ELEC 3035: Tutorial on controllability and observability

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1. *State transfer of a mechanical system* Consider the mass–damper-springer mechanical system, described in Problem 1 from the tutorial on state space and polynomial representations:

$$m\frac{\mathrm{d}^2}{\mathrm{d}t^2}y + d\frac{\mathrm{d}}{\mathrm{d}t}y + ky = u$$

or in an input/state/output form

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}x &= \underbrace{\begin{bmatrix} 0 & 1 \\ -k/m & -d/m \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 0 \\ 1/m \end{bmatrix}}_{b} u \\ y &= \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{c} x + \underbrace{0}_{d} u.\end{aligned}$$

Here m is the mass of the body, k is the elasticity constant of the spring, d is the damping factor of the damper, u is the external force applied on the body, and x is the state vector, consisting of the the body displacement y from its equilibrium position and the body velocity $\mathrm{d}y/\mathrm{d}t$.

   (a) Check whether the system is state controllable.

   (b) Check whether the system is input/output controllable.

   (c) Check whether the system is state observable.

   Do your answers depend on the values of the mass, the elasticity constant, and the damping factor?

□

2. *True–false questions* Which of the following statements are true and which are false. If the answer is "true", give an argument. If the answer is "false", give a counter example. A statement is true if it is correct for any legitimate choice of the free parameters. Otherwise, it is false.

   (a) $\mathscr{B}_{\mathrm{ss}}(A,B)$ uncontrollable implies that $B = 0$ (*i.e.*, the system is autonomous).

   (b) $\mathscr{B}_{\mathrm{ss}}(A,B)$ uncontrollable implies that the state transition problem from a given initial state x_{ini} to a given final state x_{des} in t seconds is unsolvable, *i.e.*, there is no input signal $u : [0,t] \to \mathbb{R}^{\mathtt{m}}$ that transfers the system from state $x(0) = x_{\mathrm{ini}}$ to state $x(t) = x_{\mathrm{des}}$.

   (c) $\mathscr{B}_{\mathrm{ss}}(A,B)$ discrete-time and controllable implies that the state transition problem from a given initial state x_{ini} to a given final state x_{des} in t seconds is solvable, *i.e.*, there is an input signal $u : [0,t] \to \mathbb{R}^{\mathtt{m}}$ that transfers the system from state $x(0) = x_{\mathrm{ini}}$ to state $x(t) = x_{\mathrm{des}}$.

□