

ELEC 3035: Quiz on controllability

(15 min)

Name (optional):

Consider the system $\mathcal{B} = \left\{ (u, y) \in (\mathbb{R}^2)^\mathbb{N} \mid \text{there is } x \in (\mathbb{R}^2)^\mathbb{N} \text{ such that } \sigma x = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u, y = \begin{bmatrix} 1 & 1 \end{bmatrix} x \right\}$.

1. *Transfer function, order, stability* Find transfer function and difference equation representations of \mathcal{B} . Suggest a name for \mathcal{B} .

What is the order of \mathcal{B} ? Is \mathcal{B} stable?

2. *Simulation* Find the impulse response of \mathcal{B} (i.e., the response to the unit pulse under zero initial conditions).

3. *Controllability* Is the system $\mathcal{B}_x = \{ (u, x) \in (\mathbb{R}^3)^\mathbb{N} \mid \sigma x = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \}$ controllable? Is it possible to transfer the state from $x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ to $x(2) = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$?

If so, give a control input that achieves the transfer.

4. *Singular system of equations* Solve the system $\begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} u = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$.