Signal theory, part 1, test 1 name:

1. *Step function output* (1 points)
   Write down a representation of an autonomous linear time-invariant system that can give a step function output.

2. *Sine function output* (1 points)
   Write down a representation of an autonomous LTI system that can give a sine with frequency ω output.

3. *System's order* (2 points)
   The signal $y(t) = e^t + e^{2t} + e^{3t}$ is a response of an autonomous LTI system. What can you say about the order $\mathtt{n}$ of the system?

   (a) $\mathtt{n} = 1$ (b) $\mathtt{n} = 2$ (c) $\mathtt{n} = 3$ (d) $\mathtt{n} \geq 3$

4. $y \stackrel{?}{\in} \mathscr{B}(A,C)$ (2 points)
   Check if $y_{\mathrm{d}} = (2,3,5,9,17,33,64)$ is a possible output of the system defined by the state space representation

   $$x(t+1) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} x(t), \quad y(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} x(t). \tag{SS}$$

   If y_{d} is not a possible output of the system, suggest a way of correcting it, so that the corrected signal is.

5. $y \stackrel{?}{\in} \ker(P(z))$ (2 points)
   Check if $y_{\mathrm{d}} = (2,3,5,9,17,33,64)$ is a possible output of the system defined by the difference equation

   $$2y(t) - 3y(t+1) + y(t+2) = 0. \tag{KER}$$

   If y_{d} is not a possible output of the system, suggest a way of correcting it, so that the corrected signal is.

6. $\mathscr{B}(A,C) \stackrel{?}{=} \ker(P(z))$ (2 points)
   Check if the system defined by (SS) is the same system as the one defined by (KER).

7. $\mathscr{B}(A_1,C_1) + \mathscr{B}(A_2,C_2)$ (2 points)
   Let $\mathscr{B}$ be the system obtained by adding the outputs of two autonomous LTI systems of orders n_1 and n_2. Is $\mathscr{B}$ linear time-invariant? What is its order?

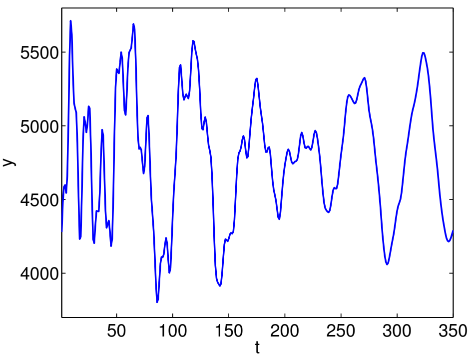
8. *Fast method for computing* A^{100} (4 points)
   How many scalar multiplications requires the direct computation of A^{100} as $\underbrace{A\cdots A}_{100}$ for a 2×2 matrix A?

   Suggest a faster method. Using the method, find a good approximation of $\begin{bmatrix} -1/4 & 1/4 \\ -3/2 & 1 \end{bmatrix}^{100}$.

9. *Is the data generating system LTI?* (4 points)

A colleague of yours shows you the signal on the right and says:

> "I think the data generating system is not linear time-invariant, because the response of such a system is a sum of terms that are exponentially decaying, exponentially growing, or periodic while the behavior of the given signal is more complicated: it is obviously not periodic and it is neither exponentially decaying nor exponentially growing."



Do you agree? If so, how would you make the argument rigorous? If not, what is wrong with the argument and how would you prove that it is wrong? You can assume that you have the observed signal numerically and a computer available to process the data.