Inverse

- `inv(A)` returns the inverse matrix A^{-1}
- Julia will issue an error if
  - A is not square
  - A is not invertible
- you can solve square set of linear equations $Ax = b$, with invertible A, using

```
b = rand(5,1)
A = rand(5,5)
x = inv(A)*b
norm(A*x-b)  # check residual
```

but there is a better way, using backslash

Pseudo-inverse

- for a $m \times n$ matrix A, `pinv(A)` will return the $n \times m$ pseudo-inverse
- if A is square and invertible
  - `pinv(A)` will return the inverse A^{-1}
- if A is tall with linearly independent columns
  - `pinv(A)` will return the left inverse $(A^T A)^{-1} A^T$
- if A is wide with linearly independent rows
  - `pinv(A)` will return the right inverse $A^T (A A^T)^{-1}$
- in other cases, `pinv(A)` returns an $m \times n$ matrix, but
  - it is not a left or right inverse of A
  - what it is is beyond the scope of this class

The backslash operator

- given A and b, the \ operator solves the linear system $Ax = b$ for x
- for a $m \times n$ matrix A and a m-vector b, `A\b` returns a n-vector x
- if A is square and invertible
  - $x = A^{-1}b$
  - the unique solution of $Ax = b$
- if A is tall with linearly independent columns
  - $x = (A^T A)^{-1} A^T b$
  - the least squares approximate solution of $Ax = b$
- if A is wide with linearly independent rows
  - $x = A^T (AA^T)^{-1} b$
  - x is the least norm solution of $Ax = b$
- in other cases, `A\b` returns an n-vector x, but what it means is beyond the scope of this class
- uses a factor and solve method similar to QR

Solving matrix systems with backslash

- solve matrix equation $AX = B$ for X, with A square
- with $X = [x_1 \cdots x_k]$, $B = [b_1 \cdots b_k]$, same as solving k linear systems

$$Ax_1 = b_1, \ldots, Ax_k = b_k$$

- `X = A\B` solves the system, doing the right thing:
  - factor A once (order n^3)
  - back substitution to get $x_i = A^{-1}b_i$, $i = 1, \ldots, k$ (order kn^2)