

ELEC 3035: Quiz on linear algebra

1. Matrix–vector and matrix–matrix products

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1+4 \\ 2+2 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 11 \\ 4 & 10 \end{bmatrix}$$

2. *Linear system of equations* Solve the following systems of equations in u_1 and u_2 . Give only the final answer. (Do not show your derivation.) If the system has a unique solution, write it down. If the solution is not unique, show the general solution with free parameter(s). If there is no solution, say so.

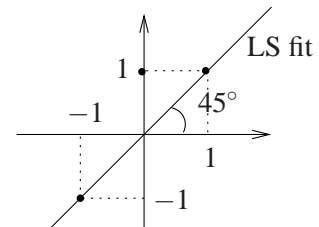
$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}, \quad \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad (\text{unique solution})$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix} \quad \text{is incompatible — there is no solution}$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}, \quad \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 4-2\alpha \\ \alpha \end{bmatrix}, \quad \text{where } \alpha \in \mathbb{R} \quad (\text{nonunique solution})$$

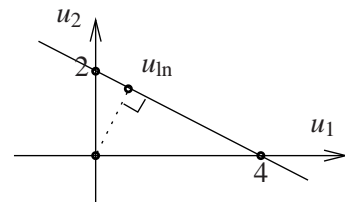
3. *Least squares solution* Find the least squares fit to the points $\begin{bmatrix} -1 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Sketch the solution.

$$\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} u = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \implies u = 2^{-1} 2 = 1$$



4. *Least norm solution* Find the solution $\begin{bmatrix} u_{\text{ln},1} \\ u_{\text{ln},2} \end{bmatrix}$ of the system $\begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = 4$ that has minimal 2-norm, i.e., $u_{\text{ln},1}^2 + u_{\text{ln},2}^2$ is as small as possible.

$$u_{\text{ln}} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} 5^{-1} \cdot 4 = \begin{bmatrix} 4/5 \\ 8/5 \end{bmatrix}$$



5. *Eigenvalues and eigenvectors* What are the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$?

Eigenvalues: **1 and 2**, corresponding eigenvectors $\begin{bmatrix} \alpha \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ \beta \end{bmatrix}$, where $\alpha \in \mathbb{R}$ and $\beta \in \mathbb{R}$.