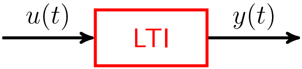
Vrije Universiteit Brussel

Block-oriented modeling

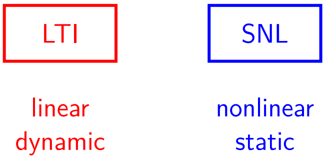
Maarten Schoukens Koen Tiels

Introduction

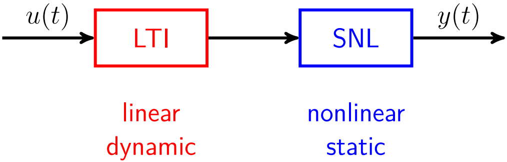


linear
dynamic

Block-oriented models consist of linear dynamics and static nonlinearities

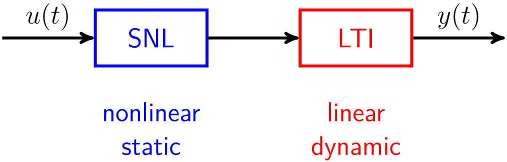


Block-oriented models consist of linear dynamics and static nonlinearities



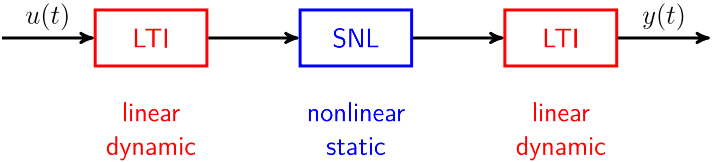
- Wiener

Block-oriented models consist of linear dynamics and static nonlinearities



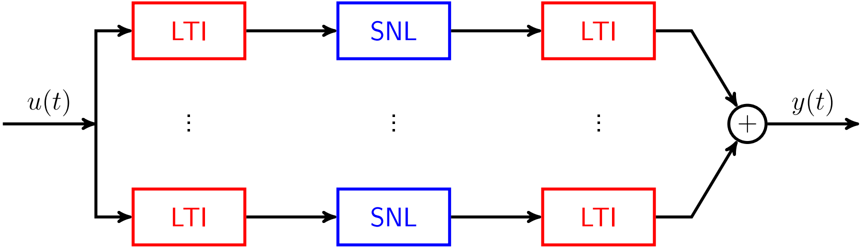
- Wiener
- Hammerstein

Block-oriented models consist of linear dynamics and static nonlinearities



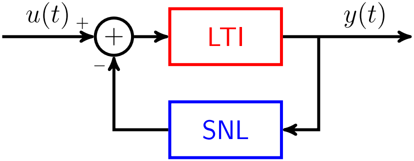
- Wiener
- Hammerstein
- Wiener-Hammerstein

Block-oriented models consist of linear dynamics and static nonlinearities



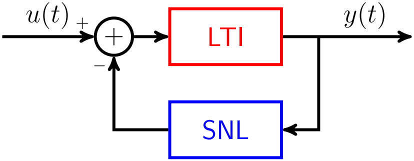
- Wiener
- Hammerstein
- Wiener-Hammerstein
- parallel Wiener-Hammerstein

Block-oriented models consist of linear dynamics and static nonlinearities



- Wiener
- Hammerstein
- Wiener-Hammerstein
- parallel Wiener-Hammerstein
- nonlinear feedback

Block-oriented models consist of linear dynamics and static nonlinearities

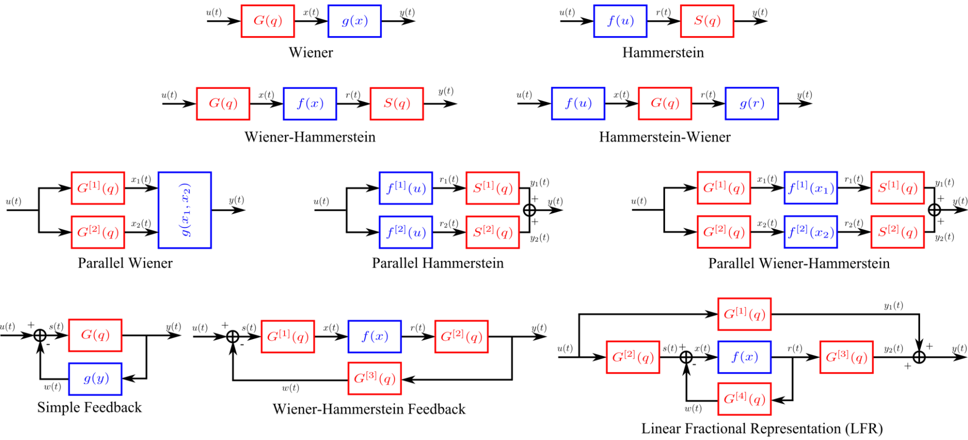


- Wiener
- Hammerstein
- Wiener-Hammerstein
- parallel Wiener-Hammerstein
- nonlinear feedback
- ...

Which model to choose?

Outline

- Which block structure to choose?
- How to identify the chosen block structure?



Wiener
Hammerstein
Wiener-Hammerstein
Hammerstein-Wiener
Parallel Wiener
Parallel Hammerstein
Parallel Wiener-Hammerstein
Simple Feedback
Wiener-Hammerstein Feedback
Linear Fractional Representation (LFR)

Structure detection

- Bussgang's theorem
- ε – approximation
- Structure detection

Bussgang's Theorem

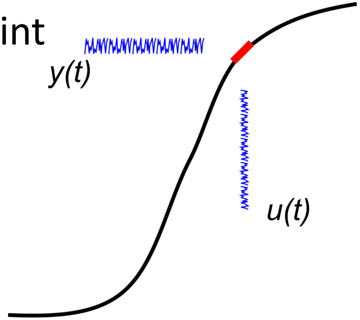
Stationary Gaussian input

→ Static nonlinearity ≈ static gain

$$\boxed{f(u) = \gamma u}$$

ε - Approximation

Small signal around a setpoint



Taylor approximation

→Static nonlinearity ≈ static gain

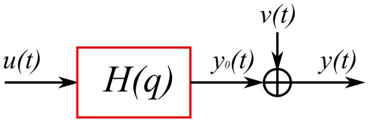
$$\boxed{f(u) = \gamma u}$$

Structure detection

- BLA, ε - Approximation @ different setpoints
  - Change offset
  - Change power spectrum

Structure detection

- Linear-Time-Invariant (LTI)



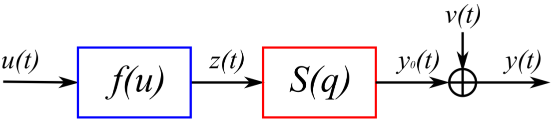
$$\boxed{G_{bla}(q) = H(q)}$$

➔ No changes

	Gain	Poles	Zeros
Linear Time Invariant	Fixed	Fixed	Fixed
Hammerstein			
Wiener			
Wiener-Hammerstein			
Parallel WH			
Feedback			
LFR			

Structure detection

- Hammerstein



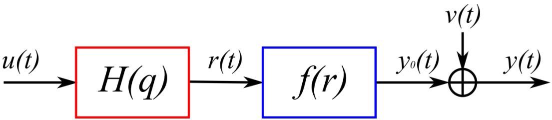
$$G_{bla}(q) = \gamma S(q)$$

➔ Only gain factor

	Gain	Poles	Zeros
Linear Time Invariant	Fixed	Fixed	Fixed
Hammerstein	Variable	Fixed	Fixed
Wiener			
Wiener-Hammerstein			
Parallel WH			
Feedback			
LFR			

Structure detection

- Wiener



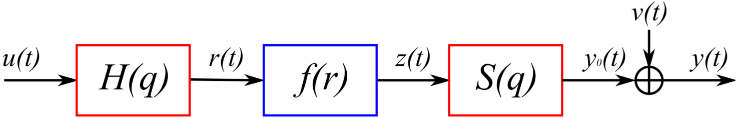
$$\boxed{G_{bla}(q) = \gamma H(q)}$$

➔ Only gain factor

	Gain	Poles	Zeros
Linear Time Invariant	Fixed	Fixed	Fixed
Hammerstein	Variable	Fixed	Fixed
Wiener	Variable	Fixed	Fixed
Wiener-Hammerstein			
Parallel WH			
Feedback			
LFR			

Structure detection

- Wiener-Hammerstein



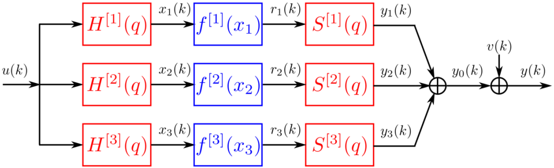
$$G_{bla}(q) = \gamma H(q)S(q)$$

➔ Only gain factor

	Gain	Poles	Zeros
Linear Time Invariant	Fixed	Fixed	Fixed
Hammerstein	Variable	Fixed	Fixed
Wiener	Variable	Fixed	Fixed
Wiener-Hammerstein	Variable	Fixed	Fixed
Parallel WH			
Feedback			
LFR			

Structure detection

- Parallel Wiener-Hammerstein



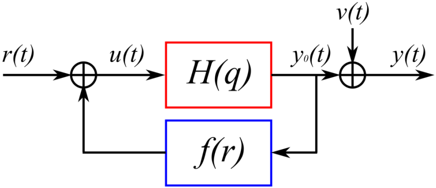
$$\boxed{G_{bla}(q) = \sum_i \gamma_i H^{[i]}(q) S^{[i]}(q)}$$

➔ Moving zeros, fixed poles

	Gain	Poles	Zeros
Linear Time Invariant	Fixed	Fixed	Fixed
Hammerstein	Variable	Fixed	Fixed
Wiener	Variable	Fixed	Fixed
Wiener-Hammerstein	Variable	Fixed	Fixed
Parallel WH	Variable	Fixed	Variable
Feedback			
LFR			

Structure detection

- Feedback system



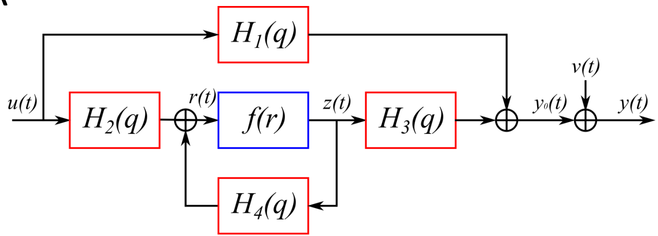
$$G_{\varepsilon}(q) = \frac{H(q)}{1 + \gamma H(q)}$$

➔ Fixed zeros, moving poles

	Gain	Poles	Zeros
Linear Time Invariant	Fixed	Fixed	Fixed
Hammerstein	Variable	Fixed	Fixed
Wiener	Variable	Fixed	Fixed
Wiener-Hammerstein	Variable	Fixed	Fixed
Parallel WH	Variable	Fixed	Variable
Feedback	Variable	Variable	Fixed
LFR			

Structure detection

- LFR



$$\boxed{G_{\varepsilon}(q) = H_1(q) + \frac{\gamma H_2(q) H_3(q)}{1 + \gamma H_4(q)}}$$

➔ Moving zeros, moving poles

	Gain	Poles	Zeros
Linear Time Invariant	Fixed	Fixed	Fixed
Hammerstein	Variable	Fixed	Fixed
Wiener	Variable	Fixed	Fixed
Wiener-Hammerstein	Variable	Fixed	Fixed
Parallel WH	Variable	Fixed	Variable
Feedback	Variable	Variable	Fixed
LFR	Variable	Variable	Variable

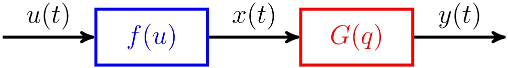
Structure detection

- BLA, ε – approximation @ ≠ setpoints
- Only gain change
  - Hammerstein, Wiener, Wiener-Hammerstein, …
- Zeros shift
  - Parallel feed-forward structure
- Poles shift
  - Feedback present

Outline

- Which block structure to choose?
- How to identify the chosen block structure?
  - Hammerstein model
  - Wiener model
  - Orthonormal basis functions

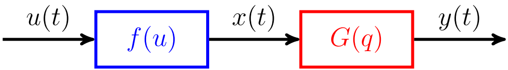
Identification of a Hammerstein model



$$f(u) = \sum_{d=0}^{D} \beta_d u^d$$

$$G(q) = \frac{B(q,\theta)}{A(q,\theta)}$$

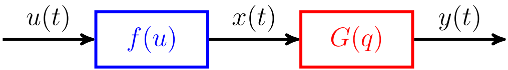
Step 1: Estimate a nonparametric BLA



Random-phase multisine excitation:

$$u^{[m]}(t) = \sum_{k=1}^{N_F} A_k \sin(2\pi k f_0 t + \phi_k^{[m]})$$

Step 1: Estimate a nonparametric BLA



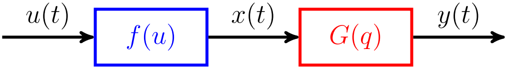
Random-phase multisine excitation:

$$u^{[m]}(t) = \sum_{k=1}^{N_F} A_k \sin(2\pi k f_0 t + \phi_k^{[m]})$$

FRF:

$$G_{BLA}(k) = \frac{1}{M} \sum_{m=1}^{M} \frac{Y^{[m]}(k)}{U^{[m]}(k)}$$

Step 1: Estimate a nonparametric BLA



Random-phase multisine excitation:

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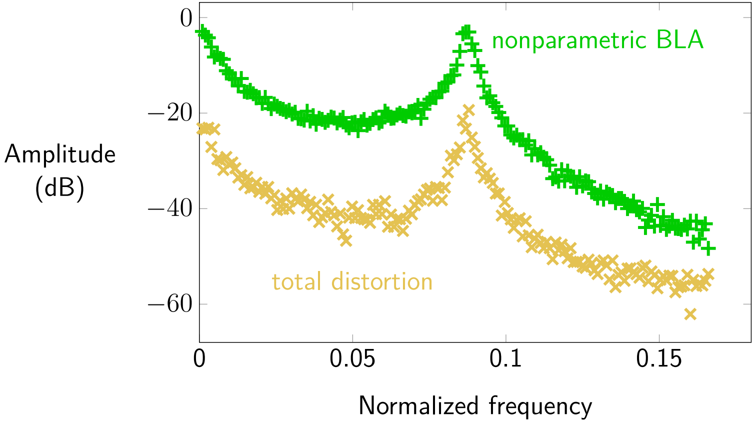
FRF:

$$G_{BLA}(k) = \frac{1}{M} \sum_{m=1}^{M} \frac{Y^{[m]}(k)}{U^{[m]}(k)}$$

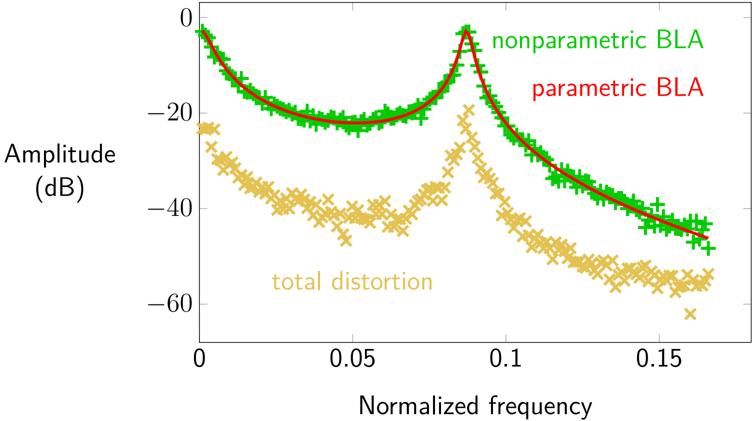
Nonparametric noise model:

$$\sigma^2_{G_{\mathrm{BLA}}}(k) = \sigma^2_{\mathrm{NL}}(k) + \sigma^2_{\mathrm{noise}}(k)$$

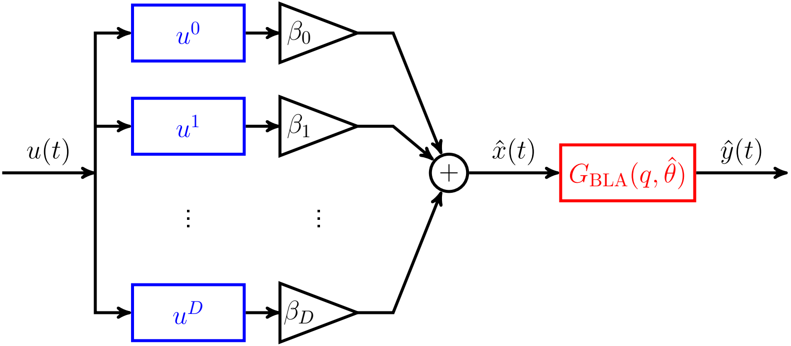
Step 2: Estimate a parametric BLA



Step 2: Estimate a parametric BLA



Step 3: Estimate the polynomial coefficients



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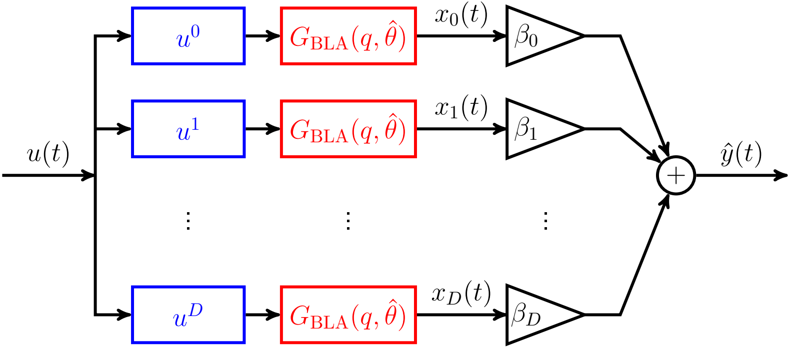
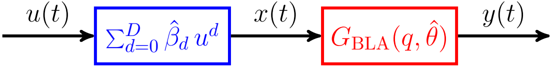


$$\hat{\beta} = \arg\min_{\beta} \left\| y(t) - \sum_{d=0}^{D} \beta_d x_d(t) \right\|_2^2$$

Step 4: Do a nonlinear optimization of all parameters

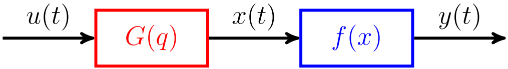


Nonlinear optimization of β and θ simultaneously.

Outline

- Which block structure to choose?
- How to identify the chosen block structure?
  - Hammerstein model
  - Wiener model
  - Orthonormal basis functions

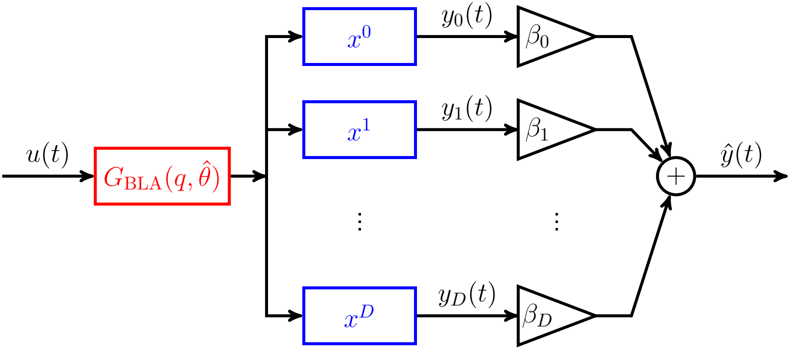
Identification of a Wiener model



$$G(q) = \frac{B(q, \theta)}{A(q, \theta)}$$

$$f(u) = \sum_{d=0}^{D} \beta_d x^d$$

Step 3: Estimate the polynomial coefficients



$$\hat{\beta} = \arg\min_{\beta} \left\| y(t) - \sum_{d=0}^{D} \beta_d y_d(t) \right\|_2^2$$

Outline

- Which block structure to choose?
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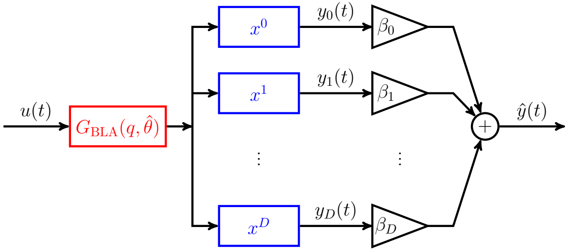
  - Hammerstein model
  - Wiener model
  - Orthonormal basis functions

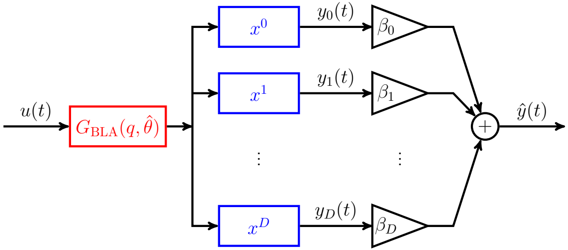
Motivation for orthonormal basis functions



$$\hat{\beta} = \arg\min_{\beta} \left\| y(t) - \sum_{d=0}^{D} \beta_d y_d(t) \right\|_2^2$$

possibly ill-conditioned

Motivation for orthonormal basis functions



$$\hat{\beta} = \arg\min_{\beta} \left\| y(t) - \sum_{d=0}^{D} \beta_d y_d(t) \right\|_2^2$$ possibly ill-conditioned

Linear dynamics	$\rightarrow$	Rational orthonormal basis functions
Static nonlinearities	$\rightarrow$	Hermite polynomials

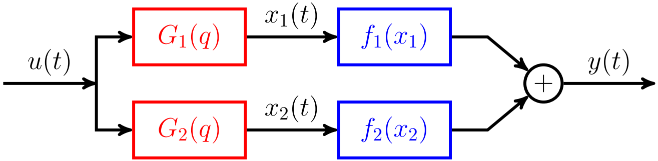
Rational OBFs are determined by their pole locations

$$F_k(q) = \frac{\sqrt{1-|\xi_k|^2}}{q-\xi_k} \prod_{i=1}^{k-1} \frac{1-\xi_i{}^* q}{q-\xi_i}$$

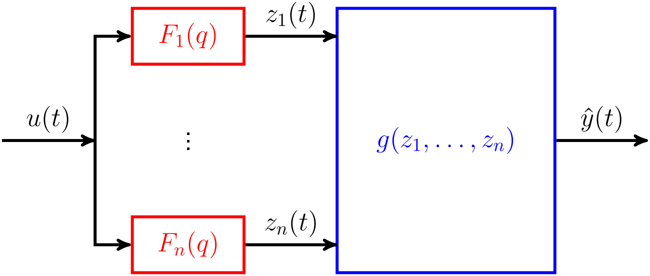
all-pass filter

Pole locations ξ_k	OBFs
origin	FIR
real pole	Laguerre
complex conjugate pair	Kautz
repeated poles	Generalized OBFs
arbitrary	Takenaka-Malmquist

Example: Approximation of a parallel Wiener system



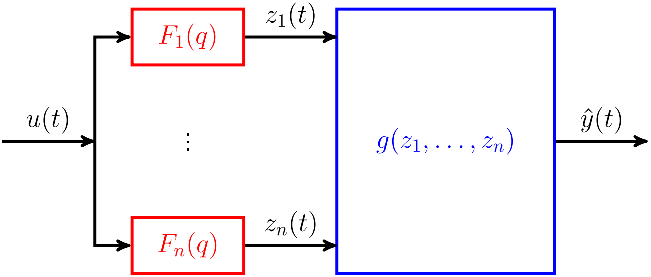
Step 1: Estimate the linear dynamics



F_k: rational orthonormal basis functions

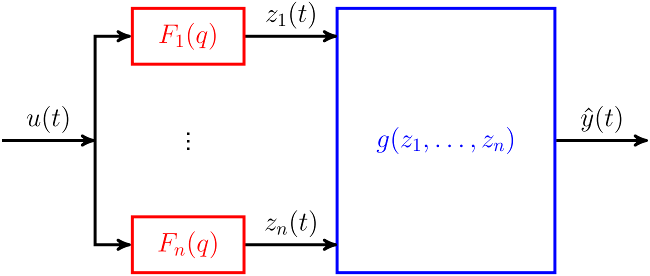
g: multivariate Hermite polynomials

Step 1: Estimate the linear dynamics



pole locations $\rightarrow$ orthonormal basis functions

Step 1: Estimate the linear dynamics

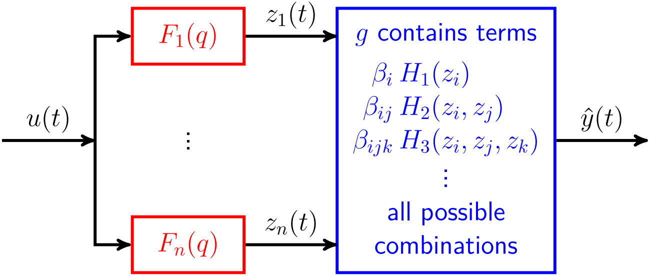


pole locations $\rightarrow$ orthonormal basis functions

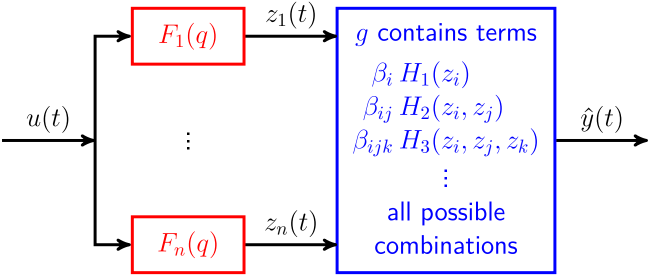
$\Downarrow$

best linear approximation

Step 2: Estimate the polynomial coefficients

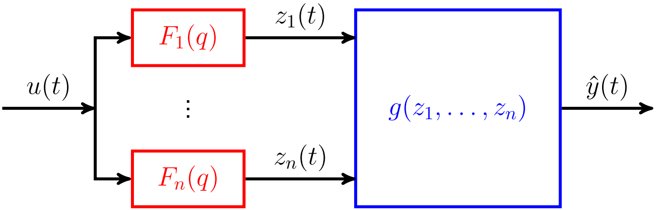


The model is linear-in-the-parameters



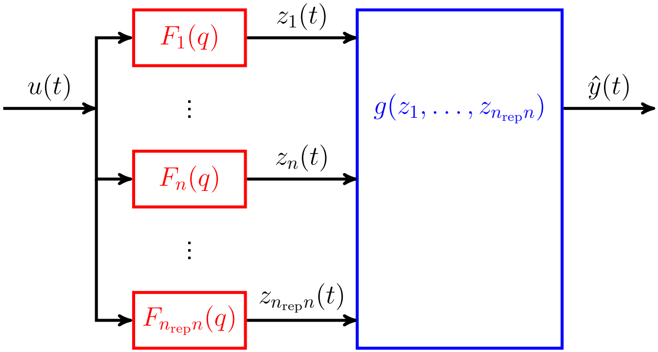
Prior knowledge can be incorporated via user-specified pole locations

$$\{\hat{p}_1, \ldots, \hat{p}_n\} \Rightarrow \{F_1, \ldots, F_n\}$$

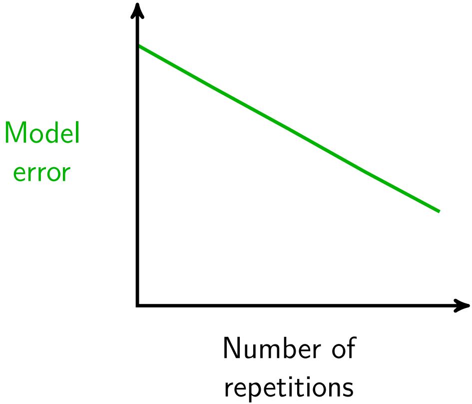


Prior knowledge can be incorporated via user-specified pole locations

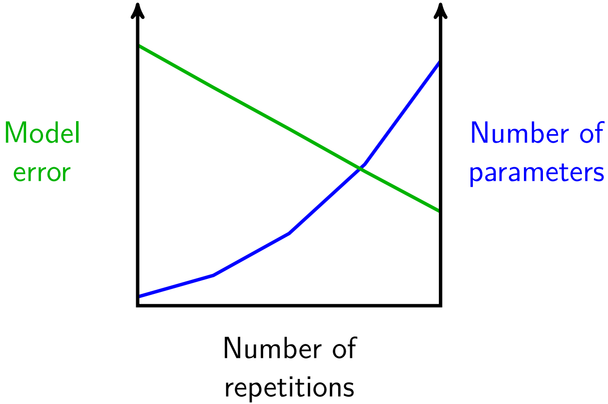
$$
\begin{aligned}
\{\hat{p}_1, \dots, \hat{p}_n\} &\Rightarrow \{F_1, \dots, F_n\} \\
\{\hat{p}_1, \dots, \hat{p}_n\} &\Rightarrow \{F_{n+1}, \dots, F_{2n}\} \\
\vdots \quad & \quad\quad \vdots
\end{aligned}
$$



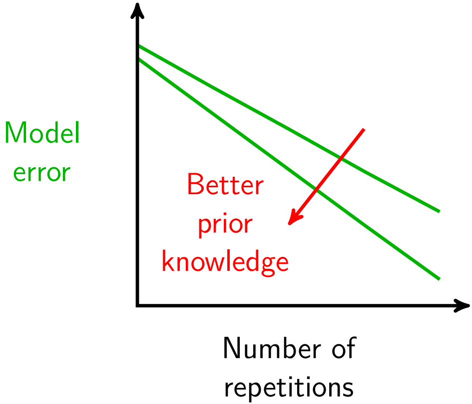
The extra basis functions compensate for a pole mismatch



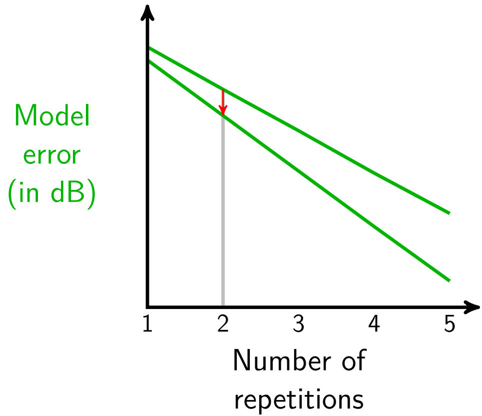
The number of parameters increases rapidly with the number of orthonormal basis functions



Better prior knowledge allows for better models



Iteratively update the pole locations



1) Estimate a high-order nonlinear model
2) Extract a low-order linear model

Overview

- Structure detection via BLA
- Identification of some block structures
  - Hammerstein
  - Wiener
  - Parallel Wiener