LEARNING KALMAN FILTERING WITH LEGO MINDSTORMS

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Research shows that, in learning science and engineering, guided project work leads to deeper understanding of theoretical concepts (as well as acquisition of hands-on skills) than the classical approach of textbook reading and attending lectures. In an approach to education based on project work, the role of the teacher is to create a stimulating learning environment and to supervise the students in accomplishing their objectives. The main challenge is to come up with projects that are engaging, diverse, and feasible in view of limited time and resources. In this paper, we describe such a signal processing project. The task is to improve the speed and accuracy characteristics of a sensor by real-time signal processing. It turns out that this is an application of Kalman filtering however the students need to identify a model of the sensor and implement the Kalman filter on a DSP. The project consists of three main tasks: 1) mathematical formalization of the problem, 2) development of solution methods, and 3) implementation and testing of the methods. The testing is done on an inexpensive laboratory setup, using the Lego Mindstorms educational kit in combination with a temperature sensor. The learning outcomes are understanding of model representations, system identification, and state estimation, as well as implementation in Matlab and C of real-time signal processing algorithms. Possible extensions include data fusion of multiple sensors, adaptive signal processing, and applications to other sensors.

Index Terms— Kalman filtering, dynamic measurement, system identification, digital signal processing

> "Success in the rapidly changing world of the future depends on being able to do well what you were not taught to do." Seymour Papert [1]

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1. INTRODUCTION

Moore's method [2] in mathematics is to let students rediscover the theory that they are learning. This is done individually or in small groups, where every student has personal contribution. The teacher's role is to supervise the work of the students, suggesting where needed possible ways of overcoming difficulties. The suggestions are hints that are just enough to direct the students into making the discovery on their own. Applied to science and engineering, Moore's method implies working on hands-on projects. The projects should be challenging for the students and make them apply theoretical knowledge that they may have already learned in a classical lectures-based course or they discover on their own by doing the project.

In [3] we have developed a set of analytical problems and laboratory setup, using Lego Mindstorms kit, for learning systems and control theory. The problems are intentionally formulated as open ended curiosity driven questions. Translating the open ended questions into well defined mathematical problems is an essential step in solving the original problems. In our experience, this step is the most challenging one for the students. Students with technical skills in solving the resulting mathematical problems often lack the experience of applying the theoretical knowledge in practice. The aim of the teaching program presented in [3] is to fill this gap in the area of system theory and control.

In this paper, we present another open ended educational project that is aimed at learning digital signal processing and Kalman filtering. The context is a problem in metrology, known as dynamic measurement [4]: Given a measurement device (sensor), improve its speed and accuracy characteristics by real-time signal processing. State-of-the-art methods use adaptive signal processing [5, 6]. As shown in [7], when the sensor dynamics is a priori known the dynamic measurement problem is equivalent to a state estimation problem for an autonomous system, so that the solution is given by the Kalman filter. The project is aimed at rediscovering this fact and exploring its practical consequences on a temperature measurement setup.

2. THE EDUCATIONAL PROJECT

The project is split into four steps.

1. Understand the high-level problem statement "improve the speed and accuracy characteristics of sensor by real-time signal processing".
2. Formalize the high-level problem by formulating a well defined mathematical problem.
3. Develop an algorithm for solving the mathematical problem.
4. Implement and test the algorithm in practice.

2.1. Understanding the problem

As a motivating exercise, the students may be presented with the following problem.

> "A thermometer reading 21°C, which has been inside a house for a long time, is taken outside. After one minute the thermometer reads 15°C; after two minutes it reads 11°C. What is the outside temperature? (According to Newton's law of cooling, an object of higher temperature than its environment cools at a rate that is proportional to the difference in temperature.)" [8, Page 53]

This problem is a simplified version of the project (first order scalar measurement process, exact data, three samples), which still shows the main idea—a "slow" processes can be made faster by data processing. The project is a generalization of the problem and its solution to higher order multivariable processes, noisy data, and real-time data acquisition.

The first task of the students is to understand the problem. They should be able to answer the questions:

- What is the given data and what is to be found?
- How are the data and the to-be-found variables related?

It is important to separate the answers of these questions from the subsequent search for solution methods.

In the dynamic measurement setup, the given data is the sensor reading. In practice, the sensor reading is sampled in time, quantized, and collected in real-time. Initially, the students may and should abstract from the complications resulting from the quantization and the real-time data collection. However, they will be addressed later; in particular, the real-time signal processing is an essential aspect in implementation of the method in practice.

The to-be-found parameter is the measured value. In metrology, it is natural to assume that it is constant over the measurement time. Students who are interested in relaxing this assumptions, *i.e.*, consider tracking of a time-varying quantity, may explore this in a follow-up project. Since the measurement process starts at an initial moment of time, there is a step change in the measured variable. This step change initiates a transient. The transient is the essential object that we analyze in the dynamic measurement problem.

The connection between the data and the measured variable is the sensor dynamics. The sensor is based on a physical process, e.g., heat exchange between the environment and the thermometer in the case of temperature measurement. This is a link between engineering and physics, which leads to modeling using the underlying physical laws. After the problem is conceptually understood, the task of the students is to formalize it mathematically.

2.2. Mathematical formalization

The link between the problem at hand — dynamic measurement — and system theory is done by modeling the sensor as a dynamical system, with input u the measured variable and output y the sensor's reading (see Figure 1). Since the measured variable is constant during the measurement period, the input is a step function $u(t) = \bar{u}s(t)$. Let G be the DC-gain of the sensor and $y_{\text{transient}}$ be the transient process. We have,

$$y = G\bar{u} + y_{\text{transient}}.$$

Assuming that the sensor dynamics is linear time-invariant, it admits a state-space representation, *i.e.*, there are matrices A and C, such that

$$\begin{aligned} x(t+1) &= Ax(t), \qquad x(0) = x_{\text{ini}} \\ y_{\text{transient}}(t) &= Cx(t). \end{aligned} \tag{1}$$

The noise-free sensor's output is then given by

$$\begin{bmatrix} \bar{y}(1) \\ y(2) \\ \vdots \\ y(T) \end{bmatrix} = \underbrace{\begin{bmatrix} G \\ G \\ \vdots \\ G \end{bmatrix}}_{\mathscr{G}} \bar{u} + \underbrace{\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{T-1} \end{bmatrix}}_{\mathscr{O}} x_{\text{ini}}. \tag{2}$$

The key observation is that (2) is a system of linear equations for the unknown input value $\bar{u}$ and the initial conditions x_{ini}.

2.3. An algorithm for solving the problem

With known sensor's model, estimation of $\bar{u}$ from the output y requires solving the overdetermined system of equations

$$\begin{bmatrix} \mathscr{G} & \mathscr{O} \end{bmatrix} \begin{bmatrix} \widehat{u} \\ \widehat{x}_{\text{ini}} \end{bmatrix} \approx y.$$

An optimal in the least-squares sense approximate solution $(\widehat{u}, \widehat{x}_{\text{ini}})$ is given in closed form. Moreover, $\widehat{u}$ is the maximum-likelihood estimator for the true value $\bar{u}$ in the case of zero mean, white, Gaussian measurement noise.

to-be-measured value u (constant during measurement) $\xrightarrow{\text{sensor}}$ measured value y (exhibits transient response)

Fig. 1. The measurement process is modeled as an input-output dynamical system, where the input is the to-be-measured variable and the output is the value measured by the sensor.

The closed form expression for the least-squares solution gives a block algorithm for estimation of $\bar{u}$. A recursive algorithm can be derived or found in the literature, *e.g.*, [9]. The recursive algorithm is initialized with a prior guess of the true value $(\bar{u}, x_{\text{ini}})$ and related covariance matrix of the initial guess uncertainty. Without prior knowledge, one can select as initial guess of $\bar{u}$ the estimate $(G^\top G)G^\top y(1)$ obtained from the first sample, and as x_{ini} a zero vector with associate high covariance matrix.

It can be shown that the recursive least-squares algorithm is the Kalman filter for the augmented system

$$x'(t+1) = \begin{bmatrix} I & \\ & A \end{bmatrix} x'(t), \qquad x'(0) = \begin{bmatrix} \bar{u} \\ x_{\text{ini}} \end{bmatrix}$$
$$y(t) = \begin{bmatrix} G & C \end{bmatrix} x'(t).$$

with uncorrelated output noise and no process noise.

Although at this point we have an algorithm for computing $\widehat{u}$, the problem is not solved yet because the algorithm requires the model's parameters A, C. Therefore, the next step is to estimate them offline. In the special case of temperature measurement the sensor dynamics is described by Newton's law of cooling. It states that the heat exchange between the environment and the thermometer is governed by a first order constant coefficients ordinary differential equation

$$\frac{\mathrm{d}}{\mathrm{d}t} y = a(u - y),$$

where $a > 0$ is a parameter (the heat transfer coefficient). This parameter depends on thermometer as well as the environment. Identifying the model is equivalent to determining a from the output data y. Note that, for the model identification the data is available off-line, so that the input's step level is known. The identification of the model can be done with existing algorithms. However, the identification problem at hand (first order system) is sufficiently simple to be solved by the students without resort of existing methods.

2.4. Practical implementation and sample results

Once the students have derived a real-time solution algorithm, they can proceed with the implementation and testing in practice. They should first implement the algorithm in a high-level programming language, such as Matlab, where bug fixes are faster and easier. Then, they can proceed by testing it on data simulated according to the hypothesis of first order linear time-invariant dynamics and zero mean, white Gaussian measurement noise.

With the testing on simulated data successfully completed, the students can proceed with a low-level implementation of the algorithm and its testing on the experimental setup (see Figure 2). For the DSP implementation we recommend the C-like language, called NXC (Not eXactly C) [10]. It is simple to use and comes with a convenient GUI development environment.

Fig. 2. The experimental setup consists of the Lego NXT brick (the DSP) and a digital Lego temperature sensor.

Sample results on data obtained from human temperature measurement and the corresponding optimal fit by the identified model are shown in Figure 3. The real-time prediction on the same data, obtained with the Kalman filter designed for the identified model, is shown in Figure 4.

Since the prediction of the Kalman filter is tested on the same data that was used for the model identification, the results are not representative for the actual performance of the method. In Figure 5 we verify the robustness of the Kalman filter, by applying it on new data—the temperature of another subject. The result similar performance of the method.

3. CONCLUSIONS AND OUTLOOK

The motivation for this work is a gap between the classical university education and the need of hands-on knowledge in solving problems in practice. A major factor for the existence of the gap is a lack of experience in translating ill-posed "real-life" problems into well defined mathematical ones. This lack of experience is due to insufficient project oriented work where students are presented with open-ended problems and are encouraged to explore freely alternative solutions.

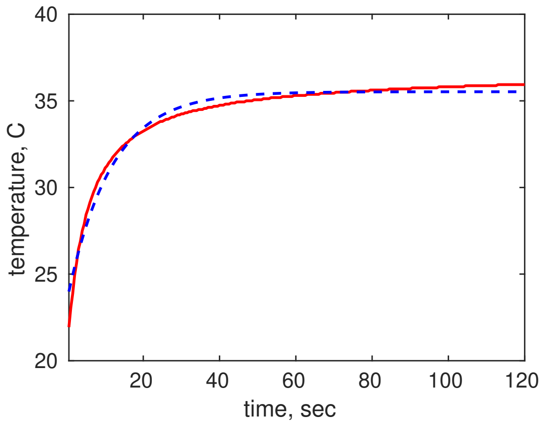

Fig. 3. The model's output (dashed line) fits the data (solid line). The data resembles an exponential function plus a trend, *i.e.*, it does not satisfy the assumption of a first order linear time-invariant dynamics.

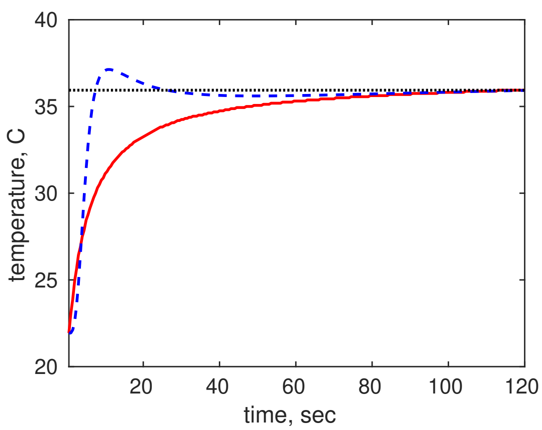

Fig. 4. The prediction of the Kalman filter (dashed line) converges to the measured temperature (dotted line) faster than the natural response of the sensor (solid line).

In this paper, we present a project that illustrates our vision for education by exploration. A variety of subproblems—data modeling, state-estimation, recursive least-squares, Kalman filter—and practical skills—implementation and validation of an algorithm on simulated data in Matlab, C programming for DSP implementation of the method—are used together for solving a metrology problem.

Some nonstandard learning outcomes of the project are: ability to identify and critically analyze assumptions, derivation of the Kalman filter using simple linear algebra only, and testing hypothesis in practice by performing experiments.

Possible extensions of the project are: using multiple sensors (data fusion), dealing with unknown sensor dynamics and time-varying measured quantity (adaptive signal processing), derivation of confidence bounds (statistical analysis), and application of the method to different types of sensors.

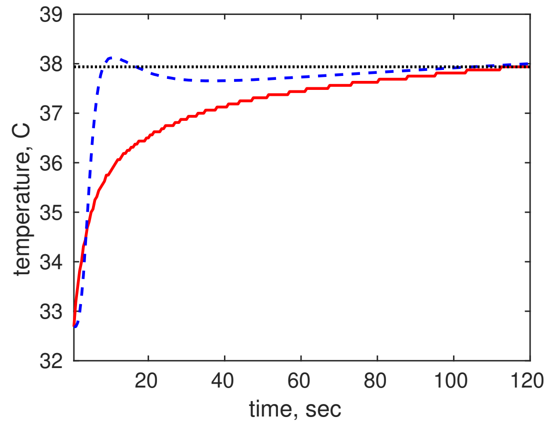

Fig. 5. Prediction of the Kalman filter (dashed line) on different data than the one used for identification still converges to the measured temperature (dotted line) faster than the natural response of the sensor (solid line). This demonstrates the robustness of the method to model uncertainty.

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