Exam questions for "Signal theory: Part 1"

Work alone. You can use printed materials but no electronic devices. Time allowed: 60 minutes.

1. (2 points) The signal $y(t) = e^t + e^{2t} + e^{3t}$ is a response of an autonomous linear time-invariant system. What can you say about the order $\mathtt{n}$ of the system?

   (a) $\mathtt{n} = 1$ (b) $\mathtt{n} = 2$ (c) $\mathtt{n} = 3$ (d) $\mathtt{n} \geq 3$

2. (3 points) Find *all* least-squares approximate solutions of the linear system of equations

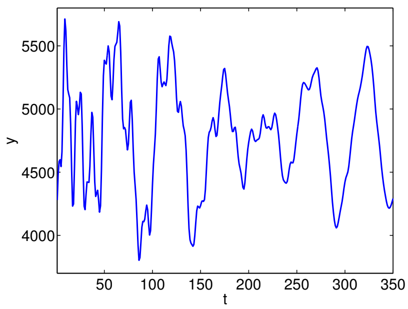
$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{bmatrix} x = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}.$$

3. (5 points) An invariant space of a state space system is a subset $\mathscr{I}$ of its state space, such that if the initial state $x(0)$ is in $\mathscr{I}$, then the state $x(t)$ remains in $\mathscr{I}$ for all $t > 0$.

   3.1. (3 points) Explain how to find all one dimensional invariant spaces of a system defined by $x(t+1) = Ax(t)$.

   3.2. (2 point) Apply the solution on the system with $A := \left[\begin{smallmatrix} 1 & 0 \\ 0 & 2 \end{smallmatrix}\right]$.

4. (5 points) A colleague of yours shows you the following signal



   and says

   > "I think the data generating system is not linear time-invariant, because the response of such a system is a sum of terms that are exponentially decaying, exponentially growing, or periodic while the behavior of the given signal is more complicated: it is obviously not periodic and it is neither exponentially decaying nor exponentially growing."

   Do you agree? If so, how would you make the argument rigorous? If not, what is wrong with the argument and how would you prove that it is wrong?

   You should give a specific answer (correct or wrong) and justify it theoretically. In addition, you can assume that you have the observed signal numerically and a computer available to process the data. Can you refine your answer by being more specific about the data generating system? What computations would you do?

5. (5 points) We discussed in the course two identification methods for autonomous linear time-invariant systems—one based on a state space representation and one based on a polynomial representation. These methods use as data a trajectory $y = \big(y(1), \dots, y(T)\big)$ and produce as an output the model parameters.

   5.1. (3 points) Extend the methods to use *two* given trajectories

$$y^{(1)} = \big(y^{(1)}(1), \dots, y^{(1)}(T_1)\big) \quad \text{and} \quad y^{(2)} = \big(y^{(2)}(1), \dots, y^{(2)}(T_2)\big)$$

   of the system instead of one trajectory. Discuss both the exact data case and the noisy data case.

   5.2. (2 points) Apply your method on the data $y^{(1)} = (0,1,1)$ and $y^{(2)} = (1,0,2)$, assuming that the data is exact and the data generating system is of second order.