Team Control Number/控制编号

For office use only

T1 ________________

T2 ________________

T3 ________________

T4 ________________

42221

Problem Chosen/选题

A

For office use only

F1 ________________

F2 ________________

F3 ________________

F4 ________________

2016 MCM/ICM Summary Sheet

Have a hot bath

This paper establishes the heat transfer model and the optimizing model to provide the person in a simple water containment vessel with the best strategy to add water.

Firstly, according to the Fourier's law, we simplify the hot water added in as a point heat source and develop a partial differential equation model of the temperature of the bathtub water in space and time. Applying ANSYS to solve the partial differential equation, we can get the change of the temperature distribution with the water inflow per unit time and the temperature of water added in.

Secondly, making the water inflow per unit time and the temperature of water added in as design variables and setting the temperature even throughout the bathtub and as close as possible to the initial temperature while without wasting too much water as targets to establish a multi-objective optimization model. Then, we transform the multiple target into single target by using the weight methods. Next, we applying the Genetic Algorithm to solve the model, and getting the results that in the case of the certain size of the bathtub and the person. When the bathing time is 30 min ,the temperature of the hot water added in is $322.71K$, *the water inflow per unit time is12* $10^{-4}m^3$, the variance of the finial temperature of the water is0.05513K^2 and the difference between finial temperature and initial temperature is -0.125K , the water wasted is 0.23m^3 .

Thirdly, we study how would our model's results change with the shape and volume of the tub, the shape/volume/ temperature of the person in the bathtub(**Table 6,7,8 ,Figure 13,14,16**), the motions made by the person(**Figure 17,18**) and the addition of the bubble additive initially(**Table 9 Figure 19**). When the shape of the tub change, in 30 min washing time the temperature of the hot water added in is 324.03K , the water inflow per unit time is $1.5\times10^{-4}m^3$ (**Table 5 Figure 12**).When the volume of the tub is getter larger, the temperature of the hot water added increases and the water inflow per unit time is the same trend(**Figure 7,8, 9,10**).

Finally, we provide a one-page non-technical explanation for users of the bathtub to explain the strategy.

Key words: heat transfer model, partial differential equation, multi-objective optimization model, Genetic Algorithm

2016 年 MCM/ICM 摘要页

洗个热水澡

本文建立的传热模型和优化模型，为人们提供一个简单的水容器，其最好的战略是加水。

首先，根据傅立叶定律，简化了作为点热源的热水的加入，建立了浴缸水在空间和时间上的偏微分方程模型。应用ANSYS 方法求解偏微分方程，我们可以得到每单位时间进水量随水的温度升高而增加。

第二，使每单位时间内水流入浴缸的量和水温增加量为设计变量，设定整个浴缸的温度且尽量靠近初始温度而又不浪费太多的水为目标，建立多目标优化模型。

第三，利用加权法将多目标转化为单目标。接下来，我们应用遗传算法求解模型，得到的结果，在人可以接受浴缸的大小。当洗澡的时间是 30 分钟时，热水温度增加 322.71K，单位时间的水量为 $12\times10^{-4}m^3$，水的最终温度变化为 0.05513K^2，最终温度和初始温度之间的差为-0.125K，浪费水是 0.23m^3。

第四，我们研究如何将我们的结果应用到不同形状和变体积的浴缸的模型里，形状/体积/浴缸里的人的体温（**表 6、7、8、图 13,14,16**），人体的运动（**图 17、18**）和最初外加的泡沫添加剂（**表 9 图 19**）。当桶形状变化时，在 30 分钟的洗涤时间内，热水温度的增加是 324.03K，单位时间内的水量是 1.5 $10^{4}m^3$(**表 5，图 12**）。当浴缸体积较大的吸气剂，热水的温度增加和每单位时间内流入的水是相同的趋势（**图 7、8、9、10**）。

最后，我们为用户提供了一个非技术性的浴缸说明战略的单页。

关键词：传热模型，偏微分方程，多目标优化模型，遗传算法

Contents

目录

1 Introduction

1.1 Background

The bathtub, which is a simple water containment vessel and uses ceramic or other adiabatic material is often used in our daily life. Taking people's comfort into consideration, the bathtub is designed as a container with a water inlet and water outlet, and when the tub reaches its capacity, excess water escapes through the overflow drain.

Aimed to get relaxed and cleansed, filling the bathtub with hot water before bathing is what we always do. With the time goes by, the temperature of the water will gradually drop. In order to maintain the suitable temperature, adding a constant trickle of hot water from the faucet to reheat the bathing water is necessary.

To have a comfortable hot bath is important for us to relax ourselves. However, there was no research on the model that can predict the temperature change of the water in the bathtub and guide people how to adjust, thus, our task is to build a model to provide a method for people to adjust the temperature of the water.

1.2 Our work

Our work begins with a heat transfer model. In this model, the heat conduction equation which uses the Fourier's law and the energy conservation law to describe the change of temperature of the water in the bathtub is established. Firstly, we only consider the circumstance that the bathtub whose shape is cuboid is filled with water and without people init. Besides, we put a person into the model and adds a boundary condition to the heat conduction equation to get the final heat transfer model.

Based on this, we set targets keep the temperature even in the bathtub and as close as possible to the initial temperature while wasting as little as possible of the water when taking a bath to optimize the heat transfer model.

Since our task is to provide people with the strategy to put the water into the bathtub, to test the model applicability in our daily life, we change the characteristic of the bathtub, the characteristic of the person in the bathtub and add the bubble additive to find how would the result change of the multi-objective optimization model.

In this paper, the finite element analysis and ANSYS are applied to solve a four- dimensional differential heat conduction equation to get the temperature distribution of the water in the bathtub.

1 问题介绍

1.1 问题背景

浴缸是一种简单的水容器，在日常
用陶瓷或其他绝热材料。考虑到人们的
被设计成带有进水口和出水口的容器，
量时，多余的水通过溢流口排出。

洗澡前，先用热水把浴缸装满热水，这样我们就可以轻松放松了。随着时间的推移，水温会逐渐下降。为了保持适宜的温度，加上一个恒定的热水的水龙头加热洗澡水是必要的。

Figure 1 The bathtub we use / 图 1 我们使用的浴缸

有一个舒适的热水浴是很重要的，我们放松自己。然而，没有一个模型可以预测浴缸中的水的温度变化来指导人们如何调整，因此，我们的任务是建立一个模型，以提供一种方法，为人们调节水的温度。

1.2 我们的任务

首先我们的工作建立传热模型。在此模型中，建立了用傅立叶定律和能量守恒定律描述浴缸水温变化的热传导方程。首先，我们只考虑浴缸的形状是充满水的长方体，没有人在里面的条件。此外，我们把人的模型，一起添加到边界条件的热传导方程，得到最终的传热模型。

在此基础上，我们设定的目标，即使保持在浴缸中的温度且尽可能接近初始温度，尽可能少的水洗澡时的优化传热模型。

因为我们的任务是为人们提供了把水倒入浴缸的策略，在我们的日常生活的模型适用性检验，我们改变了浴缸的特征、人的特征、在浴缸里添加泡沫添加剂等，找到多目标优化模型的结果的将如何变化。

在本文中，我们把 ANSYS 有限元分析应用于解决在澡盆里的水的温度分布的四维微分热传导方程。

2 Problem restatement

The problems that we need to solve in this paper are:

Develop a model of the temperature of the bathtub water in space and time to determine the best strategy the person in the bathtub can adopt to keep the temperature even throughout the bathtub and as close as possible to the initial temperature without wasting too much water.

Taking the shape and volume of the tub, the shape/volume/temperature of the person in the bathtub, the motions made by the person into consideration, to what extent does your strategy depend on these elements by using your model you build.

If a bubble additive is used initially, how would this affect the model's results?

3 Terminology

3.1 Terms

Heat transfer: the phenomena that energy transfers from a hotter object to the cooler object or heat migrates from the hotter part of the object to the cooler part of the object in the case of no work. It includes three mechanisms, which are thermal conduction, thermal radiation and thermal convection.

Thermal conduction: the heat transfer phenomena when there is no macroscopic motion within the medium, which can occur in solids, liquids and gases.

3.2 Symbols

Symbols	Descriptions
$T(x,y,z,t)$	the temperature of the object in position (x,y,z) and at time t
$F(x,y,z,t)$	the heat that add to the bathtub per unit time per unit volume of heat source
Ω	the region surrounded by the closed surface whose boundaries are the surface of the water and the bathtub
Ω_1	the region of the heat source
V_p	the volume of the water added in per unit time
T_c	the temperature of the outlet of the water
T_r	the temperature of the inlet of the water
Γ	the closed surface whose boundaries are the surface of the water and the bathtub
$\Gamma_i\ (i=1,2,3)$	the surface that the water contact with air, the water contact with the bathtub and the water contact with the person
V_Ω	the volume of the water in the bathtub
T_0	the initial water temperature that is suitable for people

4 General Assumptions

- The initial temperature of the water of each point in the bathtub is the same.
- The room is big enough thus the temperature of the room is constant and don't change over time and space.

2 问题重述

本文所要解决的问题是：

建立浴缸水的温度

的模型，以确定最佳的

的人可以采用，以保持整个浴缸的温度，尽可能接近初始温度，而不会浪费太多的水。

考虑浴缸的形状和体积，浴缸里的人的形状/体积/温度，考虑到人的动作，你的策略在何种程度上取决于你使用的模型。

如果最初使用泡沫添加剂，这会如何影响模型的结果？

3 术语

3.1 条款

热转印：能量从热物体的冷或热迁移对象的对象从较热的部分在没有工作的情况下对象的冷却器部分现象。它包括三个机制，即热传导，热辐射和热对流。

热传导：在介质中没有宏观运动的传热现象，它可以发生在固体、液体和气体中。

3.2 符号系统

符号	说明
$T(x,y,z,t)$	目标 x、y、z 的位置和 t 时间段的温度
$F\ (x,y,z,t)$	每单位体积单位时间内热源增加到浴缸的热量
Ω	表示被封闭表面包围区域，其边界是水和浴缸的表面.
Ω_1	热源的区域
V_p	单位时间内加水量
T_c	出口水的温度
T_r	入口水的温度
Γ	封闭的表面，其边界是水的表面和浴缸
$\Gamma_i\,(i=1,2,3)$	水与空气接触的表面，与浴缸的水接触和与人的水接触
V_Ω	浴缸里的水的体积
T_0	适合人体的初始水温

4 模型假设

- 在浴缸的每个点的水的初始温度是相同的。
- 房间足够大，所以房间的温度是恒定的，不会随时间和空间而改变。

- The wall of the bathtub is insulated, thus the heat transfer between it and water is ignored; because most of the bathtubs' material is ceramic, therefore, the heat preservation is better.
- Viewing the hot water injection point at the water surface as a heat source, because the water added in is trickle and can't influence the water in the bathtub.
- Regardless of the heat convection, only considering thermal conduction when hot water is added to the bathtub. Because the water we add in is a trickle, the flow of it can be ignored.
- Regardless of the heat dissipation in the process of the hot water flow into the bathtub, because the time that hot water flow into the bathtub is short.

5 The Heat Transfer Model

In this section, we build a system with bathtub, the outlet of the water, the inlet of the water, the temperature and the velocity of the hot water in it at first. Then we build the basic model which includes heat conduction equation to describe the temperature distribution of the water in the bathtub. At this part, we have two boundaries (the boundary that the surface water contact with air and the boundary that the bottom and side face of the water that contact with bathtub), so we get two boundary conditions. Based on this, we add a person into the system, and when the person is taking the shower, there is heat exchange between them. We can view the person as another medium and add another boundary condition to the heat conduction equation.

To solve the heat conduction equation which is a four-dimensional differential equation, we adopt the finite element analysis and use the ANSYS to find the temperature distribution at the end.

5.1 Local Assumption

- Function $T(x, y, z, t)$ has the second continuous partial derivatives for all independent variables (the function $T(x, y, z, t)$ shows the temperature of object in position (x, y, z) and at time t).
- The water in the bathtub is isotropic and homogeneous; because the water difference has little difference, we can view the density of water is the same in every direction.
- The person added into the bathtub can be viewed as two cuboid spliced together, because in general, the person's body is homogeneous.

5.2 Basic model

To make the problem simple, we set bathtub of a cuboid shape and only fill the water into the bathtub at first. Besides, there is no people in it to build the basic model.

The bathtub we use in this model is showed in Figure 2:

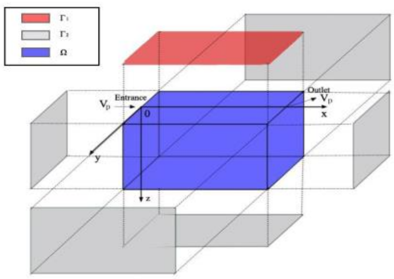

Figure 2 The bathtub we set in this model/图 2 我们建立的浴缸模型

To make the research of the problem easy, we build a three-dimensional coordinate system whose origin is the hot water injection point at the water surface, x-axis is parallel to the long side of the cuboid, y-axis is parallel to the short side of the cuboid and z-axis forms a right-handed coordinate system with x, y axis.

- The Fourier's law

The Fourier's law describes that the heat flow through an infinitesimal area along the surface normal direction in an infinitesimal period is proportional to the directional derivative of temperature along the surface normal direction:

- 浴缸的壁是绝缘的，传热可以忽视；因为的材料是陶瓷，因此
- 将热水注入点视为热缓的加入，是不会影
- 无论热对流，只考虑热传导时，热水被添加到浴缸。因为我们添加的是缓缓的水，它的流量可以忽略。
- 无论在热水进入浴缸的过程中散热，因为热水进入浴缸的时间短。

5 传热模型

在这一部分中，我们首先建立了一个系统，包括浴缸，水的出口，水的入口，在它的第一个热水的温度和流速。然后建立了描述浴缸内水温分布的热传导方程的基本模型。在这一部分，我们有两个边界（边界水与空气接触的边界和与浴缸接触的水的底部和侧面的边界），所以我们得到两个边界条件。在此基础上，我们添加一个人到系统中，当洗澡的人，有他们之间的热交换。我们可以把人看作另一种介质，并把另一个边界条件加入到热传导方程中.。

解决热传导方程的四维微分方程，采用有限元分析和利用 ANSYS 在最后找到温度分布。

5.1 局部假设

- 函数（x，Y，Z），有第二连续偏导数的所有独立变量（函数 T（x，y，z，t）显示对象的位置（X，Y，Z）的温度与时间 t）。
- 浴缸里的水是各向同性的，均匀的，因为水的差别不大，我们可以看到水的密度在每个方向都是一样的.。
- 人入浴缸可以被看作是两个长方体拼接在一起，因为在一般情况下，人的身体是均匀的。

5.2 基本模型

为了简化问题，首先，我们设置一个长方体形状的浴缸，注满水的浴缸。此外，没有人在它建立的基本模型。

我们在这个模型中使用的浴缸如图 2 所示.：

对这个问题作了简单的研究，我们建立了一个三维坐标系统的原点是热水注入点在水面上，X 轴是平行的长方体的长边平行，Y 轴和 Z 轴的长方体短边形成一个右手坐标系 x，y 轴。

- 傅立叶定律

傅立叶定律描述了在无穷小周期内沿表面法线方向的无穷小区域的热流与表面法线方向的温度方向导数成正比：

$$dQ = -\lambda \frac{\partial T(x,y,z,t)}{\partial n} dSdt \quad (1)$$

Where:

$T(x, y, z, t)$ describes the temperature of the object in position (x, y, z) and at time t ; λ is the thermal conductivity ;

dQ is the heat flow through an infinitesimal area along the surface normal direction in an infinitesimal period;

$\frac{\partial T}{\partial n}$ is the directional derivative of temperature along the surface normal direction;

dS is the infinitesimal area and dt is the infinitesimal period.

- **The heat conduction equation**

For heat transfer in the water with a heat source, the energy Conservation Law with the Fourier's law (Equation (2)) is as bellow:

$$\begin{cases} \int_{t_1}^{t_2} \iiint_{\Omega} \left[c\rho \frac{\partial T}{\partial t} - \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) - \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) - \frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right) \right] dxdydzdt - H = 0 \\ H = \int_{t_1}^{t_2} \iiint_{\Omega_1} F_1(x,y,z,t)\, dxdydzdt - \int_{t_1}^{t_2} \iiint_{\Omega_2} F_2(x,y,z,t)\, dxdydzdt \end{cases} \quad (2)$$

Where:

$\frac{\partial T}{\partial t}$ is the rate of change of temperature at a point over time;

$T(x, y, z, t)$ describes the temperature of the object in position (x, y, z) and at time t;

λ is the thermal conductivity;

c is the specific heat capacity of water and ρ is the density of water;

$F_1(x, y, z, t)$ is the heat that heat source releases per unit time per unit volume;

$F_2(x, y, z, t)$ is the heat that water outflow releases per unit time per unit volume;

Ω_1 is the region of the heat source flowing in per unit time;

Ω_2 is the region of the water flowing out the bathtub per unit time;

Ω is the region surrounded by the closed surface whose boundaries are the surface of the water and the bathtub.

As the bathtub is full of water initially, the water inflow is equal to the water outflow, we can get:

$$\int_{t_1}^{t_2} F_1 V_p dt - \int_{t_1}^{t_2} F_2 V_p dt = \int_{t_1}^{t_2} (F_1 - F_2) V_p dt = \int_{t_1}^{t_2} F V_p dt \quad (3)$$

Where:

$F(x, y, z, t)$ is the heat that add to the bathtub per unit time per unit volume of heat source.

For the temperature of the hot water added in is constant, thereby the value of the $F(x, y, z, t)$ is constant, we can put $F(x, y, z, t)$ outside of the integral sign. So we get:

$$\iiint_{\Omega} \left[c\rho \frac{\partial T}{\partial t} - \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) - \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) - \frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right) \right] dxdydz - FV_p = 0 \quad (3)$$

Where:

V_p is the volume of the water added in per unit time.

For the value of the variables t_1 and t_2 is arbitrary, and for any time interval, the equation (3) is tenable. Thus, we can get:

$$\iiint_{\Omega} \left[c\rho \frac{\partial T}{\partial t} - \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) - \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) - \frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right) \right] dxdydz - FV_p = 0 \quad (4)$$

Put forward the integral sign:

$$\iiint_{\Omega} \left[c\rho \frac{\partial T}{\partial t} - \frac{FV_p}{V_\Omega} - \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) - \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) - \frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right) \right] dxdydz = 0 \quad (5)$$

$$Q = -\lambda \frac{\partial T(x,y,z,t)}{\partial n} ($$

其中：

T（x，y，z，T）描述对

y，z）和时间 t 的温度

dQ是热流通过沿表面法

穷小面积微元段；

$\frac{\partial T}{\partial n}$是沿表面法线方向的方向导数的温度；

dS 是无穷小面积和 dt 是无穷小时间。

- **热传导方程**

与热源的水换热，用傅里叶定律能量守恒定律（方程（2））如下：

$$\begin{cases} \int_{t_1}^{t_2} \iiint_{\Omega} \left[c\rho \frac{\partial T}{\partial t} - \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) - \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) - \frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right) \right] dxdydzdt - H = 0 \\ H = \int_{t_1}^{t_2} \iiint_{\Omega_1} F_1(x,y,z,t)\, dxdydzdt - \int_{t_1}^{t_2} \iiint_{\Omega_2} F_2(x,y,z,t)\, dxdydzdt \end{cases} \quad (2)$$

其中：

$\frac{\partial T}{\partial t}$是在一点时间温度变化率；

T（x，y，z，t）描述在 t 时刻在位置（x，y，z）对象的温度；

λ 是导热系数；

c 是水的比热容量，ρ是水的密度；

$F_1(x, y, z, t)$是热源释放每单位时间每单位体积的热；

Ω_1是热源单位时间内流区；

Ω_2 是单位时间内流出浴缸的水的区域；

Ω是被封闭的表面包围的区域，其边界是水的表面和浴缸。

由于浴缸里充满了水，最初的水流入等于水流出，我们可以得到：

$$\int_{t_1}^{t_2} F_1 V_p dt - \int_{t_1}^{t_2} F_2 V_p dt = \int_{t_1}^{t_2} (F_1 - F_2) V_p dt = \int_{t_1}^{t_2} F V_p dt \quad (3)$$

其中：

$F(x, y, z, t)$是浴缸里增加单位时间每单位体积热源的热。

对于加入的热水的温度是恒定的，从而 F（x，y，z，t）的值是恒定的，我们可以把 F（x，y，z，t）以外的积分符号。所以我们得到：

$$\iiint_{\Omega} \left[c\rho \frac{\partial T}{\partial t} - \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) - \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) - \frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right) \right] dxdydz - FV_p = 0 \quad (3)$$

其中：

V_P是单位时间内增加的水的体积。

变量 t_1 和 t_2 的值是任意的，对于任何时间间隔，方程（3）都是成立的。因此，我们可以得到：

$$\iiint_{\Omega} \left[c\rho \frac{\partial T}{\partial t} - \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) - \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) - \frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right) \right] dxdydz - FV_p = 0 \quad (4)$$

提出积分符号：

$$\iiint_{\Omega} \left[c\rho \frac{\partial T}{\partial t} - \frac{FV_p}{V_\Omega} - \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) - \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) - \frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right) \right] dxdydz = 0 \quad (5)$$

Where:

V_Ω is the volume of the water in the bathtub.

F is the heat that the heat source releases per unit time per unit volume and can be calculated in the equation shown bellow:

$$F = c\rho(T_r - T_c) \tag{6}$$

Where:

T_r is the temperature of the inlet of water;

T_c is the temperature of the outlet of the water.

Eventually, the heat conduction equation we get is;

$$\frac{\partial T}{\partial t} = \frac{(T_r - T_c)V_p}{V_\Omega} + \frac{\lambda}{c\rho}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) \tag{7}$$

- **Boundary conditions**

We divide the closed surface Γ whose boundaries are the surface of the water and the bathtub into two part Γ_1, Γ_2 :

$$\Gamma = \Gamma_1 \cup \Gamma_2 \tag{8}$$

Where:

Γ_1 is the surface that the water contact with air;

Γ_2 is the surface that the water contact with bathtub.

Then we get the boundary conditions for surface Γ_1, Γ_2 :

$$\lambda_1 T + \lambda \frac{\partial T}{\partial \vec{n}_1} = \lambda_1 T_1, (x, y, z) \in \Gamma_1 \tag{9}$$

$$\lambda_2 T + \lambda \frac{\partial T}{\partial \vec{n}_2} = \lambda_2 T_2, (x, y, z) \in \Gamma_2 \tag{10}$$

Where:

λ is the thermal conductivity among water;

λ_1 is the heat transfer coefficient between water and air;

λ_2 is the heat transfer coefficient between water and bathtub;

T_1 is the temperature of the air and T_2 is the temperature of the bathtub;

T is the temperature of the surface of the water;

$\vec{n}_1, \vec{n}_2$ are the normal vectors for the upper surface of the water the faces that water contact with bathtub;

$\frac{\partial T}{\partial \vec{n}_i} (i=1,2)$ are the directional derivative of temperature along the upper surface of the water the faces that the water contact with bathtub.

$$\frac{\partial T}{\partial \vec{n}_2} = 0, (x, y, z) \in \Gamma_2 \tag{11}$$

- **Initial condition**

Besides, we provide the initial condition that at the initial time t=0 , the value of temperature in each point of bathtub is the initial temperature of the water T_0 in bathtub which is suitable for people:

$$T(x,y,z,0) = T_0 \tag{12}$$

Where:

T_0 is the initial water temperature that is suitable for people;

$T(x, y, z, 0)$ is the water temperature in position (x, y, z) at the initial time.

- **The final equations**

Adding the initial condition and the boundary condition to the heat conduction equation, we get the heat transfer model eventually. The final equations are shown bellow;

其中：

V_Ω 表示在浴缸里的水的

F 是单位时间单位体积

可以计算出以下的热

$$F = c\rho(T_r - T_c)$$

其中：

T_r 是水的入口的温度；

T_c 是水的出口温度

最终，我们得到的热传导方程是；

$$\frac{\partial T}{\partial t} = \frac{(T_r - T_c)V_p}{V_\Omega} + \frac{\lambda}{c\rho}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) \tag{7}$$

- **边界条件**

我们把封闭的表面表示为Γ，其边界是水的表面和浴缸分为两部分Γ_1，Γ_2:

$$\Gamma = \Gamma_1 \cup \Gamma_2 \tag{8}$$

其中：

Γ_1 是水与空气接触的表面；

Γ_2 是水与浴缸接触的表面。

$$\lambda_1 T + \lambda \frac{\partial T}{\partial \vec{n}_1} = \lambda_1 T_1, (x, y, z) \in \Gamma_1 \tag{9}$$

$$\lambda_2 T + \lambda \frac{\partial T}{\partial \vec{n}_2} = \lambda_2 T_2, (x, y, z) \in \Gamma_2 \tag{10}$$

其中：

λ 在水的导热系数；

λ_1是水和空气之间的热传递系数；

λ2是水和浴缸之间的传热系数；

T1是空气的温度,T2是浴缸的温度；

T是水的表面温度；

$\vec{n}_1, \vec{n}_2$分别为正常载体水的上表面，浴缸水接触面；

$\frac{\partial T}{\partial \vec{n}_i} (i=1,2)$在水的表面与水的接触面上的温度方向导数。

$$\frac{\partial T}{\partial \vec{n}_2} = 0, (x, y, z) \in \Gamma_2 \tag{11}$$

- **初始条件**

此外，我们在初始时刻τ = 0提供初始条件，温度在浴缸里每一点的值是在浴缸适合水的初始温度T_0:

$$T(x,y,z,0) = T_0 \tag{12}$$

其中：

T_0是适合人的初始水温度；

- **最终的方程**

在热传导方程中加入初始条件和边界条件，得到热导模型。最后给出波纹管；

$$\begin{cases}\dfrac{\partial T}{\partial t}=f+\dfrac{\lambda}{c\rho}\left(\dfrac{\partial^2 T}{\partial x^2}+\dfrac{\partial^2 T}{\partial y^2}+\dfrac{\partial^2 T}{\partial z^2}\right),(x,y,z)\in\Omega\\ f=\dfrac{(T_r-T_c)V_p}{V_s}\\ T(x,y,z,0)=T_0,(x,y,z)\in\Omega\\ \lambda_1 T+\lambda\dfrac{\partial T}{\partial \vec{n}_1}=\lambda_1 T_1,(x,y,z)\in\Gamma_1\\ \dfrac{\partial T}{\partial \vec{n}_2}=0,(x,y,z)\in\Gamma_2\end{cases} \tag{13}$$

Where:

$\frac{\partial T}{\partial t}$ is the rate of change of temperature at a point over time;

$T(x,y,z,t)$ describes the temperature of the object in position (x,y,z) and at time t;

λ is the thermal conductivity ;

c is the specific heat capacity of water and ρ is the density of water;

V_Ω is the volume of the water in the bathtub;

V_p is the volume of the water added in per unit time;

λ_1 is the heat transfer coefficient between water and air;

T_0 is the initial water temperature that is suitable for people;

n_1, n_2 are the normal vectors for the upper surface of the water, the faces that the water contact with bathtub;

T_1 is the temperature of the air.

5.3 Take people into consideration

Further, we take a person into consideration. The person will have the temperature exchange with the water while having the shower. To research the influence of the person to the temperature distribution of the water in the bathtub, we view the person's shape as two cuboid spliced together which is shown in **Figure 3**, the size of the cuboid bellow is $1.1m\times0.45m\times0.25m$ and the size of the cuboid above is $0.5m\times0.3m\times0.45m$

Figure 3 The placement of the person in water

The addition of the person add one boundary condition to the heat conduction equation, which is:

$$\lambda_3 T+\lambda\frac{\partial T}{\partial \vec{n}_3}=\lambda_3 T_3 \tag{14}$$

Where:

T_3 is the temperature of the person;

λ_3 is the heat transfer coefficient between water and the person;

$\vec{n}_2$ is the normal vector of the surface of the person;

Adding the boundary condition to equation (12), we get:

$$\begin{cases}\dfrac{\partial T}{\partial t}=f+\dfrac{\lambda}{c\rho}\left(\dfrac{\partial^2 T}{\partial x^2}+\dfrac{\partial^2 T}{\partial y^2}+\dfrac{\partial^2 T}{\partial z^2}\right),(x,y,z)\in\Omega\\ f=\dfrac{(T_r-T_c)V_p}{V_\Omega}\\ T(x,y,z,0)=T_0,(x,y,z)\in\Omega\\ \lambda_1 T+\lambda\dfrac{\partial T}{\partial \vec{n}_1}=\lambda_1 T_1,(x,y,z)\in\Gamma_1\\ \dfrac{\partial T}{\partial \vec{n}_2}=0,(x,y,z)\in\Gamma_2\\ \lambda_3 T+\lambda\dfrac{\partial T}{\partial \vec{n}_3}=\lambda_3 T_3,(x,y,z)\in\Gamma_3\end{cases} \tag{15}$$

$$\begin{cases}\dfrac{\partial T}{\partial t}=f+\dfrac{\lambda}{c\rho}\left(\dfrac{\partial^2 T}{\partial x^2}+\dfrac{\partial^2 T}{\partial y^2}+\dfrac{\partial^2 T}{\partial z^2}\right),(x,y,z)\in\Omega\\ f=\dfrac{(T_r-T_c)V_p}{V_s}\\ T(x,y,z,0)=T_0,(x,y,z)\in\Omega\\ \lambda_1 T+\lambda\dfrac{\partial T}{\partial \vec{n}_1}=\lambda_1 T_1,(x,y,z)\in\Gamma_1\\ \dfrac{\partial T}{\partial \vec{n}_2}=0,(x,y,z)\in\Gamma_2\end{cases}$$

其中：

$\frac{\partial T}{\partial t}$是在一点时间温度变化率；

T（x，y，z，t）描述在 t 时刻在位置（x，y，z）对象的温度；

λ 是导热系数；

c 是水的比热容量，　是水的密度；

V_Ω 是浴缸里的水的体积；

V_P 是单位时间内增加的水的体积；

$_1$ 是水和空气之间的热传递系数；

T_0 是适合人的初始水温度；

$\vec{n}_1, \vec{n}_2$分别为正常载体水的上表面，浴缸水接触面；

T_1 是空气的温度。

5.3 考虑有人的情况

此外，我们考虑到一个人。淋浴时，人将与水交换温度。研究人在浴缸里的水的温度分布的影响，我们认为，人的形状为长方体拼接在一起，如图 3 所示，在长方体的大小低于 $1.1m\times0.45m\times0.25m$ 和长方体的大小为 $0.5m\times0.3m\times0.45m$.

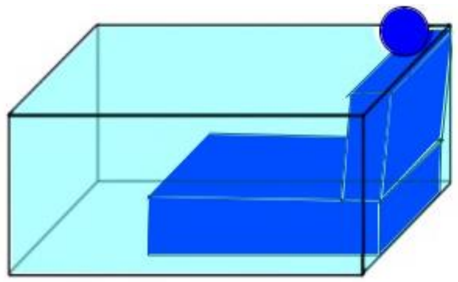

图 3 有人的水缸

添加一个增加人的边界条件的热传导方程，是：

$$\lambda_3 T+\lambda\frac{\partial T}{\partial \vec{n}_3}=\lambda_3 T_3 \tag{14}$$

其中：

T_3 是人的温度；

λ_3 是水和人之间的传热系数；

$\vec{n}_2$是人的表面法线向量；

将边界条件加入到方程（12）中，得到：

$$\begin{cases}\dfrac{\partial T}{\partial t}=f+\dfrac{\lambda}{c\rho}\left(\dfrac{\partial^2 T}{\partial x^2}+\dfrac{\partial^2 T}{\partial y^2}+\dfrac{\partial^2 T}{\partial z^2}\right),(x,y,z)\in\Omega\\ f=\dfrac{(T_r-T_c)V_p}{V_\Omega}\\ T(x,y,z,0)=T_0,(x,y,z)\in\Omega\\ \lambda_1 T+\lambda\dfrac{\partial T}{\partial \vec{n}_1}=\lambda_1 T_1,(x,y,z)\in\Gamma_1\\ \dfrac{\partial T}{\partial \vec{n}_2}=0,(x,y,z)\in\Gamma_2\\ \lambda_3 T+\lambda\dfrac{\partial T}{\partial \vec{n}_3}=\lambda_3 T_3,(x,y,z)\in\Gamma_3\end{cases} \tag{15}$$

Where:

$\frac{\partial T}{\partial t}$ is the rate of change of temperature at a point over time;

$T(x,y,z,t)$ describes the temperature of the object in position (x,y,z) and at time t;

λ is the thermal conductivity ;

c is the specific heat capacity of water and ρ is the density of water;

V_Ω is the volume of the water in the bathtub;

$\lambda_i\ (i=1,3)$ is the heat transfer coefficient between water and air and between water and the person;

T_0 is the initial water temperature that is suitable for people

$\vec{n}_1,\vec{n}_2,\vec{n}_3$ are the normal vectors for the upper surface of the water, the faces that the water contact with bathtub and the surface of the person;

$T_i\,(i=1,3)$ is the temperature of the air and temperature of the person.

5.4 ANSYS simulation

Given a certain circumstance, in which the bathing time is 30 min , the size of the bathtub is $1.5\times0.5\times0.7$ and the values of other variables we set are shown in the **Table 1**.

Table 1 The values of the parameters

Parameter	f	λ_1	T_1	c	T_0
Value	$100J$	$15J/(Ksm)$	$308K$	$4200J/(kg\cdot K)$	$321K$
Parameter	ρ	λ	λ_3	T_3	
Value	$1000kg/m^3$	$0.64J/(Ksm)$	$200J/(Ksm)$	$320K$	

we use the heat transfer model and apply ANSYS to simulate the temperature distribution of the water with a person in the bathtub. The process of the temperature distribution for 2-D dimensional is shown in Figure 3(the 2-D dimensional plane we choose in the bathtub is parallel to the horizontal plane)

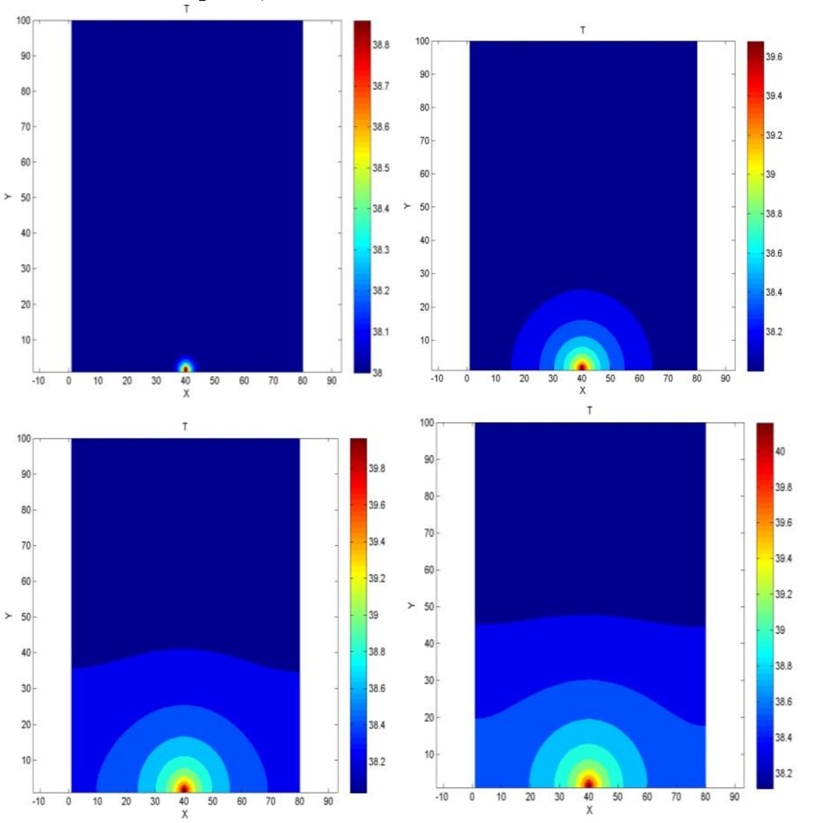

Figure 4 The process of the temperature distribution for 2-D dimensional

The temperature distribution of the water in the bathtub can be seen in Figure 5:

Figure 5 The temperature distribution in certain circumstance

其中：

$\frac{\partial T}{\partial t}$是在一点时间温度变

T（x，y，z，t）描述在

（x，y，z）对象的温度

λ 是导热系数；

c 是水的比热容量，ρ是水的密度；

V_Ω 是浴缸里的水的体积；

$\lambda_i\,(i=1,3)$是水和空气，水和人之间的传热系数；

T_0 是适合人的初始水温度；

$\vec{n}_1,\vec{n}_2,\vec{n}_3$分别为正常载体水的上表面，浴缸水接触面，人的表面接触的表面；

$T_i\,(i=1,3)$是空气的温度和人的温度。

5.4 ANSYS 模拟

在一定的情况下，在洗澡的时间是 30 分钟，浴缸的尺寸是 $1.5\times0.5\times0.7$ 和其他变量的值，我们在表 1 所示。

表 1 参数的值表

参数	f	λ_1	T_1	c	T_0
值	$100J$	$15J/(Ksm)$	$308K$	$4200J/(kg$	321 K
参数	ρ	λ	λ_3	T_3	
值	$1000kg/m^3$	$0.64J/(Ksm)$	$200J/(Ksm)$	$320K$	

我们使用的传热模型，采用 ANSYS 在浴缸里一个人模拟水的温度分布。二维尺寸的温度分布过程如图 3 所示（我们在浴缸中选择的二维尺寸平面与水平面平行）

图 4 二维温度分布的过程图

浴缸里的水温度分布见图 5:

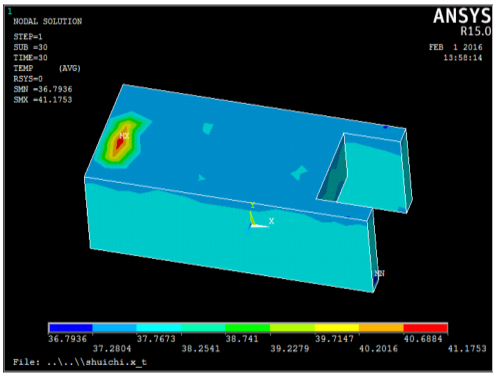

图 5 特定情况下的温度分布

6 Multi-objective optimization model

In this section, with the targets of wasting as little water as possible, keeping the temperature even and as close as possible to the initial temperature, we apply a multi- objective optimization to the heat transfer model to get the best strategy that can be applied for people in our daily life.

6.1 The build of the multi-objective optimization model

To make the temperature in the bathtub even and as close as possible to the initial temperature, we need to calculate the finial average temperature of the water after taking the shower, which is:

$$\overline{T}_z = \frac{1}{V_\Omega}\iiint_\Omega T(x,y,z,t_z)dxdydz \tag{16}$$

Where:

t_z is the time that the person finish the bathing;

$\overline{T}_z$ is the finial average temperature of the water after taking the shower;

To make the temperature as evenly as possible. The first objective function is:

$$\min \quad \sigma^2 = \frac{1}{V_\Omega}\iiint_\Omega (T(x,y,z,t_z)-\overline{T}_z)^2 dxdydz \tag{17}$$

Where:

σ^2 is the variance of the finial temperature of the water.

To make the temperature of the water as close as possible to the initial temperature. The second objective function is:

$$\text{Min } \Delta T_z = T_z - T_0 \tag{18}$$

Where:

ΔT_z is the temperature difference between final average temperature and initial temperature of the water in bathtub.

As the bathtub is filled with water at the initial time, the water inflow is equal to the water outflow. We can minimize the water inflow to get the minimum wasting water. The third objective function is:

$$\text{Min } V_p \tag{19}$$

The range of the temperature we added in is from 317k to 323k, and the range of the volume of the hot water added in per unit time is from 0 to $2\times10^{-4}m^3$, which are the restriction conditions:

$$st.\begin{cases}317K \le T_r \le 323K \\ 0 \le V_p \le 2\times10^{-4}m^3\end{cases} \tag{20}$$

Eventually, we get the finial multi-objective optimization model, which is:

$$\begin{aligned}
&find \quad T_r \quad V_p \\
&\min \quad \sigma^2 = \frac{1}{V_\Omega}\iiint_\Omega (T(x,y,z,t_z)-\overline{T}_z)^2 dxdydz \\
&\min \quad \Delta T_z = \overline{T}_z - T_0 \\
&\min \quad V_p \\
&st.\begin{cases}317K \le T_r \le 323K \\ 0 \le V_p \le 2\times10^{-4}m^3\end{cases}
\end{aligned} \tag{21}$$

Where:

T_z is the finial average temperature of the water after taking the shower;

t_z is the time that the person finish the bathing;

σ^2 is the variance of the finial temperature of the water;

$T(x,y,z,t_z)$ is the temperature of the water in positon (x,y,z) and at time t;

ΔT_z is the temperature difference between final average temperature and initial temperature of the water in bathtub;

T_0 is the initial water temperature that is suitable for people;

T_r is the temperature of the inlet of water;

6 多目标优化模型

在这一部分中，以尽可能
保持温度尽可能接近初始
将多目标优化应用到传热
了适合人们日常生活的

6.1 多目标优化模型的建立

使温度在浴缸里甚至和尽可能的初始温度，我们需要洗澡后的水最终平均温度的计算，为：

$$\overline{T}_z = \frac{1}{V_\Omega}\iiint_\Omega T(x,y,z,t_z)dxdydz \tag{16}$$

其中：

t_z是人完成洗澡所用的时间

T_z是洗澡后的水最终平均温度；

使温度尽可能稳定。第一个目标函数是：

$$\min \quad \sigma^2 = \frac{1}{V_\Omega}\iiint_\Omega (T(x,y,z,t_z)-\overline{T}_z)^2 dxdydz \tag{17}$$

其中：

σ^2是水的最终温度的变化。

使水的温度尽可能接近初始温度。第二个目标函数是：

$$\text{Min } \Delta T_z = T_z - T_0 \tag{18}$$

其中：

ΔT_z浴缸内最终平均温度与初始水温的温差。

当浴缸充满水的初始时间，流入的水等于流出。我们可以尽量减少水的流入，以获得最小的浪费水。第三个目标函数是：

$$\text{Min } V_p \tag{19}$$

我们增加了在温度范围从317k到323K，在单位时间内增加的热水量的范围是从 0 到 $2\times10^{-4}m^3$，这个限制条件是：

$$st.\begin{cases}317K \le T_r \le 323K \\ 0 \le V_p \le 2\times10^{-4}m^3\end{cases} \tag{20}$$

最终，我们得出了最终的多目标优化模型是：

$$\begin{aligned}
&find \quad T_r \quad V_p \\
&\min \quad \sigma^2 = \frac{1}{V_\Omega}\iiint_\Omega (T(x,y,z,t_z)-\overline{T}_z)^2 dxdydz \\
&\min \quad \Delta T_z = \overline{T}_z - T_0 \\
&\min \quad V_p \\
&st.\begin{cases}317K \le T_r \le 323K \\ 0 \le V_p \le 2\times10^{-4}m^3\end{cases}
\end{aligned} \tag{21}$$

其中：

T_z是洗澡后的水最终平均温度；

t_z是人完成洗澡的时间；

σ^2是水的最终温度的变化；

T（x，y，z，t_z）是位置的水的温度（x，y，z）和在时间 t；

ΔT_z在浴缸最终平均温度和水的初始温度之间的温差；

T_0是初始水温度是适合的人；

T_r是水入口温度；

T_c is the temperature of the outlet of the water;
V_p is the volume of the water added in per unit time.
To solve the multi-objective optimization model, we set different weight to the variance of the final temperature and the temperature difference between final average temperature and initial temperature, and change the multi-objective to the single objective optimization model:

$$
\begin{aligned}
& find \quad T_r \quad V_p \\
& \min \quad \omega_1\sigma^2+\omega_2\Delta T_z+\omega_3 V_p \\
& st.\begin{cases} 317K \le T_r \le 323K \\ 0 \le V_p \le 2\times10^{-4}m^3 \\ \omega_1+\omega_2+\omega_3=1 \\ \omega_i(i=1,2,3)\ge 0 \end{cases}
\end{aligned}
\tag{22}
$$

Where:
V_p is the volume of the water added in per unit time;
T_r is the temperature of the inlet of water;
σ^2 is the variance of the finial temperature of the water;
ΔT_z is the temperature difference between final average temperature and initial temperature of the water in bathtub;
ω_1 is the weight of the the variance of the finial temperature of the water;
ω_2 is the weight of the temperature difference between final average temperature and initial temperature of the water in bathtub;
ω_3 is the weight of the water wasted per unit time.
Using this model, we can calculate the temperature and the velocity of the hot water we need to add in with the certain targets, thereby, a strategy to add water can be provided for people.

6.2 The strategy to add water

To solve the multi-objective optimization model in order to propose the strategy to add water, we use the Genetic Algorithm and address the problem in four steps:
(1) At first step, we select countless values from the range of the temperature and the range of the volume of the hot water added in per unit time we set in equation (20)
(2) At second step, using the objective function Min $\omega_1\sigma^2+\omega_2\Delta T+\omega_3V_p$ to filter the selected values, and then get the values that satisfied the objective function
(3) At third step, based on the values we picked out, using the objective function to filter.
(4) At fourth step, repeat the third step until find out the best strategy.
We set the values of the parameters in the Genetic Algorithm, which is shown in Table 2:

Table 2 The values of the parameters in Genetic Algorithm

Parameter	Evolutionary times	Crossover probabilit	Mutation probability	Sample size
Value	8	0.4	0.2	41
Paramet	ω_1	ω_2	ω_3	
Value	0.6	0.4	0	

Using Matlab to programme and call the ANSYS to calculate the best strategy to add water. The values of the variables in the model shown in the **Table 3** and the strategy how to add the water is shown in **Table 4.**

Table 3 The values of the variables in the model

Paramet	λ_1	T_1	c	T_0	Size of the bathtub
Value	$15J/(Ksm)$	$308K$	$4200J/(kg\cdot K)$	$321K$	$1.5\times0.5\times0.7$
Paramet	ρ	λ	λ_3	T_3	Shower time
Value	$1000kg/m^3$	$0.64J/(Ksm)$	$200J/(Ksm)$	$320K$	30 min

T_c是水出口的温度；
V_p是单位时间内的水的体
为了解决多目标优化模型
的权重的最终温度的方差
温度和初始温度之间的温
目标到单目标优化模型：

$$
\begin{aligned}
& find \quad T_r \quad V_p \\
& \min \quad \omega_1\sigma^2+\omega_2\Delta T_z+\omega_3 V_p \\
& st.\begin{cases} 317K \le T_r \le 323K \\ 0 \le V_p \le 2\times10^{-4}m^3 \\ \omega_1+\omega_2+\omega_3=1 \\ \omega_i(i=1,2,3)\ge 0 \end{cases}
\end{aligned}
\tag{22}
$$

其中：
V_p是单位时间内的水的体积。
T_r是水入口温度；
σ^2是水的最终温度的变化；
ΔT_Z在浴缸最终平均温度和水的初始温度之间的温度差；
ω_1是水的最终温度的权重；
ω_2是浴缸内平均温度与初始水温的温差；
ω_3是单位时间浪费的水的权重。
利用这个模型，我们可以计算出的温度和流速的热水，我们需要添加在一定的目标，从而为人们提供了一个加水的策略。

6.2 加水策略

为了解决多目标优化模型，以提出加水的策略，我们使用遗传算法和解决问题的四个步骤：
（1）在第一步中，我们从温度和温度的范围内选择无数个数值，在我们设定的单位时间内加入热水。
（2）第二步，使用目标函数 Min $\omega_1\sigma^2+\omega_2\Delta T+\omega_3V_p$ 过滤选定的值，进而得到满足目标函数值
（3）在第三步，根据我们挑选出来的值，使用目标函数进行滤波。
（4）在第四步，重复第第三步，直到找到最佳策略。
我们在遗传算法中设置参数的值，如表 2 所示：

表 2 遗传算法中参数的取值

参数	进化时代	交叉概率	变异概率	样本大小
值	8	0.4	0.2	41
参数	ω_1	ω_2	ω_3	
值	0.6	0.4	0	

用 matlab 编程，调用 ANSYS 计算补充水分的最佳策略。表 3 中所示的变量的值以及如何添加水的策略如表 4 所示。

表 3 模型中变量的值

参数	λ_1	T_1	c	T_0	浴缸大小
值	$15J/(Ksm)$	$308K$	$4200J/(kg\cdot K)$	$321K$	$1.5\times0.5\times0.7$
参数	ρ	λ	λ_3	T_3	淋浴的时间
值	$1000kg/m^3$	$0.64J/(Ksm)$	$200J/(Ksm)$	$320K$	30 min

Table 4 The best strategy to add water

Variable	σ^2	ΔT_z	V	T_r	V_p
Value	0.05513	$-0.125K$	$0.23m^3$	$322.71K$	$1.2\times10^{-4}\ m^3$

Where:

V is the water we waste.

We get the final temperature distribution after shower of the best strategy in Figure 6.

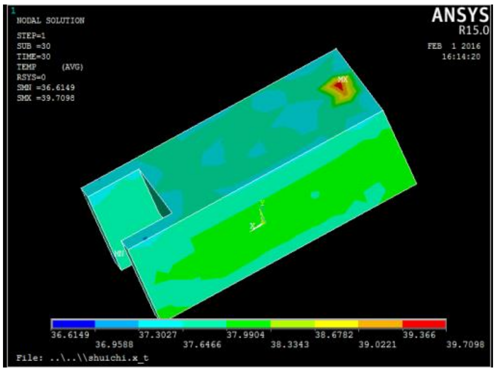

Figure 6 The temperature distribution of the best strategy

7 Sensitivity Analysis

To provide people with strategy to add water better. In this part, we find the influence of the shape and volume of the tub, the shape/volume/temperature of the person taking a bathing and the bubble additive to the multi-objective optimization model.

First step, we research the influence of the characteristic of the tub. We change the size of the length, width and height of the tub of a cuboid shape respectively to the multi-objective optimization and we design the shape of the bathtub closer to life to find the better strategy to add water.

Next step, we research the influence of the characteristic of the person. We change the size of the person and the temperature to find the results of the multi-objective optimization model. Besides, we take the motion of the person in the water into consideration. The movement of the person leads to the movement of the water, thus we can think that the forced convection happens to the water which makes the heat transfer coefficient between water and air and between water and the person get changed. At the end, we add the bubble bath additive initially, which only change the heat transfer coefficient between water and air λ_1, to find how the strategy changes.

Lastly, we research the influence of the addition of the bubble additive which changes the heat transfer coefficient between water and air. Changing the heat transfer coefficient between water and air to get the best strategy to add water.

7.1 Change the characteristic of the bathtub

- **Change the area of the bathtub's upper surface**

We change the area of the bathtub's upper surface to change the volume of the bathtub, and get the change of optimal temperature of the water added in and the optimal water inflow per unit time with the area, the result is shown on **Figure 7,8**

Figure 7 The change of the optimal temperature of the water added in with the area

Figure 8 The change of the optimal water inflow per unit time with area

表 4 添加水的最

变量	σ^2	ΔT_z	V
值	0.05513	$-0.125K$	$0.23m^3$

其中：

V 是我们浪费的水。

我们得到最终的温度分布淋浴后的最佳策略图 6。

图 6 最优策略的温度分布

7 敏感性分析

为人们提供更好的加水策略。在这一部分中，我们发现的形状和体积的浴缸，洗澡/气泡添加剂的形状/体积/温度的影响的多目标优化模型。

第一步，我们研究了浴缸的特性的影响。我们改变长度的大小，一个长方体的形状分别为多目标优化的槽的宽度和高度，我们设计的浴缸更接近生活中找到更好的策略添加水的形状。

下一步，我们研究人的特性的影响。我们改变的人的大小和温度，以找到结果的多目标优化模型。此外，我们考虑在水中的人的议案。人的运动导致水的运动，因此我们可以认为，强迫对流发生的水，使水和空气之间的传热系数和水和人得到改变。最后，我们把泡泡浴添加剂最初，只改变水和空气 1 之间的换热系数，找出策略的变化。

最后研究了气泡添加剂的加入对水与空气换热系数的影响.。改变水和空气之间的传热系数，以获得最佳的加水策略。

7.1 改变浴缸的特点

- **改变浴缸上表面的面积**

我们改变了浴缸的上表面面积改变浴缸的体积，得到的水的最佳温度的变化增加和优化水流入单位时间和面积，结果显示在图 7、8

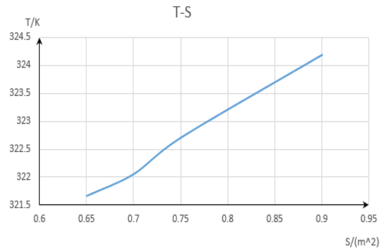

图 7 加入该区域的水的最佳温度的变化

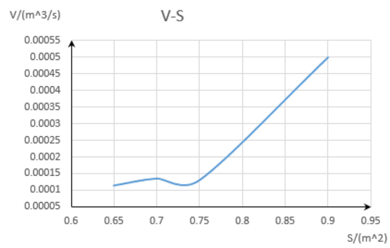

图 8 每单位时间的最佳进水量随面积的变化

Where:

x-axis in **Figure 7,8** represents the change of the area;

y-axis in **Figure** 7 represents the change of the optimal temperature of the water added in;

y-axis in **Figure 8** represents the change of the optimal volume of the water inflow per unit time

The increase of the upper surface's area causes the increase of the heat exchange (between water and area), moreover the heat added in increases. We can find both optimal temperature and optimal water inflow per unit time are increasing with the increase of the area.

- **Change the height of the bathtub**

We change the height of the bathtub to change the volume of the bathtub, and get the change of optimal temperature of the water added in and the optimal water inflow per unit time with the height, the result is shown on Figure 9,10.

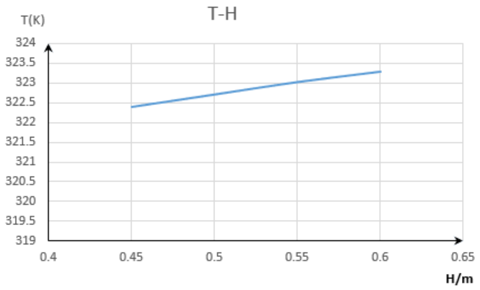

Figure 9 The change of the optimal temperature with the height

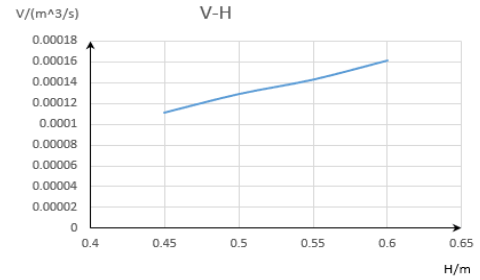

Figure 10 The change of the optimal water inflow per unit time with the height

Where:

x-axis in Figure 9,10 represents the change of the height;

y-axis in Figure 9 represents the change of the optimal temperature of the water added in;

y-axis in Figure 10 represents the change of the optimal water inflow per unit time.

The increase of the height only causes the increase of the volume of the bathtub, while the area of the water contact with air is not influenced, furthermore, the heat loss isn't influenced. Thus the curves in the Figure 9,10 are relatively flat.

- **Change the shape of the bathtub**

Based on the multi-objective optimization model, and keeping the volume of the bathtub invariant, we design the shape of the bathtub closer to life which is presented in Figure 11 to find the best strategy to add water(Table 5) and the final temperature distribution(Figure 12)

Table 5 The best strategy to add water after changing the shape of the bathtub

Variable	σ^2	ΔT_z	V	T_r	V_p
Value	0.0676	$-0.1503K$	$0.27m^3$	$324.03K$	$1.5\times10^{-4}\ m^3$

Figure 11 The shape of the bathtub after changing

Figure 12 The temperature distribution after changing the shape of the bathtub

其中：

图 7、8 X 轴代表该地区

在**图 7** 中，Y 轴代表了水加；

在**图 8** 中，Y 轴代表单位佳体积的变化

上表面的面积的增加会导致热交换（水和区域之间）的增加，而且增加的热量增加。我们可以发现最佳的温度和最佳的单位时间的进水量随面积的增加而增加。

- **浴缸的高度变化**

我们改变了浴缸的高度改变浴缸的体积，得到的水的最佳温度的变化增加和优化水流入每单位时间的高度，其结果是在图 9 所示。

图 9 最佳温度随高度的变化

图 10 单位时间内最佳进水量随高度的变化

其中：

图 9，X 轴代表高度的变化；

在图 9 中，Y 轴代表了水的温度变化增加；

在图 10 中，Y 轴代表单位时间的最优水流入的变化。

高度的增加只会增加浴缸的体积，而不影响与空气接触的水的面积，而且不影响热损失。因此在图 9 曲线相对平坦。

- **改变浴缸的形状**

基于多目标优化模型，并保持不变的浴缸的体积，我们设计的浴缸的形状更贴近生活，如图 11 所示，以找到最佳的策略，以补充水（表 5）和最终的温度分布（图 12）

表 5 更换浴缸形状后加水的最佳策略

变量	σ^2	ΔT_z	V	T_r	V_p
值	0.0676	$-0.1503K$	$0.27m^3$	$324.03K$	$1.5\times10^{-4}\ m^3$

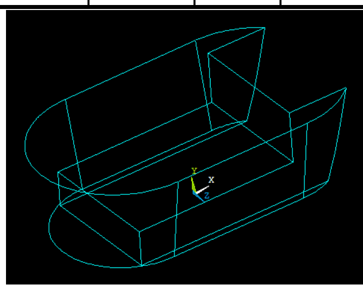

图 11 浴缸形状变化后

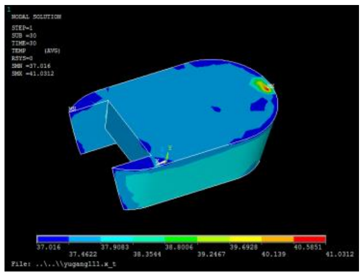

图 12 浴缸形状变化后的温度分布

From the figures above, we can find that the temperature of the hot water added in is higher, and the water inflow per unit time is larger than the bathtub of the cuboid shape. With the same volume the area of the upper surface of the bathtub is bigger and the heat loss increases, thus more heat needs to be added in.

7.2 Change the characteristic of the person

- **Change the volume of the person**

We change the volume of the person which influence the boundary condition of the person and get the change of optimal temperature of the water added in and the optimal water inflow per unit time with the volume of the person, the result is shown on Figure 13,14.

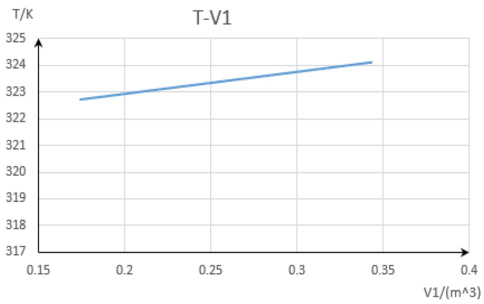

Figure 13 The change of the optimal temperature with the volume of the person

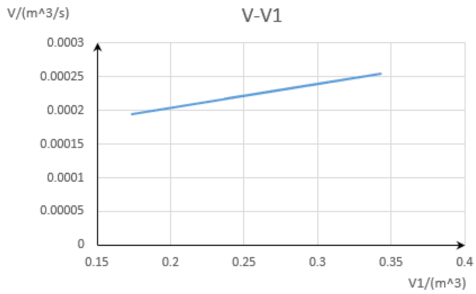

Figure 14 The change of the optimal water inflow per unit time with the volume of the person

Where:

x-axis in Figure 13,14 represents the change of the volume of the person;

y-axis in Figure 13 represents the change of the optimal temperature of the water added in;

y-axis in Figure 14 represents the change of the optimal water inflow per unit time.

From the figures, we find that the strategy to add water change a little with the increase of the volume of the person. For the increase of the volume of the person resulting in the

surface people contact with water. Furthermore, the heat exchange between people and water get a little increase, so the heat added in has slight increase.

- **Change the shape of the person**

Keeping the volume of the person invariant and we change the shape of the person into the two cylinders spliced together, which is shown in Figure 15, to find the best strategy to add water(Table 6) and the final temperature distribution(Figure 16).

Table 6 The best strategy to add water when change the shape of the person

Variable	σ^2	ΔT_z	V	T_r	V_p
Value	0.0266	$-0.158K$	$0.358m^3$	$323.135K$	1.99×10^{-4} m^3

Figure 15 the shape of the person after change

Figure 16 the temperature distribution after changing the shape of the person

从以上数字，我们可以发度增加更高，单位时间内长方体形状的浴缸。同样上表面面积较大，热损失要增加更多的热量。

7.2 改变人的特性

改变人的数量

我们改变的人，影响人的边界条件，得到的体积和水的最佳温度的变化增加和优化水流入单位时间与人的量，结果显示在图 13,14。

图 13 最佳温度随体积的变化

图 14 单位时间的最佳进水量随人员体积的变化

其中：

图 13、14 X 轴代表人的体积变化；

在图 13 中， Y 轴代表了水的温度变化增加；

图 14 表示每单位时间的最佳进水量变化的 Y 轴。

从这些数字来看，我们发现加水的策略随着人的数量的增加有点变化.。为增加的体积的人造成

表面人与水接触。此外，人与水之间的热交换略有增加，所以增加的热量略有增加。

- 改变人的形状

保持不变的人的体积，我们改变了人的两圆柱拼接在一起的形状， 如图 15 所示，找到最好的补充水分的策略(表 6）和最后的温度分布（图 16）。

表 6 改变人物形状时加水的最佳策略

变量	σ^2	ΔT_z	V	T_r	Vp
值	0.0266	$-0.158K$	$0.358m^3$	$323.135K$	1.99×10^{-4} m^3

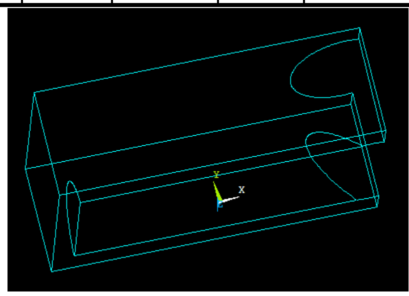

图 15 更改后的人的形状

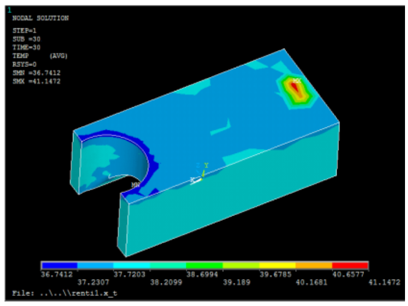

图 16 改变人的形状后的温度分布

From the result, we can find that when the shape of the person is close to reality, the difference between the final temperature and the initial temperature, the wasting water and the water inflow per unit time get some increase. According to the mathematical knowledge, in the same volume, the cylinder's surface area is bigger than the cuboid's surface area thus the area the person contact with water increases which leads to more heat exchange. Hence, more heat is needed.

- **Change the temperature of the person**

The range of the temperature change of the person is relatively small, thus we only study the strategy to add water when the temperature are 319.5K and 320.5K which is shown in Table 7,8.

Table 7 The best strategy to add water when the temperature of the person is 319.5K

Variable	σ^2	ΔT_z	V	T_r	V_p
Value	0.0487	$-0.1176K$	$0.361m^3$	$319.5K$	$2.01\times10^{-4}\ m^3$

Table 8 The best strategy to add water when the temperature of the person is 320.5K

Variable	σ^2	ΔT_z	V	T_r	V_p
Value	0.0413	$-0.1215K$	$0.342m^3$	$320.5K$	$1.90\times10^{-4}\ m^3$

Although the temperature of the surface of the body maybe change for some reasons, for example, getting ill, the range is relatively small. And the temperature of body is close to the temperature of water, so the heat exchange is small. Thus, the change of the temperature of the person has little influence on the strategy to add water.

7.3 Add the motion of the person in the bathtub

Considering the actual situation, we add the motion of the person in the bathtub to obtain the strategy to add water. The motion of the person only change the heat transfer coefficient between water and air λ_1 and the heat transfer coefficient between water and the person λ_3 and other variables are the same as the values of the variables we choose in Table 3.

We use percentage to measure the intensity of the motion, and obtain the change of the optimal temperature of the water added in and the optimal water inflow per unit time with the percentage, the result is shown on Figure 17,18.

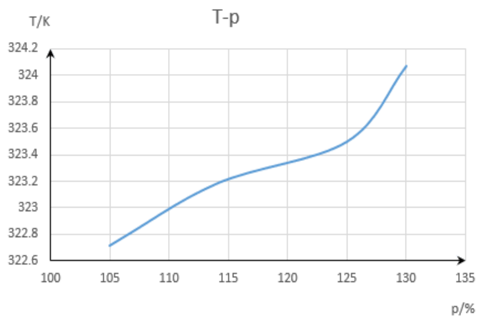

Figure 17 The change of optimal temperature with the percentage

Figure 18 The change of the optimal water inflow per unit time with the percentage

Where:

x-axis in Figure 17,18 represents the change of the percentage which measures the intensity of the motion;

y-axis in Figure 17 represents the change of the optimal temperature of the water added in;

y-axis in Figure 18 represents the change of the optimal water inflow per unit time.

We can find that the optimal temperature of the water added in and the optimal water inflow per unit time are get larger with the increase of the intensity of the motion. Because the intenser the movement is, the more heat dissipates. If we want to keep the temperature even and as close as possible to the initial temperature, more heat will be added in the bathtub.

结果表明，当人的形状接终温度与初始温度、单位量和进水量之间的差异得的提高。根据数学知识，在圆柱的表面积大于长方体域水增加导致更多的人接此，需要更多的热量。

- **改变人的温度**

人的温度变化范围比较小，因此我们只研究加水时，温度 319.5k 和 320.5k 是表 7、8 所示的策略。

表 7 最佳策略添加水时，人的温度 319.5k

变量	σ^2	ΔT_z	V	T_r	V_p
值	0.0487	$-0.1176K$	$0.361m^3$	$319.5K$	$2.01\times10^{-4}\ m^3$

表 8 最佳策略添加水时，人的温度 320.5k

变量	σ^2	ΔT_z	V	T_r	V_p
值	0.0413	$-0.1215K$	$0.342m^3$	$320.5K$	$1.90\times10^{-4}\ m^3$

虽然身体表面的温度可能因某些原因改变，例如生病，范围相对较小。而且体温接近水的温度，所以热交换小。因此，人的温度变化对加水的策略影响不大。

7.3 加入浴缸里的人的动作

考虑到实际情况，我们在浴缸中添加人的运动，以获得加水的策略。运动的人改变的只有水和空气 1、水和人 3 和其他变量之间的传热系数之间的传热系数作为我们选择表 3 中的变量的值相同。

我们用百分比来衡量运动强度，得到的水的温度的变化增加和最佳进水的每单位时间的比例，结果显示在图 18。

图 17 最佳温度随百分比的变化

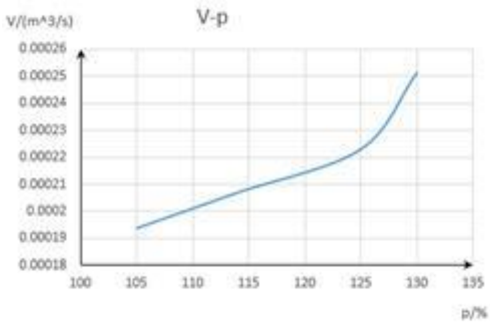

图 18 单位时间最优进水量随百分率的变化

其中：

图 18 x 轴代表的百分比衡量运动强度的变化；

在图 17 中，y 轴代表了水的温度变化增加；

在图 18 中，y 轴代表单位时间的最优水流入的变化。

我们可以发现，最佳的温度增加的水和每单位时间的最佳进水量随着运动强度的增加变得更大。因为激烈的运动，更多的热量。如果我们想保持温度和尽可能接近初始温度，将增加更多的热量在浴缸里。

7.4 Add the bubble additive to the bathwater

The addition of the bubble additive only change the heat transfer coefficient between water and air λ_1, the values of the other variables in the multi-objective optimization we choose are the same as what we choose in Table 3. We set the value of λ_1 as 11J / (Ksm) ,and obtain the strategy to add water (Table 9). At the same time, the final temperature distribution is presented in Figure 19.

Table 9 The best strategy to add water when the bubble additive is added initially

Variable	σ^2	ΔT_z	V	T_r	V_p
Value	0.0189	$-0.1144K$	$0.186m^3$	$321.02K$	$1.59\times10^{-4}\ m^3$

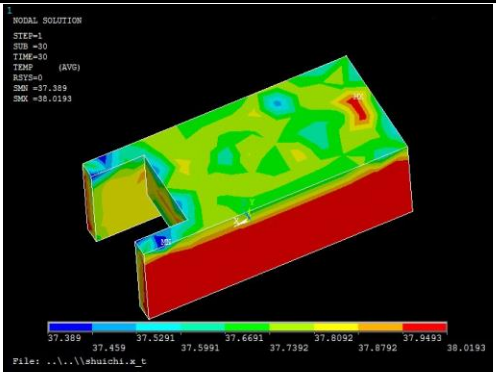

Figure 19 The final temperature distribution when the bubble additive is added initially

We come to the conclusion that compared with the circumstance without the bubble additive in the water, the variance of the finial temperature of the water and the water inflow per unit time increase, while the temperature of the inlet of water and the temperature difference between final average temperature and initial temperature of the water in bathtub decrease.

8 Strengths and Weaknesses

8.1 Strengths

(1) The main strength of our model is close to the reality. The heat exchange between people and water and between water and air is considered when research the temperature distribution in the bathtub.

(2) To test the application of the model in our daily life, we take the characteristic of the person and the motions of the person into discussion.

(3) Besides, considering the comfort of the person, we also change the cross section of the bathtub to isosceles trapezoid and change the sloped angle of the trapezoid to discuss the temperature distribution.

8.2 Weaknesses

(1) The calculation of our model is complex, we need to solve a four-dimensional differential equation.

(2) The bathroom of small size whose room temperature is changed is not taken into consideration.

(3) When we change the shape of the person in the bathtub in the sensitivity analysis of our model, we only consider the change based on the shape of a cuboid. But other changes such as cylinder is not taken into consideration.

9 Explanation for users

The bathtub we use in our daily life is a simple water containment vessel, in order to take a shower comfortably, the person needs to add a constant trickle of the hot water from the faucet. Here, we give the explanation of the strategy to add water researched in this paper.

If the bathtub you use is a little larger than the normal size, you should increase the temperature of the hot water added in by14.28K per unit cubic meter. You'd batter to use the bathtub whose upper surface is not very big. Besides, if you are fat, maybe you should increase the temperature of the hot water a little (about 0.5K).

7.4 添加泡沫添加齐

气泡添加剂的加入只改
之间的传热系数λ_1，在我
标优化的其他变量的值
选择在表 3。设定的λ_1的
（KSM），并获得战略补充水分（表 9）。在同一时间，最终的温度分布图 19 所示。

表 9 初始添加泡沫添加剂时加水的最佳策略

变量	σ^2	ΔT_z	V	T_r	V_p
值	0.0189	$-0.1144K$	$0.286m^3$	$321.02K$	$1.59\times10^{-4}\ m^3$

图 19 初始添加气泡添加剂时的最终温度分布

我们得出的结论是，没有泡在水中的添加剂的情况相比，水的最终温度的变化和单位时间增加的水流入，而水的进口温度和降低最终的平均温度和浴缸的水的初始温度之间的温度差。

8 优势和劣势

8.1 大优势

（1）我们模型的主要力量接近现实。在研究浴缸内温度分布时，考虑了人与水、水与空气之间的热交换。

（2）为了检验模型在我们日常生活中的应用，我们将人的特性和人的行为。

（3）此外，考虑到人的舒适性，我们也改变了浴缸的截面等腰梯形改变梯形倾斜角度讨论温度分布。

8.2 个弱点

（1）我们模型的计算是复杂的，我们需要解决一个四维微分方程。

（2）不考虑房间温度变化的小型浴室。

（3）当我们改变我们的模型的敏感性分析在浴缸里的人的形状，我们只考虑基于一个长方体形状的变化。但其他的变化，如气缸没有考虑到。

9 用户解释

我们日常生活中使用的浴缸是一个简单的水容器，为了舒适地淋浴，人们需要从水龙头里不断地加入热水.。在这里，我们给出了解释的策略，本文研究加水。

如果你用浴缸比正常尺寸大一点，你应该增加每单位立方米 by14.28k 添加热水的温度。你会习惯用浴缸的上表面不是很大。此外，如果你很胖，你也许应该增加温度的热一点的水（约 0.5K）。

If the temperature of your body surface is a little higher, to decrease the temperature of the hot water. Some children like playing while taking the shower, you need to increase the temperature of the water according to the intensity of his movement; if he is crazy, you need to increase the temperature by 2K . Bubble additive is our preference; don't forget to decrease the temperature if you want to have a comfortable washing.

While, getting an evenly maintained temperature throughout the bath water is difficult.

We have the reasons as follow:

(1) Without hot water added in, the surface of the water contact with air will have the heat loss making the temperature of the upper layer and lower layer different. Besides, the motion of the person is complex, which leads to the complexity of the heat loss

(2) With hot water added in, the temperature of the inlet is different with the temperature of the outlet. When the volume of the bathtub gets bigger, with the restriction of the water inflow(the maximum flow of the faucet is definite), the heat that the water in the outlet absorbs is litter which makes the temperature gradient from inlet to the outlet becomes large.

如果你的身体表面温度稍
低热水的温度。一些孩子
耍，你需要根据自己的运
水的温度；如果他入迷了
2K 增加温度。泡沫添加
选，如果你想洗个舒服的水，别忘了降低温度。

同时，在洗澡水中获得均匀保持的温度是困难的。

我们有理由如下：

（1）没有热水加入，与空气接触的水表面会产生热量损失，使上层和下层的温度不同。此外，人的运动是复杂的，这导致的复杂性的热损失

（2）加入热水后，入口温度与出口温度不同。当浴缸的体积变大，受进水的限制（水龙头的最大流量为一定）时，出口处的水吸收的热量是垃圾，使得入口到出口的温度梯度变大。

Reference

[1] GU CHaohao&LI Daqian & CHEN SHuxing & ZHEN Songmu & TAN Yongji.
Equations of Mathematical Physics[M].Beijing: Higher Education Press,2012.7

[2] K.Deb.Multi-objective Genetic Algorithms: Problem Difficulties and Construction of Test Problems. Evolutionary Computation .(1999)

[3] XU Lei. Multi-objective optimization problem based on genetic algorithm research and application[D]. Central South University,(2007).

[4] Finite Element Analysis and Optimal Design Based on ANSYS in a XH2408 Gantry Style NC Machining Center[J]. International Journal of Plant Engineering and Management,2010,03:188-192.

参考文献

[1]谷超豪，李大潜，陈恕行，郑颂穆和谭永基.数学物理方程[M].北京：高等教育出版社，2012.7

[2] k.deb.多目标遗传算法的测试问题的困难和建设问题.进化计算（1999）

[3]徐雷.基于遗传算法的多目标优化问题研究与应用[D].中南大学（2007）。

[4]有限元分析和优化的基础上在 XH2408 龙门加工中心软件设计[J].国际设备工程与管理杂志，2010,03:188-192。