Homework "Signal theory: Part 1"

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1 Introduction

Homework

reading assignment

- notes Leo part 1, sections 1.1–4
- if needed, follow MATLAB tutorials

2 Signals and systems

Homework

reading assignment

- section 1.1 (classification of signals), 1.2 (classification of systems), and chapter 2 (representation of signals and systems) from A. Oppenheim and A. Willsky, *Signals and Systems*, Prentice Hall, 1996

problems

- Periodicity in discrete-time When is $a\cos(\omega t + \phi)$, $t \in \mathbb{Z}$ periodic?

  Is $a\cos(\omega t + \phi)$, $t \in \mathbb{Z}$ periodic when $\phi = 0$ and 1) $\omega = 2\pi/12$, 2) $\omega = 8\pi/31$, 3) $\omega = 1/6$?

- Relation between impulse and step functions Find relations between the impulse δ and step s functions.
- System classification Give specific examples of:
  - linear static system
  - nonlinear static system
  - linear time-invariant dynamical systems
    - finite impulse response (FIR)
    - infinite impulse response (IIR)
    - scalar
    - multivariable
  - linear time-varying dynamical systems
  - nonlinear time-invariant dynamical systems
  - nonlinear time-varying dynamical systems
- Peak and RMS values

  Find the peak and RMS values of $x(t) := a\cos(\omega t + \phi)$, for $t \geq 0$.

- Response of 1st and 2nd order LTI system

  Find analytically the response of 1st and 2nd order linear time-invariant autonomous systems.

- Multiple poles

  Consider the autonomous system represented by a difference equation

  $$y(t+2) - 2ay(t+1) + a^2y(t) = 0.$$

  (Its characteristic polynomial has both roots equal to $\lambda = a$.)

  1. Show that both $y(t) = a^t$ and $y(t) = ta^t$ are solutions.
  2. Find the trajectory generated from the initial conditions $y(0) = 1$ and $y(1) = 0$.

- $(A,B,C,D) \mapsto$ impulse response

  Find the impulse response of the linear time-invariant system

  $$\mathscr{B}(A,B,C,D) := \{\, (u,y) \mid \text{there is } x \text{, such that } \sigma x = Ax + Bu,\ y = Cx + Bu \,\}.$$

- $(A,B,C,D) \mapsto$ transfer function

  Find the transfer function of the linear time-invariant system $\mathscr{B}(A,B,C,D)$.

3 Representations of LTI systems

Homework

additional reading

Chapters 1 (behavioral models) and 4 (state-space representation) from

`http://wwwhome.math.utwente.nl/~poldermanjw/onderwijs/DISC/mathmod/book.pdf`

problems

- Matrix representation of the convolution operation

  Find a matrix representation of the discrete-time convolution operation.

- Matrix representation of the discrete Fourier transform

  Find a matrix representation of the discrete Fourier transform.

- Prediction using a model (seperate document "exercise autonomous models")

4 Stochastic models

Homework

reading assignment

- notes Leo part 1 sections 1.5–1.8 and notes Leo part 2

problems

- Wiener-Khintchine theorem

  For a discrete-time signal y, let

  - $\phi_y := |F(y)|^2$, where $F(y)$ be a Fourier transform of y, and
  - $r_y := \sum_{t=1}^{T} y(t)y(t-\tau)$.

  Show that $\phi_y = F(r_y)$.

5 Least-squares estimation

Homework

reading assignment

- notes Leo part 3, sections 3.1–3.3

problems

- Orthogonality principle for least-squares estimation

  Show that

  1. $\widehat{x}$ being a least squares approximate solution of the system $Ax = b$, and
  2. $\widehat{x}$ being such that $b - A\widehat{x}$ is orthogonal to the span of the columns of A,

  are equivalent. (This result is known as the orthogonality principle for least squares approximation.)

- Weighted least-squares approximate solution

  For a given positive definite matrix $W \in \mathbb{R}^{m\times m}$, define the weighted 2-norm

  $$\|e\|_W = e^\top We.$$

  The weighted least-squares approximation problem is

  $$\text{minimize} \quad \text{over } \widehat{x} \in \mathbb{R}^n \quad \|A\widehat{x} - b\|_W. \tag{WLS}$$

  When does a solution exist and when is it unique? Under the assumptions of existence and uniqueness, derive a closed form expression for the least squares approximate solution.