ELEC 3035: Lab 3 — Autonomous systems

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1. *Phase plane* For a random second order continuous-time system (choose random A and C matrices), plot (on the same figure) a few state trajectories, starting from different initial conditions. Repeat for different random systems.

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2. *Interpretation of the eigenvalues and eigenvectors* Find i) analytically and ii) using Matlab's function `eig`, the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 0.5 & 1 \\ 0 & 0.9 \end{bmatrix}.$$

   Find i) analytically and ii) using Matlab, the state trajectory of the system $\sigma x = Ax$ (discrete-time) starting from initial conditions:

   (a) $x(0) = v_1$, where $Av_1 = 0.5v_1$ and $\|v_1\| = 1$,

   (b) $x(0) = v_2$, where $Av_2 = 1.1v_2$ and $\|v_2\| = 1$,

   (c) $x_0 = \left[\begin{smallmatrix} 1 \\ 1 \end{smallmatrix}\right]$.

   Plot the resulting trajectories in the phase plane. Comment on the results. □

3. *Harmonic oscillator* Find the response of the harmonic oscillator of page 4 from Lecture 3 to initial condition $x_{\text{ini}} = \begin{bmatrix} 1 & 0 \end{bmatrix}^\top$, i) analytically, and ii) using Matlab. Make sure that the results are equivalent.

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4. *Companion form* Using `rss`, generate a 5th order continuous-time random system. Convert the state space representation, returned by `rss`, to companion form, i) using the function `canon`, ii) by computing the characteristic polynomial of the A matrix, using `poly`. Is the system stable? Check this in two different ways: i) by computing the eigenvalues of A, using the function `eig`, ii) by computing the roots of the characteristic polynomial, using the function `roots`.

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5. *Cayley-Hamilton theorem* Verify numerically the Cayley-Hamilton theorem for a random square matrix A.

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