In search for useful extensions of the STLS:
problems the method can solve and problems we aim at

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Promoters

Sabine Van Huffel and Bart De Moor

Brain storming meeting, Leuven, 16 June 2003

About my PhD

topic: errors-in-variables (EIV) estimation problems

includes: system identification, filtering, TLS, STLS

motivation: consider measurement errors on the available data

equation error and measurement error in the linear model $Ax = b$

$$Ax = b + e, \; e \text{ equation error}, \; b \text{ is noisy, but } A \text{ is exact}$$
$$(A + \tilde{A})x = b + \tilde{b}, \; \tilde{A}, \tilde{b} \text{ measurement noises, all the data } [A \; b] \text{ is noisy}$$

tools: linear algebra, optimization, systems theory, statistics

collaborations: A. Kukush (Kiev), A. Premoli (Milano), M-L. Rastello (Torino) P. Rapisarda (Univ. Maastricht), R. Pintelon (VUB)

current financing: research council scholarship

Problems and publications

- Linear parametric design

  "Linear parametric design", TR#01-39

  "H2-optimal linear parametric design", TR#01-40

  "Multi-model system parameter estimation", TR#02-40, *SMC 2002*

- Estimation in bilinear EIV model $AXB \approx C$

  "Consistent estimation in the bilinear multivariate EIV model", TR#01-72, *Metrika*

  application in computer vision: fundamental matrix estimation

  "Consistent fundamental matrix estimation in a quadratic measurement error model arising in motion analysis", TR#01-64, *CSDA*

- Ellipsoid estimation

  "Consistent estimation in an implicit quadratic measurement error model", TR#02-115

  "Consistent least squares fitting of ellipsoids", TR#02-116, *Numerische Mathematik*

Problems and publications (cont.)

- Element-wise weighted total least squares problem

  "The element-wise weighted total least squares problem", TR#02-48

- Errors-in-variables Kalman filtering

  "Continuous-time Errors-in-variables Filtering", TR#02–41, *CDC 2002*

  "Linear dynamic filtering with noisy input and output", TR#02–191, *SYSID 2003*

- Structured total least squares

  "Consistency of the STLS estimator in a multivariate EIV model", TR#02–192

  "On the computation of the structured total least squares estimator", TR#02–203

- Stability of reduced order models in subspace identification

Methodologies and algorithms

- adjusted least squares

  inspired by consistency consideration
  results in simple and reliable computational method (just as LS)
  requires Gaussian noise model with known covariance matrix
  noise variance estimation is possible but results in iterative methods
  less efficient than the maximum likelihood method

- maximum likelihood estimation

  nice properties but results in nonconvex optimization problem
  develop efficient local optimization methods

- subspace methods

  robust and efficient computation
  less efficient than the maximum likelihood method

The structured total least squares problem

$$\boxed{\boxed{\min_{\Delta A, \Delta B, X} \left\| \begin{bmatrix} \Delta A & \Delta B \end{bmatrix} \right\|_F^2 \quad \text{s.t.} \quad (A - \Delta A)X = B - \Delta B} \\ \text{and } \begin{bmatrix} \Delta A & \Delta B \end{bmatrix} \text{ has the same structure as } \begin{bmatrix} A & B \end{bmatrix}} \tag{1}$$

assume affine structured data matrix $C := \begin{bmatrix} A & B \end{bmatrix}$

$$C = \mathcal{S}(p) := S_0 + \sum_{l=1}^{n_p} S_l p_l$$

then (1) becomes the following optimization problem

$$\min_{X, \Delta p} \left\| V^{-1/2} \Delta p \right\|_2^2 \quad \text{s.t.} \quad \mathcal{S}(p - \Delta p) \begin{bmatrix} X \\ -I_d \end{bmatrix} = 0$$

Previous work in SISTA on the STLS problem

- B. De Moor, Rimmanian SVD approach

  "Structured total least squares and L_2 approximation problems"
  gives many applications

- Ph.D. thesis P. Lemmerling

  overview of computational methods
  algorithm for deconvolution, the structure of C is `[T U]`
  algorithm for speech compression, C is Hankel

- Ph.D. thesis N. Mastronardi

  efficient (meaning $O(m)$, m sample size) computational algorithms

Errors-in-variables system identification

STLS problem with two blocks structured data matrix $C =$ `[T T]`

with noise-free input or output, C is `[T F]`

for MIMO systems, C has block-Toeplitz/Hankel blocks

in order to cover these (and other) problems, we consider data matrices

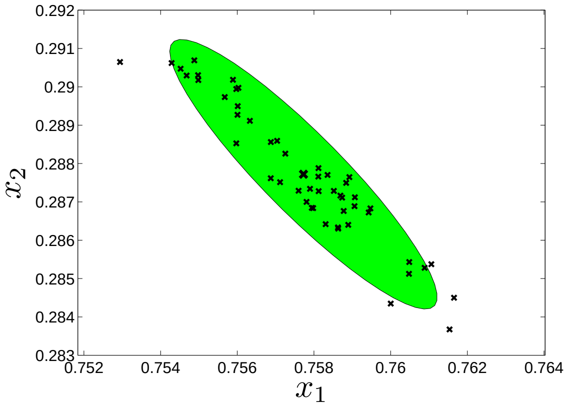
$$C = \mathcal{S}(p) = \begin{bmatrix} C_1 \cdots C_q \end{bmatrix}$$

where C_i is Toeplitz (`T`), Hankel (`H`), unstructured (`U`), or noise free (`F`)

efficient computation is still possible using an alternative approach,
see TR#02–203

Other extensions

- regularized STLS needed in, *e.g.*, image deblurring
- computation of confidence ellipsoids, useful additional information



Other extensions and application

the following extensions are more difficult:

- treat known initial conditions of the time-series model
- allow for more general weighting matrix V

  motivation: different input and output noise variances
- design of optimized algorithms for particular structures

applications:

- improvement of the balanced model reduction/subspace estimate
- image deblurring