ELEC 3035: Practice problems for part 1
(Academic year 2010/11)

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1. *State-space representation of a system defined by a differential equation* Find a state space representation for a dynamical system defined by the differential equation

$$m\ddot{p} + \alpha\dot{p} + \beta p = u, \tag{1}$$

where p and u are vector valued signals (of the same dimension), and m, α, β are positive constants. Give real-life examples of systems, described by (1). What is the physical meaning of p, u, m, α, and β in the examples?

□

2. *Drawing a circle by a dynamical system (state-space approach)* Find a state space equation $\dot{x} = Ax$ (*i.e.*, specify the matrix A) and an initial condition $x(0)$, such that the corresponding trajectory x is a circle in $\mathbb{R}^2$ with centre $(0,0)$ and radius r.

□

3. *Drawing a circle by a dynamical system (transfer function approach)* Find transfer functions H_1 and H_2 and corresponding inputs u_1 and u_2, such that if y_1 and y_2 are the outputs of the systems defined by H_1 and H_2, respectively, the locus of the points $\big(y_1(t), y_2(t)\big)$ is a circle with centre $(0,0)$ and radius r.

□

4. *Discretization* Discretize the system defined by the equation $m\ddot{p} = u$, assuming that the input u is constant in between the discretization points with value equal to the value at the left point (zero-order hold). Find the corresponding continuous-time and discrete-time state-space representations.

□

5. *Controllability* Is it possible to transfer the continuous-time system of problem 4 from any initial condition $\big(p(0), \dot{p}(0)\big)$ to any final condition $\big(p(T), \dot{p}(T)\big)$, where $T > 0$? Is the same true for the discrete-time system (in which case T is a positive integer—the discrete-time index). What is the smallest T, for which the transfer can be achieved?

□

6. *State transfer* Design an input u for the discrete-time system of problem 4, such that starting from given initial condition $\big(p(0), \dot{p}(0)\big)$, $p(t_i) = d_i$, for $i = 1, \dots, N$, *i.e.*, the trajectory hits a number of targets. ($t_1 > t_2 > \cdots > t_N$ are given moments of time, and d_i are given vectors.)

□

7. *Impact of uncertainty in the initial condition on the trajectory* Consider problem 6, when the horizontal component $p_1(0)$ of the initial position $p(0)$ is uncertain: it is only known to belong to an interval with width 2Δ around a value y_{ini}, *i.e.*, $p_1(0) \in [y_{\text{ini}} - \Delta, y_{\text{ini}} + \Delta]$. Find the maximum deviation of $p(t_i)$ from its target d_i due to the uncertainty in the initial condition. Propose a method to increase the accuracy in achieving the targets.

□

8. Do the skipped exercises from

    `http://www.ecs.soton.ac.uk/~im/elec3035/exercises.pdf` and think of what we did in class, *e.g.*, dynamics of an object (with or without gravity, with or without friction) in case of constant or periodically changing wind. Get a feeling how the basic setup, described in of the exercises, can get arbitrary complicated but the approach for solving the new problems remains the same: 1) derive a state space representation of the model and 2) use the methods (matrix exponential, convolution, observability, controllability) covered in the course to solve the problem.

9. Do the *ELEC 3035: Practice problems for part 1 from the 2009/10 academic year*

    `http://www.ecs.soton.ac.uk/~im/elec3035/practice-problems.pdf`

10. The the following *exercises from [ÅM08]:* 2.1, 2.3, 2.6, 5.4, 5.8(a–c), 8.1, 8.2, 8.4, 8.6.

References

[ÅM08] K. Åström and R. Murray. *Feedback systems: An introduction for scientists and engineers*. Princeton University Press, 2008.