ELEC 3035, Overview

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- Administrative issues
- Multivariable dynamical models
- Control design
- Example

Administrative issues

- Course web page:

  http://users.ecs.soton.ac.uk/im/elec3035.html

  Announcements, lecture slides, tutorial sheets, exams.

- Evaluation: 100% on a written exam.

- Suggested texts for part I of the course:

  Topic 1: Introduction to linear algebra, G. Strang

  Topics 2–5: Introduction to mathematical systems theory
  K. Astrom and R. Murray

  Topics 5–7: Computer-controlled systems: theory and design
  K. Astrom and B. Wittenmark

Part I: Linear system design

1. Review of linear algebra
2. Introduction to state space and polynomial methods
3. Autonomous systems and stability
4. Controllability and observability
5. Design by pole placement and observer design
6. Linear quadratic control and Kalman filter
7. System identification

Part II: Nonlinear system design

- Mathematical modelling of nonlinear systems
- Lyapunov stability analysis
- Describing functions
- Feedback linearization
- Adaptive control

Continuous and discrete-time signals

- A signal w is a function of an indep. variable t, called time.

  Notation: w — function, $w(t)$ — the value of w at time t

- If t is a real number, $t \in \mathbb{R}$, w is caled continuous-time.
- If t is an integer, $t \in \{\dots, -1, 0, 1, \dots\}$, w is called discrete-time.
- Most physical phenomena are continuous-time

  however, they can be modelled in discrete-time.

- Most economic phenomena are intrinsically discrete-time.

Dynamical systems

- The term "dynamical" means "changing in time".
- The antonym is "static" which means "constant in time".
- Dynamical systems are collections of signals w.
- The system imposes relations that the variables w must satisfy.
- Mathematically dynamical systems are represented by
  - differential equation in continuous-time
  - difference equation in discrete-time

Vector-valued signals and multivariable systems

- w is called vector-valued if $w(t)$ is a vector, *i.e.*, $w(t) = \begin{bmatrix} w_1(t) \\ \vdots \\ w_q(t) \end{bmatrix}$.
- The antonym of vector-valued is scalar-valued.
  (Vector-valued signal is collection of scalar-valued signals.)
- A system describing vector-valued signals is called multivariable.
- The antonym of multivariable is univariate.
  (Multivariable system is more than collection of univariate systems.)
- The multivariable aspect of the system is as important as the dynamical one.
- Mathematical theory used to describe multivariable systems is linear algebra.

Control of nonlinear multivariable systems

- ELEC 1011, 2019 cover linear scalar dynamical systems
- ELEC 3035, Part I covers linear multivariable systems
  Part II nonlinear systems
- In addition, in ELEC 3035 we use state space representations
  (*i.e.*, first order vector differential/difference equations)
- The use of state space methods is characteristic for the "modern control" (pole placement and LQ optimal control, *etc*).
- Transfer functions are characteristic for the "classical control" (PID, root locus, and lead/lag control, *etc*).

High level definition of controller design

Given a system, called a plant, and a desired behaviour,

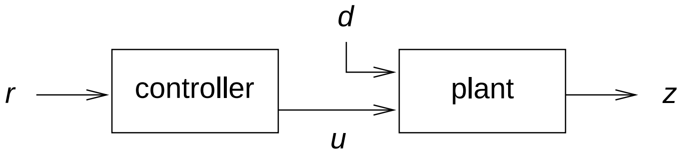
find a system, called a controller, such that the interconnected plant–controller system achieves the desired behaviour.

Usually the variables are separated into inputs and outputs
(inputs are free, outputs are bound)

Inputs are connected only to outputs $\rightsquigarrow$ two basic interconnections:

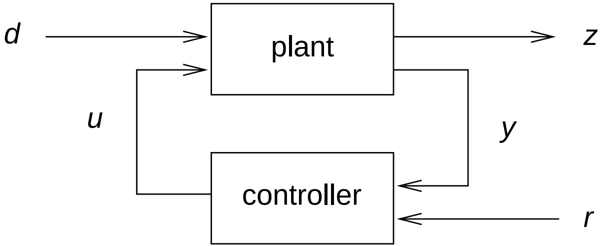
- feedforward
- feedback

Feedforward interconnection



- r — reference signal
- u — control signal
- d — disturbance
- z — performance criterion

Feedback interconnection



- y — measurement signal

Example

Let $(\sigma w)(t) := w(t+1)$ be the shift operator.

Consider a discrete-time linear time-invariant system

$$\mathscr{B} = \{ w = (u,y) \mid \exists\, x \text{ such that } \sigma x = Ax + Bu,\ y = Cx + Du \}$$

- x — state, u — input, y — output
- $A \in \mathbb{R}^{\mathtt{n}\times\mathtt{n}}$, $B \in \mathbb{R}^{\mathtt{n}\times\mathtt{m}}$, $C \in \mathbb{R}^{\mathtt{p}\times\mathtt{n}}$, $D \in \mathbb{R}^{\mathtt{p}\times\mathtt{m}}$ are parameters of $\mathscr{B}$

(called input/state/output or state space representation)

In the example, $\mathtt{m} = 2$ inputs, $\mathtt{p} = 2$ outputs, and $\mathtt{n} = 15$ states.

Aim: choose u, so that y obtains a desired value $y_{\text{des}} := \left[\begin{smallmatrix} 1 \\ -1 \end{smallmatrix}\right]$.

A simple approach

Consider a steady state regime — u, y, x constant, so that

$$\sigma x = x = Ax + Bu \quad \Longrightarrow \quad x = (I - A)^{-1} Bu$$

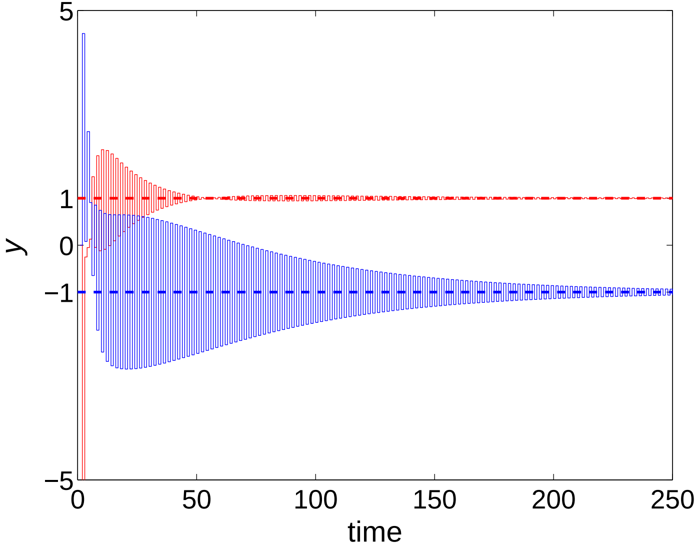
Substitute in $y = Cx + Du$ to obtain

$$u = \left(C(I - A)^{-1}B + D\right)^{-1} y.$$

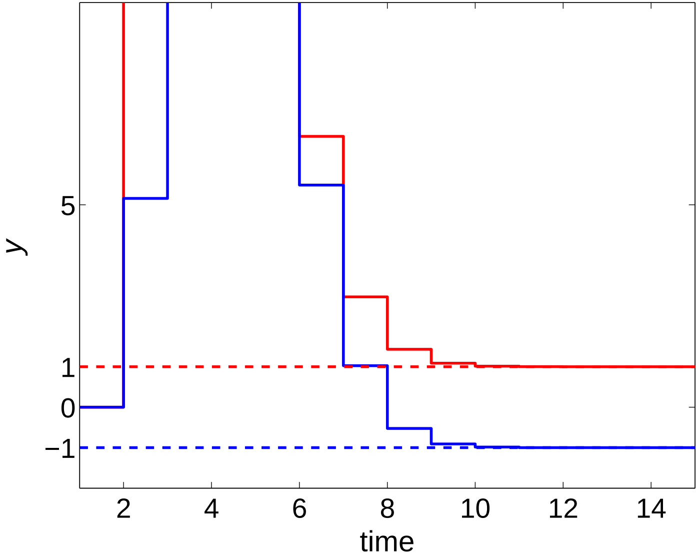
For $y = y_{\text{des}}$, we obtain the desired constant control input

$$u_{\text{c}} = \left(C(I - A)^{-1}B + D\right)^{-1} y_{\text{des}}.$$

Constant control u_c



Dynamic (deadbeat) control



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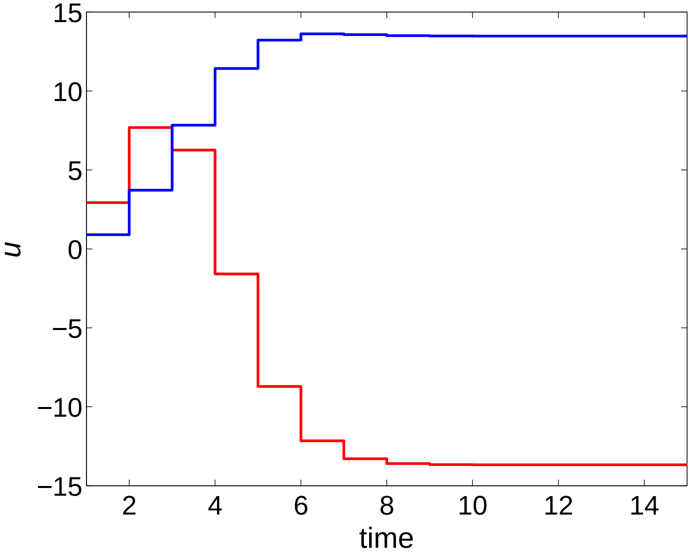

Classical, modern, and behavioural approaches

- Classical control 1920–1960, Bode, Nyquist, Shannon, . . .

  SISO systems, gives intuition, no systematic methods

- Modern control 1960–1990, Kalman, Doyle, . . .

  state-space vs transfer function (polynomial) approach

  Main topics: pole placement, LQ control,
  Kalman filtering, $\mathscr{H}_2/\mathscr{H}_\infty$ optimisation

- Behavioural approach 1990–present, Willems

  the system is the set of all possible trajectories (the behaviour)

  control is interconnection of the plant and a controller

Links with other courses and prerequisites

ELEC 3035 Control system design builds on

- ELEC 1011 Communications and control
- ELEC 2019 Control and systems engineering

A prerequisit for this course is basic knowledge of

- linear algebra
- differential equations

at the level of MATH 1013, MATH 1017, MATH 2021, and MATH 2022.

In addition, we will use extensively MATLAB.