Mini-projects for the SOCN course
"Behavioral approach to system theory"

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"The challenge is finding a balance between exploring new ideas and acquiring the skills to realize them."

Ken Robinson

As we've seen in the exercises, in case of exact data the data-driven methods solve the problems exactly, *i.e.*, they give equivalent results to model-based methods that use the data-generating system $\mathscr{B}$ instead of the data $w_{\rm d}$. The equivalence between data-driven and model-based method no longer holds true when $w_{\rm d}$ is inexact. Indeed, the true data-generating system can not be recovered from inexact data. Moreover, different identification methods deliver different models. The fundamental difficulty in dealing with inexact data is that the problems are nonconvex. All currently known methods can be viewed as heuristics for solving the underlying nonconvex optimization problem.

The mini-projects in Section 1 explore the performance of the methods in case of inexact data $w_{\rm d}$. We use different types of prior knowledge and observe its effect on the accuracy. The theoretically best performance is the one of methods using the true data-generating system (or equivalently exact data). Of particular interest is a comparison of the direct and indirect approaches. For the indirect approach, you can use methods implemented in the System Identification Toolbox of Matlab in order to obtain the model from the data. Section 2 has two projects related to practical applications. One is from the field of metrology and the other is about free fall in a gravitational field. Section 3 lists three current research projects of mine: recursive algorithm for exact identification, data-driven frequency response estimation, and computing distance measures between systems. You can also propose to work on a mini-project of your own choice (for example, a subproblem appearing in your own research topic).

1 Further experiments with direct data-driven methods

- Noisy data in the errors-in-variables setup
- High-order linear time-invariant system
- Unstable systems
- Nonminimum-phase systems
- Stiff systems
- Real-data from DAISY

Using a trajectory matrix with $\ell+1$ block-rows leads to an unstructured data matrix, so that in this case approximation by truncation of the singular value decomposition yields optimal approximation. This observation allows us to avoid the nonconvex optimization of the structured low-rank approximation problem, however, data from multiple short experiments is less informative than data from one long experiment with the same total number of samples. Empirical results show that overall low-rank approximation of the Hankel matrix gives more accurate model parameter estimates than low-rank approximation of the trajectory matrix. This contradicts empirical results reported in [**?**, **?**] on using the trajectory matrix in data-driven MPC control so that a further research in this direction is needed.

2 Practical applications

Sensor speed-up

Free fall in a gravitational field

3 Research projects

Distance between systems

Recursive algorithm for exact identification

For details about the MPUM and its computation, see [**?**].

We did not delve into numerical computations issues related to the implementation of Algorithm **??**. This topic has connections with work on numerical linear algebra methods for Hankel structured matrices, see for example [**?**, **?**, **?**]. In particular incorporating approximation in Algorithm **??** is a interesting topic for further work.

Data-driven frequency response estimation

computationally efficiency

modifications for noisy data

statistical analysis

Data-driven interpolation and approximation

computationally efficiency

modifications for noisy data

statistical analysis

Spurious annihilators

Algorithm **??** detects the annihilators of degree up to $\ell_{\max} := L_{\max} - 1$. Assume as in the identifiability problem that the data is generated by a linear time-invariant system $\bar{\mathscr{B}} = \ker R(\sigma) \in \partial \mathscr{L}^{q,n}_{m,\ell}$. Then, the identified model $\widehat{\mathscr{B}} := \mathscr{B}_{\text{MPUM}}(\mathscr{W}_{\text{d}})$ contains the annihilators $R^1, \dots, R^g$ of $\bar{\mathscr{B}}$ of degree up to $\ell_{\max}$. Depending on the data $\mathscr{W}_{\text{d}}$, however, $\widehat{\mathscr{B}}$ may contain additional annihilators of degree up to $\ell_{\max}$ that are not annihilators of $\bar{\mathscr{B}}$. We call these annihilators *spurious*.

Definition 1 (Spurious annihilators)**.** Let the data (**??**) be generated by a system $\bar{\mathscr{B}} \in \partial \mathscr{L}^{q,n}_{m,\ell}$, *i.e.*, (**??**), $\bar{\mathscr{R}}$ be the set of annihilators of $\bar{\mathscr{B}}$, and $\widehat{\mathscr{R}}$ be the set of annihilators of $\mathscr{B}_{\text{MPUM}}(\mathscr{W}_{\text{d}})$. The elements of the set difference $\widehat{\mathscr{R}} \setminus \bar{\mathscr{R}}$ are spurious annihilators of $\mathscr{B}_{\text{MPUM}}(\mathscr{W}_{\text{d}})$ (with respect to $\bar{\mathscr{B}}$).

Existence of spurious annihilators prevents identifiability. Indeed, by definition, $\mathscr{B}_{\text{MPUM}}(\mathscr{W}_{\text{d}}) = \bar{\mathscr{B}}$ if and only if $\mathscr{B}_{\text{MPUM}}(\mathscr{W}_{\text{d}})$ has no spurious annihilators. Consequently, all identifiability results can be understood as giving conditions ensuring that the model $\widehat{\mathscr{B}}$ does not include spurious annihilators. A prototypical example of a spurious annihilator is an annihilator for a set of input variables. In the fundamental lemma such annihilators are avoided by the persistency of excitation assumption (**??**) for the inputs. Indeed, persistency of excitation of u_{d} of order L implies that there are no annihilators of order $L-1$. The fundamental lemma uses a separation of the annihilators into spurious and non-spurious based on degree: any annihilator of degree larger than an a priori known threshold degree $\ell_{\max}$ is spurious.

We envisage a range of other identifiability conditions that filter the spurious annihilators based on other types of prior knowledge about the true system $\bar{\mathscr{B}}$.

In classical identifiability results such, as the fundamental lemma, the spurious annihilators are distinguished from the true system's annihilators based on a degree separation. New identifiability conditions can be derived using more general separation criteria.

Fundamental lemma

Robustifying the persistency of excitation conditions For (u_{d}) to hold true 1) $a_i \neq 0$, for all i and 2) $\lambda_{u,i} \neq \lambda_{u,j}$, for all $i \neq j$. A way of robustifying these conditions is 1) $a_i > \varepsilon$, for some user defined tolerance ε, and 2) choose the $\lambda_{u,i}$'s "well spread".

Input design using the input model $\mathscr{B}_u$ Using the input model representation of a persistently exciting signal (see, Figure **??**), the freedom of choosing an input that satisfies the conditions of the fundamental lemma is equivalent to choosing the input model $\mathscr{B}_u$ and the initial conditions $x_{u,\mathrm{ini}}$. The input model representation of the class of sufficiently exciting inputs can be used then for input design under user defined specifications, such as frequency band, maximum/minimum value bounds, *etc*.