z} = s^*(\tilde{x_1}))}{2Pr(\tilde{z} = s^*(\tilde{x_1})) + (\tilde{x_1} + s^*(\tilde{x_1}))\frac{dPr(\tilde{z} = s^*(\tilde{x_1}))}{ds}} \leq 0$$

We are now ready to show the existence of x_1^* and x_1^{**}. The first derivative becomes strictly positive if $\tilde{x_1}$ is extremely negative and $s \in (z_d, z_u)$. Hence, $s = z_u$ dominates $s \in (z_d, z_u)$ in this case. Moreover, $s = z_u$ also dominates $s = 0$ if and only if $l(\tilde{x_1} + z_u) + cz_u \leq 0$. Therefore there exists x_1^* claimed in the lemma by the intermediate value theorem.

It's obvious to see that if given some $\tilde{x_1}$, the manipulator prefers $s = s^*(\tilde{x_1})$ to $s = 0$, then given any smaller $\tilde{x_1}' > x_1^*$, the manipulator would still prefer $s = s^*(\tilde{x_1}')$ to $s = 0$ because the partial derivative of $-l(\tilde{x_1} + s)\mathbb{P}[\tilde{Z} \leq s] - cs$ with respect to $\tilde{x_1}$ is negative, and the local maximum $s^*(\tilde{x_1}')$ leads to even greater expected utility than the expected utility with the fixed $s^*(\tilde{x_1})$.

Observe that $s = 0$ dominates all choices when $\tilde{x_1} = 0$. Hence, if there exists some $\tilde{x_1}$ such that $s = s^*(\tilde{x_1})$ strictly dominates both $s = 0$ and $s = z_d$, then there exists $x_1^* < x_1^{**}$ claimed in the lemma by the intermediate value theorem. On the other hand, if there does not exist $\tilde{x_1}$ such that $s^*(\tilde{x_1})$ strictly dominates both $s = 0$ and $s = z_d$ then $x_1^* = x_1^{**}$. $\square$

The structure in Lemma 3.1 is similar to that of Lemma 2.2: if the manipulator shorts a large position in the first trade, he is more likely to transport a great amount of commodity to the settlement location in order to raise the probability of a market squeeze, despite a higher cost of doing so. In choosing a greater $\tilde{s}$, the manipulator faces a trade-off: a gain from the marginal probability of a market squeeze and a loss associated with setting up a market squeeze, including c and a decrease in the established short position. The first-order condition holds when $\tilde{x_1} \in (x_1^*, x_1^{**})$, reflecting the optimal trade-off. Notice that $\tilde{S}$ is continuous at $\tilde{x_1} = x_1^*$ but jumps at $\tilde{x_1} = x_1^{**}$, the latter is due to zero density between $[0, z_d)$. We now consider the separating equilibrium without a market squeeze, as in the case of Theorem 2.3.

Theorem 3.2. *Suppose random variable $\tilde{z}$ with a compact support $[z_d, z_u]$ has a differentiable, weakly decreasing probability density function on (z_d, z_u). If $(h_1 - h_0) \leq -x_1^{**}$, then there exists an equilibrium such that no squeeze occurs and $p_1 = \mathbb{E}[\tilde{v}]$ almost surely.*

Proof. We claim that the following strategy profile (24), (25), (26), (27), and (28) constitute an equilibrium with the property claimed if $(h_1 - h_0) \leq -x_1^{**}$.

$$\tilde{X_1} = 0 \tag{24}$$

$$P_1 = \begin{cases} \mathbb{E}[\tilde{v}] - l, & \text{if } \tilde{y_1} < h_0 \\ \mathbb{E}[\tilde{v}], & \text{if } \tilde{y_1} \geq h_0 \end{cases} \tag{25}$$

$$\tilde{S} = \begin{cases} z_u, & \text{if } \tilde{x_1} \leq x_1^* \\ s^* \text{ uniquely defined by } Pr(\tilde{Z} \leq s^*) + (\tilde{x_1} + s^*)Pr(\tilde{Z} = s^*) + \frac{c}{l} = 0, & \text{if } x_1^* < \tilde{x_1} < x_1^{**} \\ 0, & \text{if } \tilde{x_1} \geq x_1^{**} \end{cases} \tag{26}$$

$$\tilde{X_2} = -\tilde{x_1} - \tilde{s} \tag{27}$$

$$P_2 = \begin{cases} \tilde{v} - l, & \text{if } \tilde{z} \leq \tilde{s} \\ \tilde{v}, & \text{if } \tilde{z} > \tilde{s} \end{cases} \tag{28}$$

We verify that the strategy profile satisfies conditions (2), (3), and (4). Condition (4) is trivially true from (28). (24) and (26) imply that $\tilde{x_1} = 0$ almost surely and that $\tilde{y_1} \in \{h_0, h_1\}$ and $\tilde{s} = 0$ almost surely, so (25) fulfills condition (3). Lemma 2.1 and Lemma 3.1 simplifies condition (2): we only need to check whether (24) maximizes $\mathbb{E}[\tilde{\pi}(\tilde{X_1}, P_1, \tilde{S}, \tilde{X_2}, P_2)]$ given (25), (26), (27), and (28). The h_0-type manipulator receives zero expected utility under the claimed equilibrium; for any $\tilde{x_1} > 0$, he would still receive zero expected utility; for any $\tilde{x_1} < 0$, he would receive negative expected utility due to the pricing scheme (25). On the other hand, the h_1-type manipulator receives zero expected utility under the claimed equilibrium; for any $\tilde{x_1} \geq -(h_1 - h_0)$, he would receive zero expected utility due to the assumption $(h_1 - h_0) \leq -x_1^{**}$; for any $\tilde{x_1} \leq -(h_1 - h_0)$, he would receive negative expected utility due to the pricing scheme 259). Hence, the strategy profile meets condition (2). Moreover, $p_1 = \mathbb{E}[\tilde{v}]$ almost surely due to (24) and (25), and no squeeze occurs almost surely because $\tilde{s} = 0$ almost surely. The assertion is proved. □

This result is almost identical to Theorem 2.3. Simply put, $(h_1 - h_0)$ is not wide enough to allow the h_1-type manipulator to build a sufficiently large short position in the first trade, so no market squeeze occurs in the equilibrium. In fact, the condition $(h_1 - h_0) \leq -x_1^{**}$ is also necessary: the market makers know the realization of $\tilde{H}$ in the separating equilibrium, so both types of manipulator strictly prefer to set $\tilde{x_1} = 0$, and the condition is required to prevent h_1-type manipulator from deviation. Theorem 3.2 holds for a broader class of random variable $\tilde{z}$ as long as an optimal $\tilde{S}$ in the subgame has the form depicted in (23).

We now turn to the pooling equilibrium described in Theorem 2.4. Unfortunately, we do not obtain any interesting existence results like Theorem 2.4, mainly because the expected utility of h_1-type manipulator with respect to $\tilde{y_1}$ and p_1 is not quasi-concave here. However, the pooling equilibrium still exists if $(h_1 - h_0)$ is sufficiently large. Instead, we will present relevant necessary conditions for the pooling equilibrium, allowing us to examine in the last theorem the relationship between equilibrium behavior and a continuous uniform random variable $\tilde{z}$.

Theorem 3.3. *Suppose random variable $\tilde{z}$ with a compact support $[z_d, z_u]$ has a differentiable, weakly decreasing probability density function on (z_d, z_u). In the pooling equilibrium where the market makers always observe the same order imbalance and $\tilde{S}$, $\tilde{X_2}$ and P_2 are defined respectively as (26), (27), and (28), then $\tilde{y_1} = \lambda h_1 + (1-\lambda)h_0$ almost surely, and $p_1 = \mathbb{E}[\tilde{v}] - \lambda l Pr[\tilde{z} \leq \tilde{S}(\tilde{x_1} = (1-\lambda)(h_0 - h_1)]$ almost surely.*

Proof. At least one type of manipulator must earn a strictly positive expected utility in the pooling equilibrium; otherwise, both types will set $\tilde{x_1} = 0$, which is a contradiction because $h_1 > h_0$. Thus, market squeeze must occur with strictly positive probability in equilibrium, suggesting that equilibrium futures price strictly less than $\mathbb{E}[v]$.

Observe that equilibrium order imbalance $y^* \geq h_0$ because, otherwise, h_0-type manipulator would receive strictly negative expected profit. By $y^* \geq h_0$ and (26), h_0-type manipulator set $\tilde{s} = 0$ almost surely and h_1-type manipulator always chooses $\tilde{s} > 0$ attempting to squeeze the market. Rationally expecting this, the market makers set the futures price $p_1 = \mathbb{E}[\tilde{v}] - \lambda l Pr[\tilde{z} \leq \tilde{S}(\tilde{x_1} = y^* - h_1]$. Condition (2) implies that the h_0-type manipulator's indifference curve is tangent to the h_1-type manipulator's indifference curve on the (y, p)-plane at the equilibrium order imbalance y^* and equilibrium futures price p^*. The following equation must hold:

$$\frac{\lambda Pr[\tilde{z} \leq \tilde{S}(\tilde{x_1} = y^* - h_1)]l}{y^* - h_0} = \frac{(\lambda - 1)Pr[\tilde{z} \leq \tilde{S}(\tilde{x_1} = y^* - h_1)]l}{y^* - h_1}$$

The left (right) hand side is the slope of the $h_0(h_1)$-type manipulator's indifferent curve at (y^*, p^*). The equation implies $y^* = \lambda h_1 + (1-\lambda)h_0$. In other words, there does not exist P_1 satisfying (2) and (3) if $y^* \neq \lambda h_1 + (1-\lambda)h_0$. Hence, $p_1 = \mathbb{E}[\tilde{v}] - \lambda l Pr[\tilde{z} \leq \tilde{S}(\tilde{x_1} = (1-\lambda)(h_0 - h_1)]$ almost surely. □

In the pooling equilibrium described in Theorem 3.3, we can first compute the equilibrium order imbalance and then the equilibrium futures price, simplifying the process of analyzing the pooling equilibrium, as opposed to working with a complex system of entangled equations. After setting $\tilde{x_1} = -(1-\lambda)(h_1 - h_0)$ in the first trade, the h_1-type manipulator transports strictly positive units of the commodity to the settlement location, causing a market squeeze to occur with strictly positive probability. Rationally expecting this, the market makers set the equilibrium futures price strictly lower than $\mathbb{E}[\tilde{v}]$. Notice that the model degenerates into the case in the previous section when $(h_1 - h_0)$ is extremely large. More specifically, the h_1 type manipulator would set $\tilde{s} = z_u$ almost surely in equilibrium if $(h_1 - h_0) > -\frac{x^{**}}{1-\lambda}$. In general, the pooling equilibrium still exists if $(h_1 - h_0)$ is sufficiently large.

Let's now examine how the location and scale parameters of a uniform random variable $\tilde{z}$ affect the pooling equilibrium futures price and its order imbalance in the interesting case where $\tilde{S} \in (z_d, z_u)$ almost surely if $\tilde{H} = h_1$. The uniform random variable is parameterized with the support $[\mathbb{E}[\tilde{z}] - \sigma, \mathbb{E}[\tilde{z}] + \sigma]$, where $\mathbb{E}[\tilde{z}]$ and σ are proxies for the location and scale parameters. "Local" in the following theorem means the property holds under the assumption that the interesting case persists after the change.

Theorem 3.4. *Suppose random variable $\tilde{z}$ is uniform on the support $[\mathbb{E}[\tilde{z}]-\sigma, \mathbb{E}[\tilde{z}]+\sigma]$. In the pooling equilibrium described in Theorem 3.3 and $\tilde{S} \in (\mathbb{E}[\tilde{z}]-\sigma, \mathbb{E}[\tilde{z}]+\sigma)$ almost surely if $\tilde{H} = h_1$, then the following statements hold locally:*

- *an increase (decrease) in $\mathbb{E}[z]$ makes the pooling equilibrium order imbalance unchanged and its futures price higher (lower)*
- *an increase (decrease) in σ makes the pooling equilibrium order imbalance unchanged and its futures price higher (lower) if $(1-\lambda)(h_1-h_0) \geq \mathbb{E}[z]$*
- *an increase (decrease) in σ makes the pooling equilibrium order imbalance unchanged and its futures price lower (higher) if $(1-\lambda)(h_1-h_0) < \mathbb{E}[z]$*

Proof. From Theorem 3.3, the pooling equilibrium order imbalance $y^* = \lambda h_1 + (1-\lambda)h_0$ regardless of the random variable $\tilde{z}$'s characteristics. However, they change $\tilde{S}$ defined in (26) and the probability measure of $\tilde{z}$, both varying the equilibrium futures price. (26) together with uniform random variable $\tilde{z}$ gives:

$$\tilde{S} = \begin{cases} z_u, & \text{if } \tilde{x_1} \leq x_1^* \\ -\frac{\tilde{x_1}}{2} + \frac{\mathbb{E}[z]}{2} - \frac{2c+l}{2l}\sigma, & \text{if } x_1^* < \tilde{x_1} < x_1^{**} \\ 0, & \text{if } \tilde{x_1} \geq x_1^{**} \end{cases}$$

In the equilibrium, the h_1-type manipulator sets $\tilde{x_1} = -(1-\lambda)(h_1-h_0)$ almost surely. The hypothesis suggests that $\tilde{x_1} \in (x_1^*, x_1^{**})$ and $\tilde{s} = -\frac{\tilde{x_1}}{2} + \frac{\mathbb{E}[z]}{2} - \frac{2c+l}{2l}\sigma$ almost surely for the h_1 type manipulator. Hence, $Pr[\tilde{z} \leq \tilde{S}(\tilde{x_1} = (1-\lambda)(h_0-h_1))] = \frac{(1-\lambda)(h_1-h_0)-\mathbb{E}[z]}{4\sigma} + \frac{l-2c}{4l}$, the derivative of which with respect to σ is $\frac{4(\mathbb{E}[z]-(1-\lambda)(h_1-h_0))}{16\sigma^2}$. Therefore, the equilibrium futures price increases (decreases) as $\mathbb{E}[z]$ increases (decreases); the equilibrium futures price increases (decreases) as σ increase (decrease) if $(1-\lambda)(h_1-h_0) \geq \mathbb{E}[z]$; the equilibrium futures price increases (decreases) as σ decreases (increases) if $(1-\lambda)(h_1-h_0) < \mathbb{E}[z]$. ☐

A significant change in location or scale can vary x^* and x^{**} to the extent that $\tilde{S} \notin (z_d, z_u)$ almost surely in equilibrium. Regardless of the random variable's property, the equilibrium order imbalance must be $\lambda h_1+(1-\lambda)h_0$. An increase in location induces a greater $\tilde{S}$; however, not enough to offset the impact of the density shift to the right, resulting in a higher futures price.

An increase in scale makes the density lower but more dispersed, with the location unchanged. From the perspective of the h_1-type manipulator, the cost due to a decrease in the established short position outweighs the gain from the marginal probability of a market squeeze if he sticks to the original strategy. Hence, he would adjust $\tilde{s}$ downward, leading to a lower probability of a market squeeze if the probability measure is fixed. Another effect is that the cumulative distribution function transforms non-linearly: for all $z < \mathbb{E}[z]$, $Pr[\tilde{z} \leq z]$ becomes greater; for any $z > \mathbb{E}[z]$, $Pr[\tilde{z} \leq z]$ becomes smaller. When (h_1-h_0) is significantly large, the h_1-type manipulator would short a substantial position in the initial trade and transport a significant amount of the commodity to the settlement location. In this case, the two effects would have the same direction. However, if $(1-\lambda)(h_1-h_0) < \mathbb{E}[z]$, the non-linear transformation dominates.

4 Conclusion and Regulatory Implications

The options on space and on time characterize the commodities from other financial markets. By incorporating logistics and storage into our long squeeze model, we expand upon the work of Kyle (1984) to better capture the occasional crashes in futures prices observed in commodity markets. Privately knowing the exogenous trading activity, the manipulator can profit by manipulating the futures market through the camouflage offered by the uncertainty of the exogenous trading activity and transporting a significant amount of the commodity to stockpile the storage at the delivery point to create a long squeeze. In the pooling equilibrium, the manipulator profits in two ways: if the exogenous trades are low, he goes long at a deflated price; on the other hand, if the exogenous trades are high, he goes short at an inflated price and transports a significant amount of commodity to the delivery location to squeeze the market. A symmetric pattern is observed between the long squeeze in our paper and the short squeeze discussed in Kyle (1984). Specifically, as the range of exogenous trades increases, the equilibrium shifts from separating to pooling in both works. Furthermore, we examine the local relationship between random storage supply and the pooling equilibrium. Our analysis reveals that the equilibrium order imbalance remains constant while the futures price increases with the location parameter. Additionally, when the range of exogenous trades is (not) sufficiently large, an increase in the scale parameter leads to an increase (decrease) in the futures price.

To prevent a long squeeze in the commodity market, we propose three potential solutions. The first and most obvious solution is to increase the storage capacity at the delivery point. This not only facilitates the separating equilibrium but also decreases the probability of a market squeeze in the semi-pooling and pooling equilibrium. The second solution is to increase the outflow logistics capacity while decreasing the inflow logistics capacity. In our model, the former decreases l, and the latter increases c, both of which facilitate the separating equilibrium. The third solution is to impose well-designed position limits for all market participants except for hedgers. By setting limits at $-x_1^{**}$, the separating equilibrium is ensured, as per Theorem 3.2.

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