Vrije
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Block-oriented modeling

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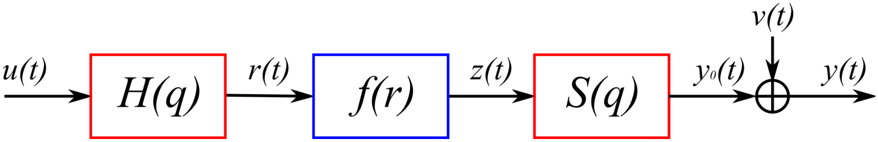
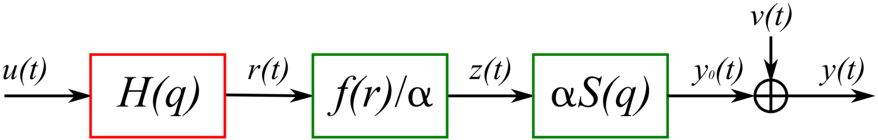
Overview

- Structure detection via BLA / ε-approximation
- Identification of some block structures
  - Hammerstein
  - Wiener
  - Parallel Wiener
  - Wiener-Hammerstein
  - Parallel Wiener-Hammerstein
  - Nonlinear feedback

Wiener-Hammerstein

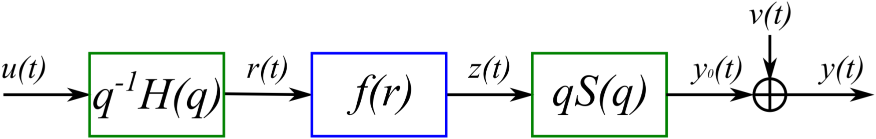


Identifiability



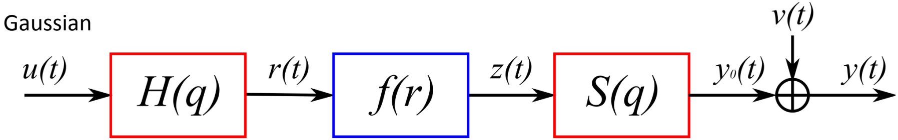
- Gain exchange

Identifiability



- Gain exchange
- Delay exchange

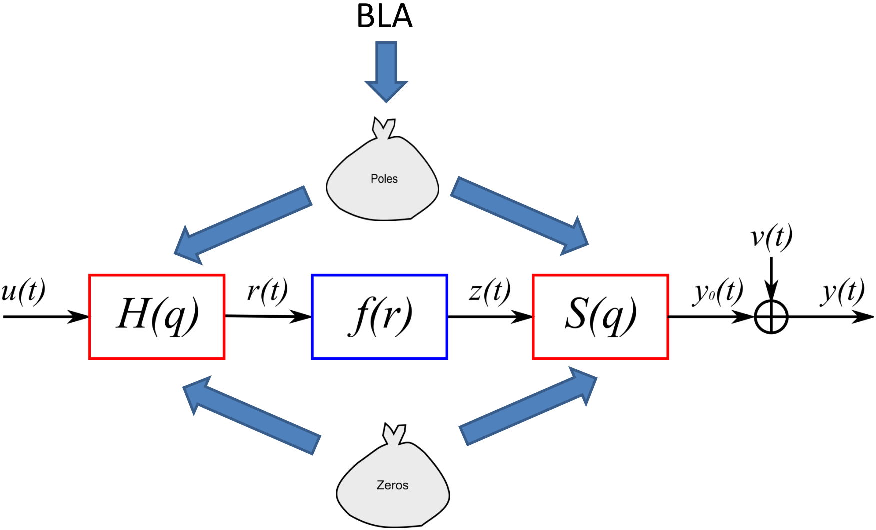
Best Linear Approximation



$$G_{bla}(q) = \gamma H(q) S(q)$$

➔ poles, zeros BLA = poles, zeros system

Partition the Dynamics



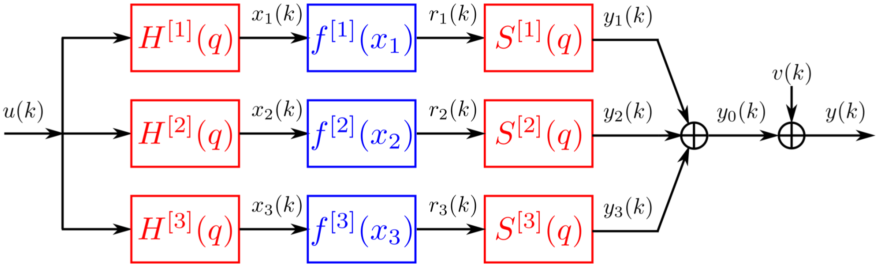
Nonlinear optimization

- Initial parameter values

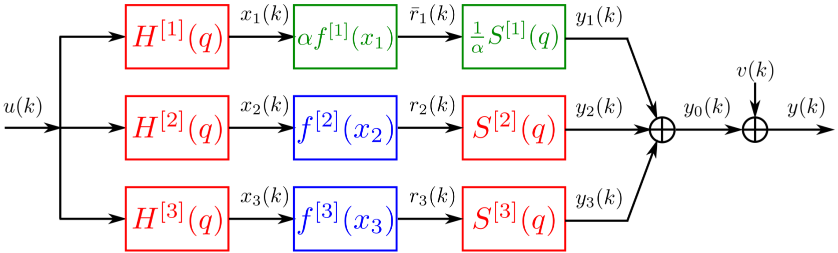
  → Optimization of all parameters together

  → Levenberg-Marquardt algorithm

Parallel Wiener-Hammerstein

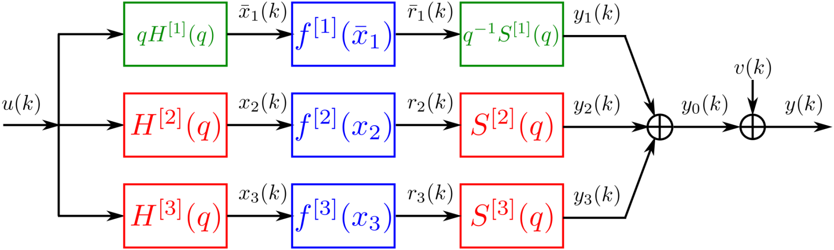


Identifiability



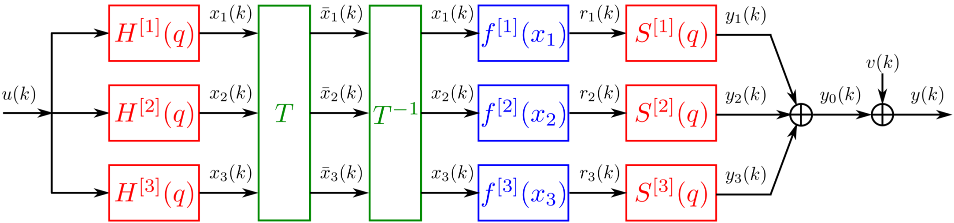
- Gain exchange

Identifiability



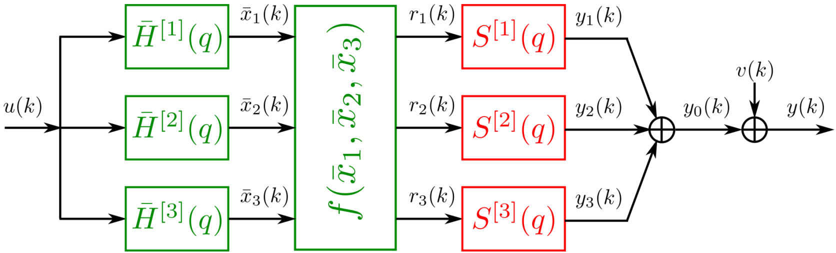
- Gain exchange
- Delay exchange

Identifiability



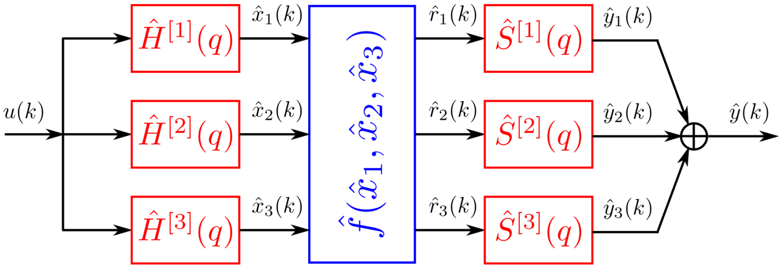
- Gain exchange
- Delay exchange
- Full rank linear transform

Identifiability



- Gain exchange
- Delay exchange
- Full rank linear transform

Model structure



Identification approach

$$G_{bla}^{[R]}(q)$$

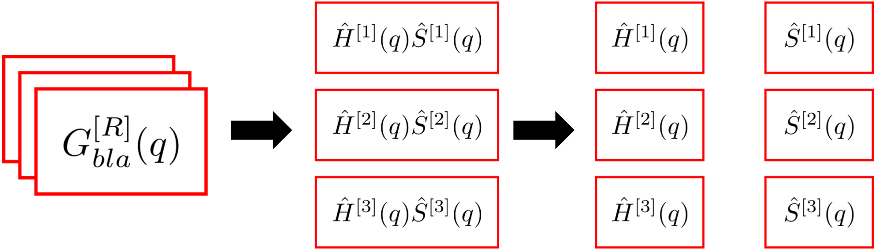
- Estimate overall dynamics

Identification approach

$$\boxed{G_{bla}^{[R]}(q)} \Rightarrow \begin{array}{c} \boxed{\hat{H}^{[1]}(q)\hat{S}^{[1]}(q)} \\ \boxed{\hat{H}^{[2]}(q)\hat{S}^{[2]}(q)} \\ \boxed{\hat{H}^{[3]}(q)\hat{S}^{[3]}(q)} \end{array}$$

- Estimate overall dynamics
- Decompose the dynamics over the parallel branches

Identification approach

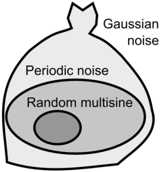


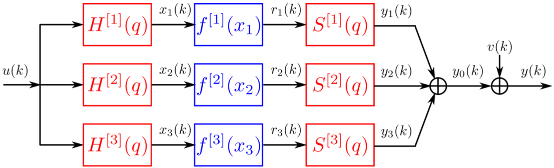
- Estimate overall dynamics
- Decompose the dynamics over the parallel branches
- Partition the dynamics to the front and back

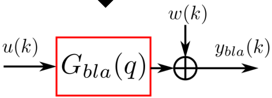
Identification approach

- Identifying the overall dynamics
  - ➔ Best Linear Approximation (BLA)
- Decomposing the dynamics
  - ➔ Singular Value Decomposition (SVD) of the BLAs
- Partition the dynamics
  - ➔ Pole and zero allocation scan

Best Linear Approximation



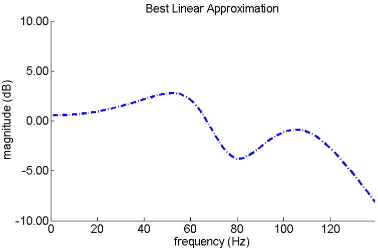


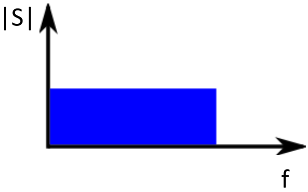


Combination of dynamics!

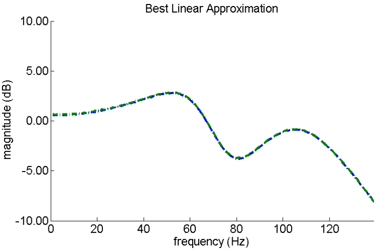
$$G_{bla}(j\omega) = \sum_i \gamma_i H^{[i]}(j\omega) S^{[i]}(j\omega)$$

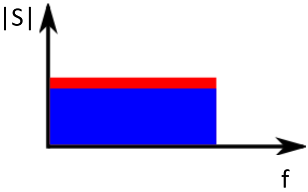
Best Linear Approximation



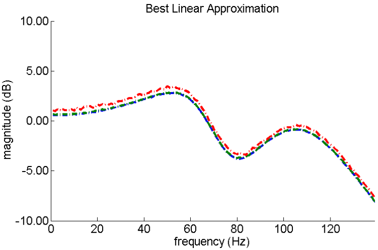


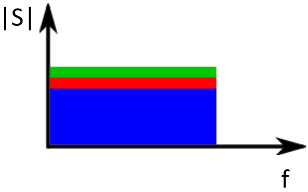
Best Linear Approximation



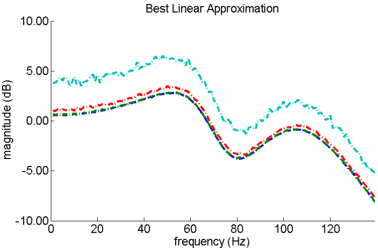


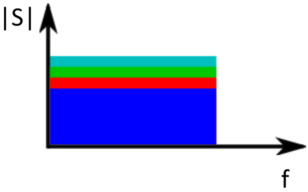
Best Linear Approximation



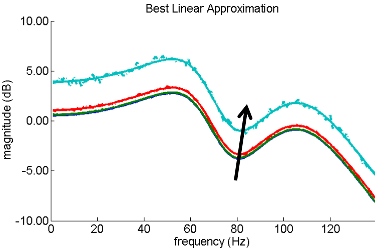


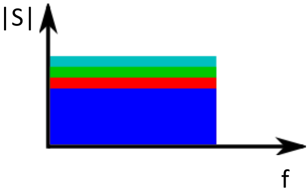
Best Linear Approximation





Best Linear Approximation



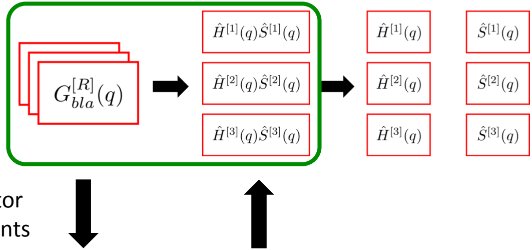


$$G_{bla}(j\omega) = \sum_i \gamma_i H^{[i]}(j\omega) S^{[i]}(j\omega)$$

$$\hat{G}_{bla}^{[i]} = \frac{\hat{d}_0^{[i]} + \hat{d}_1^{[i]} q^{-1} + \ldots + \hat{d}_{n_d}^{[i]} q^{-n_d}}{\hat{c}_0 + \hat{c}_1 q^{-1} + \ldots + \hat{c}_{n_c} q^{-n_c}}$$

- Common denominator
  - Fixed poles
  - Moving zeros

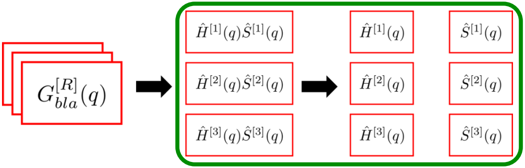
Decomposing the dynamics



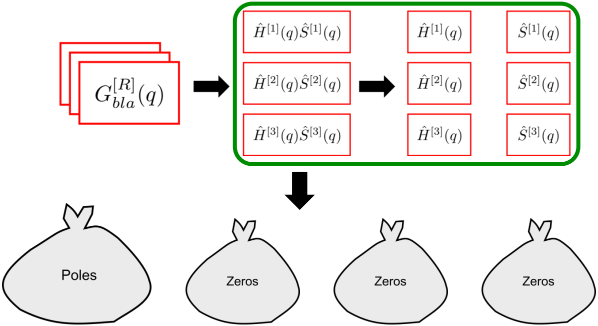
numerator coefficients

$$\underbrace{\begin{bmatrix} \hat{d}_0^{[1]} & \hat{d}_0^{[2]} & \cdots & \hat{d}_0^{[K]} \\ \hat{d}_1^{[1]} & \hat{d}_1^{[2]} & \cdots & \hat{d}_1^{[K]} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{d}_{n_d}^{[1]} & \hat{d}_{n_d}^{[2]} & \cdots & \hat{d}_{n_d}^{[K]} \end{bmatrix}}_{\text{D}} = \underbrace{\begin{bmatrix} \hat{b}_0^{[1]} & \hat{b}_0^{[2]} & \cdots & \hat{b}_0^{[K]} \\ \hat{b}_1^{[1]} & \hat{b}_1^{[2]} & \cdots & \hat{b}_1^{[K]} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{b}_{n_d}^{[1]} & \hat{b}_{n_d}^{[2]} & \cdots & \hat{b}_{n_d}^{[K]} \end{bmatrix}}_{\text{U}} \underbrace{\begin{bmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_K \end{bmatrix}}_{\Sigma} V^H$$

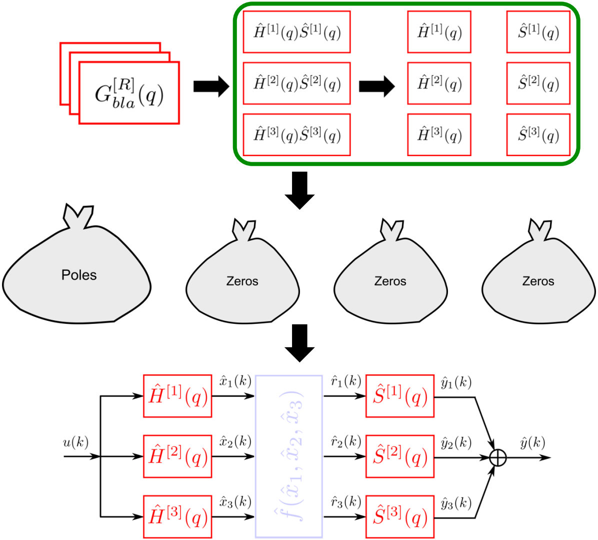
Partition the dynamics



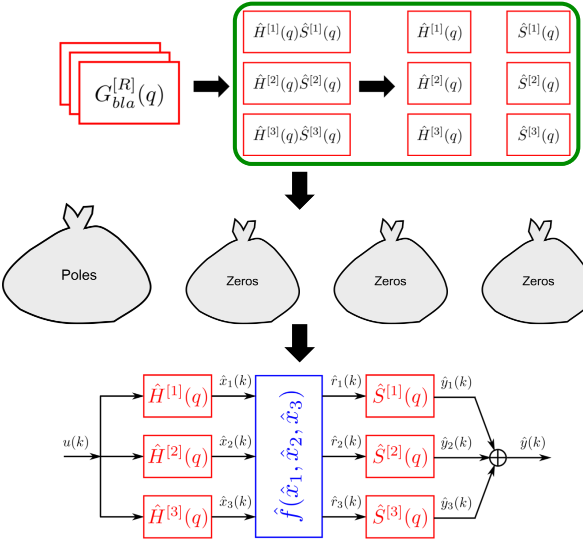
Partition the dynamics



Partition the dynamics



Partition the dynamics



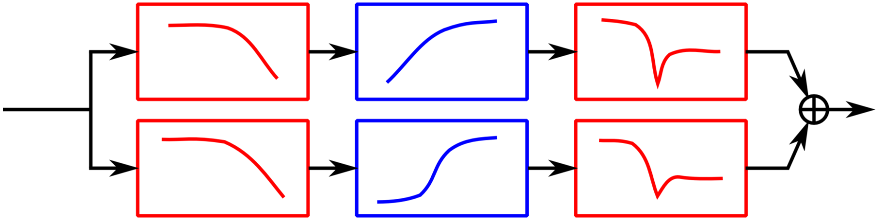
Nonlinear optimization

- Initial parameter values

  → Optimization of all parameters together

  → Levenberg-Marquardt algorithm

Example: test system

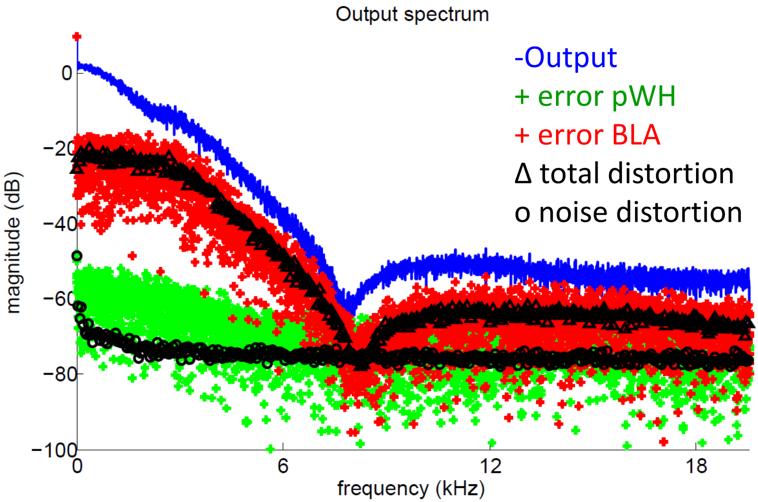


Multisine input:
5 amplitudes
20 realizations
2 periods
16384 samples

System:
Custom built circuit
12^{th} order dynamics
Diode-resistor NL

Model:
2 branches
10 neurons nn NL

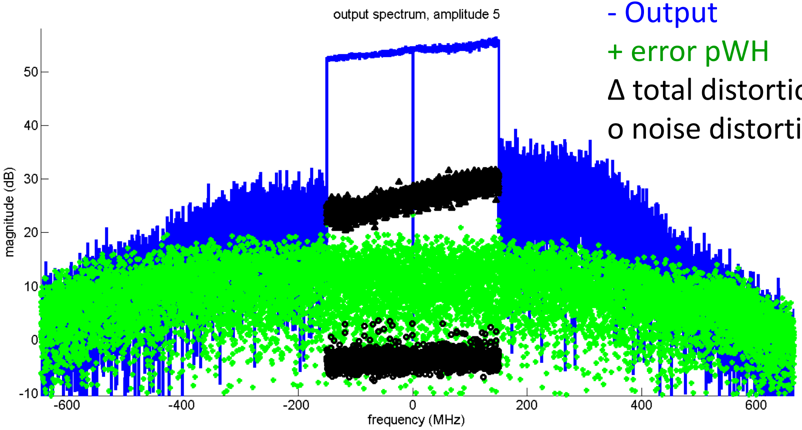
Example: test system



Example: Doherty PA

- Doherty PA
- Input:
  - Multitone, 5 amplitudes, 20 realizations
  - Bandwidth: 300MHz @ 3.45GHz
- Model
  - 2 branches
  - 10 tap FIR BLA
  - 7^{th} order NL

Example: Doherty PA



- Output

+ error pWH

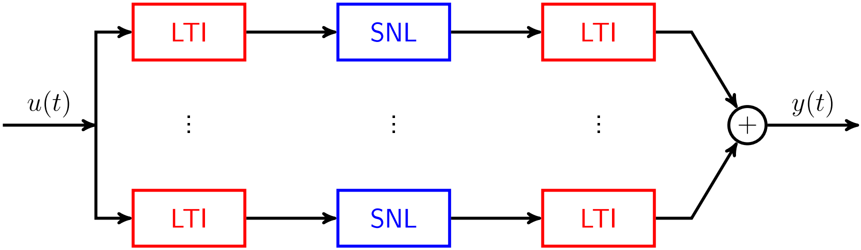
Δ total distortion

o noise distortion

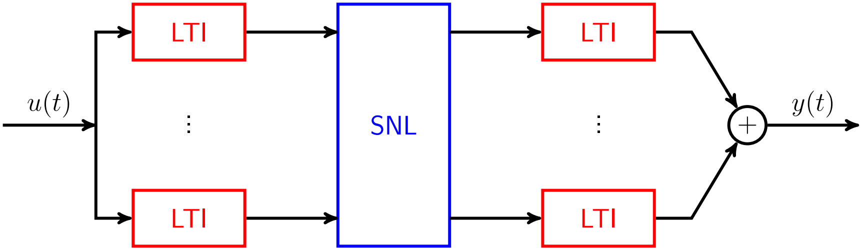
Outline

- Structure detection via BLA / ε-approximation
- Identification of some block structures
  - Hammerstein
  - Wiener
  - Parallel Wiener
  - Wiener-Hammerstein
  - Parallel Wiener-Hammerstein
  - Nonlinear feedback

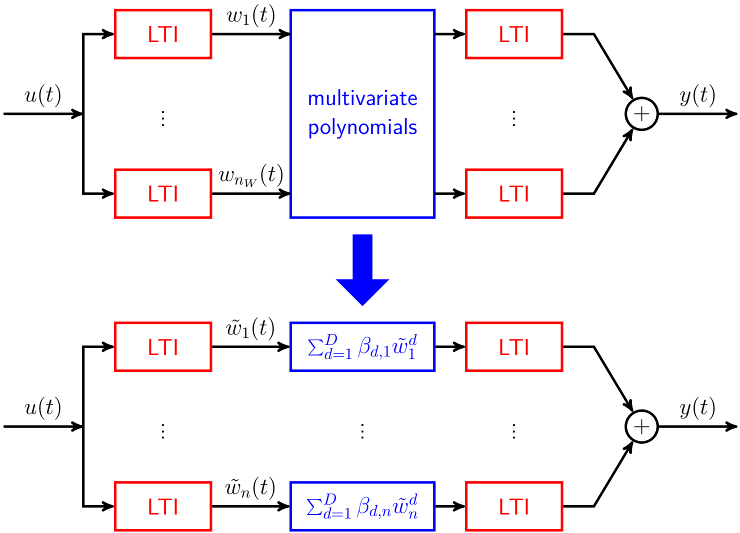
Parallel Wiener-Hammerstein models have good approximation properties



Identifiability issues require a MIMO static nonlinearity



Goal: Eliminate the cross-terms



Homogeneous polynomials are bijectively related to symmetric tensors

- quadratic polynomials $\leftrightarrow$ symmetric matrices

$$a_{11}w_1^2 + 2a_{12}w_1w_2 + a_{22}w_2^2$$

Homogeneous polynomials are bijectively related to symmetric tensors

- quadratic polynomials $\leftrightarrow$ symmetric matrices

$$\begin{aligned} a_{11}w_1^2 + 2a_{12}w_1w_2 + a_{22}w_2^2 &= \mathbf{w}^T\mathbf{A}\mathbf{w} \\ &= \begin{bmatrix} w_1 & w_2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \\ &= \mathbf{A} \;\bar{\times}_1\; \mathbf{w} \;\bar{\times}_2\; \mathbf{w} \end{aligned}$$

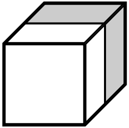
Homogeneous polynomials are bijectively related to symmetric tensors

- quadratic polynomials $\leftrightarrow$ symmetric matrices

$$\begin{aligned} a_{11}w_1^2 + 2a_{12}w_1w_2 + a_{22}w_2^2 &= \mathbf{w}^T\mathbf{A}\mathbf{w} \\ &= \begin{bmatrix} w_1 & w_2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \\ &= \mathbf{A} \;\bar{\times}_1\; \mathbf{w} \;\bar{\times}_2\; \mathbf{w} \end{aligned}$$

- cubic polynomials $\leftrightarrow$ symmetric third-order tensors

$$\begin{aligned} x_{111}w_1^3 + 3x_{112}w_1^2w_2 + 3x_{122}w_1w_2^2 + x_{222}w_2^3 \\ = \boldsymbol{\mathcal{X}} \;\bar{\times}_1\; \mathbf{w} \;\bar{\times}_2\; \mathbf{w} \;\bar{\times}_3\; \mathbf{w} \end{aligned}$$

$\boldsymbol{\mathcal{X}} =$  $\rightarrow \left[\begin{array}{cc|cc} x_{111} & x_{112} & x_{112} & x_{122} \\ x_{112} & x_{122} & x_{122} & x_{222} \end{array}\right]$

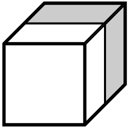
Homogeneous polynomials are bijectively related to symmetric tensors

- quadratic polynomials $\leftrightarrow$ symmetric matrices

$$
\begin{aligned}
a_{11} w_1^2 + \cancel{2 a_{12} w_1 w_2} + a_{22} w_2^2 &= \mathbf{w}^T \mathbf{A} \mathbf{w} \\
&= \begin{bmatrix} w_1 & w_2 \end{bmatrix} \begin{bmatrix} a_{11} & \cancel{a_{12}} \\ \cancel{a_{12}} & a_{22} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \\
&= \mathbf{A} \bar{\times}_1 \mathbf{w} \bar{\times}_2 \mathbf{w}
\end{aligned}
$$

- cubic polynomials $\leftrightarrow$ symmetric third-order tensors

$$
\begin{aligned}
x_{111} w_1^3 + \cancel{3 x_{112} w_1^2 w_2} + \cancel{3 x_{122} w_1 w_2^2} + x_{222} w_2^3 \\
= \boldsymbol{\mathcal{X}} \bar{\times}_1 \mathbf{w} \bar{\times}_2 \mathbf{w} \bar{\times}_3 \mathbf{w}
\end{aligned}
$$

$\boldsymbol{\mathcal{X}} =$  $\rightarrow \left[\begin{array}{cc|cc} x_{111} & \cancel{x_{112}} & \cancel{x_{112}} & \cancel{x_{122}} \\ \cancel{x_{112}} & \cancel{x_{122}} & \cancel{x_{122}} & x_{222} \end{array} \right]$

Decouple quadratic polynomials via an eigenvalue decomposition (EVD)

$$
\begin{aligned}
a_{11} w_1^2 + 2 a_{12} w_1 w_2 + a_{22} w_2^2 &= \mathbf{w}^T \mathbf{A} \mathbf{w} \\
&\overset{EVD}{=} \mathbf{w}^T \left(\mathbf{V} \mathbf{D} \mathbf{V}^T \right) \mathbf{w} \\
&= \left(\mathbf{w}^T \mathbf{V} \right) \mathbf{D} \left(\mathbf{V}^T \mathbf{w} \right) \\
&= \tilde{\mathbf{w}}^T \mathbf{D} \tilde{\mathbf{w}} \\
&= \sum_{r=1}^{2} d_r \left(v_{1r} w_1 + v_{2r} w_2 \right)^2 \\
&= d_1 \tilde{w}_1^2 + d_2 \tilde{w}_2^2
\end{aligned}
$$

A similar decomposition for tensors?

- Eigenvalue decomposition (EVD):

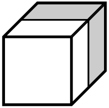
$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix}$$
$$= d_1 \begin{bmatrix} v_{11} \\ v_{21} \end{bmatrix} \begin{bmatrix} v_{11} & v_{21} \end{bmatrix} + d_2 \begin{bmatrix} v_{12} \\ v_{22} \end{bmatrix} \begin{bmatrix} v_{12} & v_{22} \end{bmatrix}$$

A similar decomposition for tensors: the CPD

- Eigenvalue decomposition (EVD):

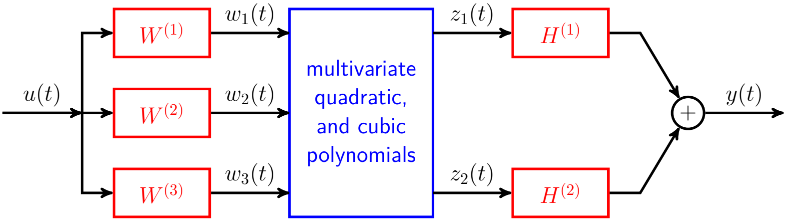
$$
\begin{aligned}
\mathbf{A} &= \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix} \\
&= d_1 \begin{bmatrix} v_{11} \\ v_{21} \end{bmatrix} \begin{bmatrix} v_{11} & v_{21} \end{bmatrix} + d_2 \begin{bmatrix} v_{12} \\ v_{22} \end{bmatrix} \begin{bmatrix} v_{12} & v_{22} \end{bmatrix}
\end{aligned}
$$

- (Symmetric) canonical polyadic decomposition (CPD):

$\boldsymbol{\mathcal{X}} =$  $\rightarrow \left[\begin{array}{cc|cc} x_{111} & x_{112} & x_{112} & x_{122} \\ x_{112} & x_{122} & x_{122} & x_{222} \end{array}\right]$

$$
\approx \lambda_1 \overset{b_{11}\ \ b_{21}}{\begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} \begin{bmatrix} b_{11} & b_{21} \end{bmatrix}} + \ldots + \lambda_R \overset{b_{1R}\ \ b_{2R}}{\begin{bmatrix} b_{1R} \\ b_{2R} \end{bmatrix} \begin{bmatrix} b_{1R} & b_{2R} \end{bmatrix}}
$$

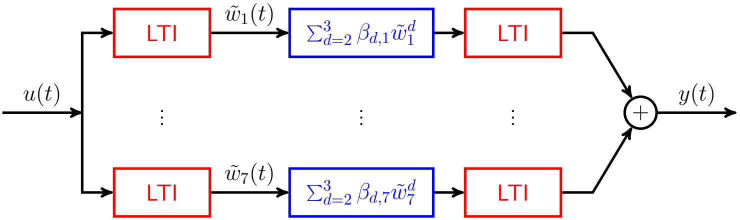
Example



$$a^{(k)}_{i_1 i_2} \text{ and } x^{(k)}_{i_1 i_2 i_3} \in \mathcal{N}(0,1)$$

There are 32 polynomial coefficients.

Example



There are only 14 polynomial coefficients.

Outline

- Structure detection via BLA / ε-approximation
- Identification of some block structures
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  - Wiener-Hammerstein
  - Parallel Wiener-Hammerstein
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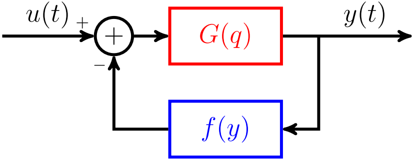
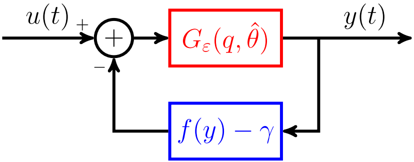
Identification of a nonlinear feedback model

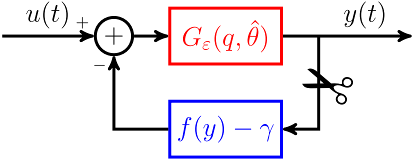


Step 1: Estimate the linear dynamics

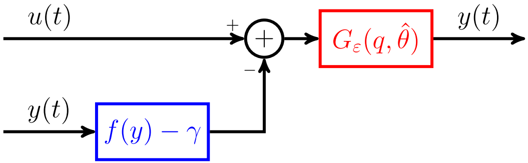


$$G_\varepsilon(q, \hat{\theta}) = \frac{B_\varepsilon(q, \hat{\theta})}{A_\varepsilon(q, \hat{\theta})}$$

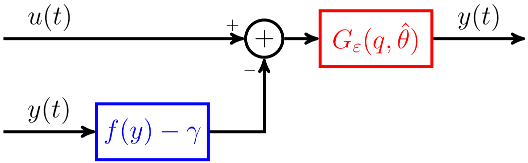
Step 2: Estimate the static nonlinearity



Step 2: Estimate the static nonlinearity



Step 2: Estimate the static nonlinearity

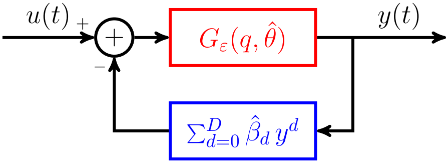


$$w(t) = y(t) - G_\varepsilon(q, \hat{\theta})u(t)$$

$$\hat{w}(t) = -G_\varepsilon(q, \hat{\theta}) \left(\sum_{d=0}^{D} \beta_d y^d(t) \right)$$

$$\hat{\beta} = \arg\min_{\beta} \| w(t) - \hat{w}(t) \|_2^2$$

Step 3: Optimize all model parameters



Nonlinear optimization of β and θ simultaneously.

Overview

- Structure detection via BLA / ε-approximation
- Identification of some block structures
  - Hammerstein
  - Wiener
  - Parallel Wiener
  - Wiener-Hammerstein
  - Parallel Wiener-Hammerstein
  - Nonlinear feedback

Overview

- Structure detection via BLA / ε-approximation
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- Guidelines

Model structure selection

Do I need a nonlinear model?
Best linear approximation framework:

Noise distortions $\leftrightarrow$ Nonlinear distortions

Which block-oriented model is suited?
Best linear approximation at different setpoints:

- Shifting poles $\Rightarrow$ Nonlinear feedback
- Shifting zeros $\Rightarrow$ Parallel nonlinear signal paths

Input design

What inputs are well suited?

- Model errors ⇒ use realistic excitations (amplitude/frequency)
- BLA framework ⇒ preferably periodic random signals
  $\sigma^2_{noise} \sim O\left(\frac{1}{\#\text{periods } \#\text{realizations}}\right)$
  $\sigma^2_{nonlinear} \sim O\left(\frac{1}{\#\text{realizations}}\right)$
- Multisine excitation/perturbation signals
  ⇒ full control of amplitude spectrum