

## IMPORTANT NOTE:

**Date, time, and room for the final exam will change!**

But I don't know how  
→ watch the official website!



Practicals: fill up ~~rooms~~ in that order: 452 - 402 (-402)

## Graphics 2008/2009, period 1

### Lecture 11

### Radiosity and shadows



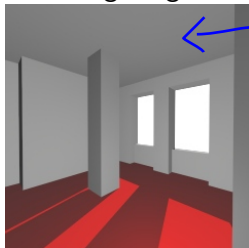
Note: some of the images used here  
have been taken from the following website:

→ <http://freespace.virgin.net/hugo.elias/radiosity/radiosity.htm>  
(copyright restrictions may apply)

Tutorial

# Direct vs. global lighting

→ Direct lighting:



Global lighting:



# Ray tracing

*Ray tracing* traditionally considers **diffuse reflections** only in the **local** lighting calculations.

- The **recursive step** only deals with perfect specular reflection.



- *Monte-Carlo methods* **approximate** global diffuse reflections, but if the sampling density is low, then the results are noisy, and if the sampling density is high, the method takes lots of time.

# Ambient shading

Parts of **diffusely reflecting objects** that are not directly lit by a light source appear completely black in traditional ray tracing, unless we resort to a standard trick: **ambient lighting**.

Remember the **basic idea**: simulate “global light” by adding a constant color term to the color of each object, i.e.

$$\rightarrow c = c_r c_l \max(0, l \cdot n)$$

becomes

$$c = c_r (c_a + c_l \max(0, l \cdot n))$$

Limitations: moving an object close to a **bright surface** has no effect, and **color bleeding** does not occur.

## View dependent vs. independent lighting calculations

Moreover, ray tracing is a **view dependent** rendering method. Moving the virtual camera even just slightly means that we have to recompute the entire image.

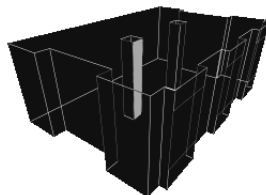
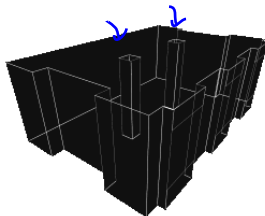
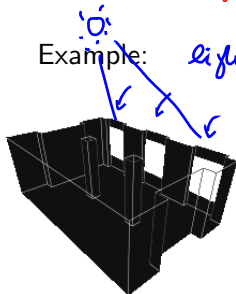
**Diffuse reflections** are **view independent**. If only there was a way to compute diffuse global illumination for diffuse reflections . . .

- For scenes with mainly diffusely reflecting objects, we could pre-compute the diffuse global illumination, and do walk-throughs in real time.
- ↪ We could also replace the ambient lighting in ray tracing by more realistic global diffuse illumination.

# Radiosity

Such a method for **global illumination** with **diffuse reflections** only exists: **Radiosity**.

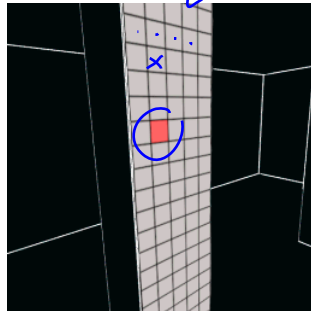
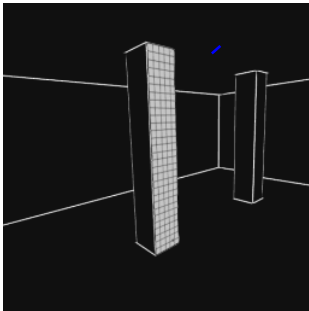
Example: *light*



# Radiosity

*thermal heat transfer*

Radiosity doesn't compute light transport along a discrete number of rays, but instead focuses on energy transfer between patches, into which the polygons in the model are subdivided.





## Radiosity equations

We define the radiosity of a patch as the amount of energy that leaves the patch per unit time per unit area. It is measured in  $W/m^2$ .

Informally, the radiosity of a patch is a measure for its **brightness**.

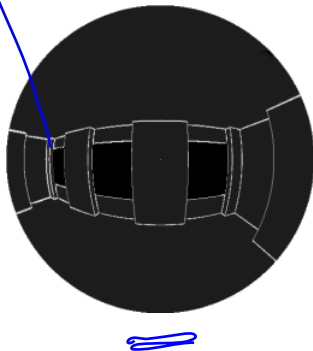
The radiosity of a patch not only depends on the energy it <sup>light</sup> emits, but also on the energy that it **reflects** (where does this energy come from?)  
*reflections from incoming light*

In the radiosity method, every patch can be a **light source**; there are no separate point light sources.

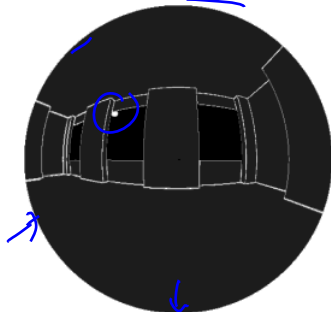
# Radiosity



View from a patch

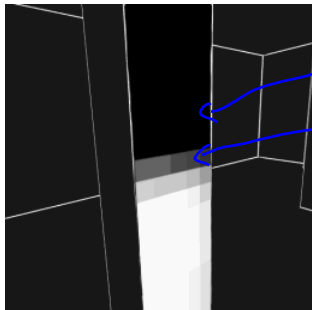


View from a lower patch

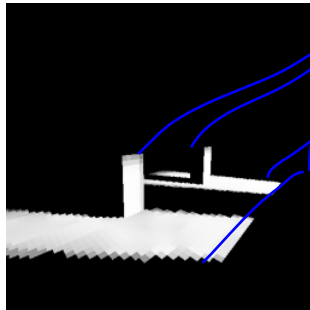


# Radiosity

Pillar and ...



... entire room ...



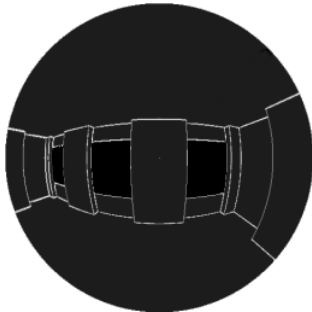
... after emission from the direct light source is considered.

7 But what about the reflections from enlightened objects?

# Radiosity

Compare view from the upper patch ...

... before ...



... and after ...

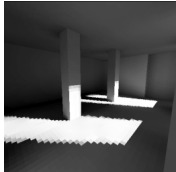


... emission from the direct light source is considered.

# Radiosity

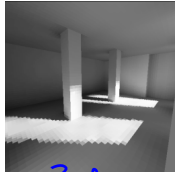
Ah, there's more light! Let's consider that as well ...

... and again



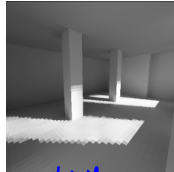
2nd

... and again



3rd

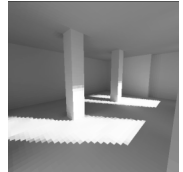
... and again



4th

...

... and again



16th pass

... until it converges (or we are satisfied with the result).

## Radiosity equations

Now, if we denote the radiosity of patch  $A_i$  with  $B_i$ , the energy it emits by  $E_i$ , and its reflectivity by  $\rho_i$ , then we can write

Radiosity  $B_i$  of patch  $A_i$       Emitted energy      Reflected energy

$$B_i = E_i + \rho_i \sum_{j \neq i} B_j F_{ji} \frac{A_j}{A_i}$$

(Handwritten annotations: Blue arrows point from the labels to the corresponding terms in the equation. A blue arrow points from  $B_i$  to the left-hand side. A blue arrow points from  $E_i$  to the first term on the right. A blue arrow points from  $\rho_i$  to the reflectivity coefficient. A blue arrow points from  $B_j$  to the summation term. A blue arrow points from  $F_{ji} \frac{A_j}{A_i}$  to the form factor term. A blue arrow points from  $j \neq i$  to the summation index.)

where  $F_{ji}$  depends on the shapes of patches  $A_i$  and  $A_j$ , their distance, their orientation, etc: not all energy that leaves  $A_j$  reaches  $A_i$ .  $F_{ji}$  is called the form factor from  $A_j$  to  $A_i$ .

## Form factors

Form factors are **dimensionless**, and specify the **fraction of the energy** leaving one patch that arrives at the other patch. They are given by

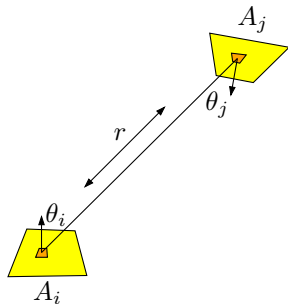
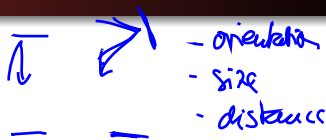
$$\rightarrow F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos \theta_i \cos \theta_j}{\pi r^2} dA_j dA_i$$

There's symmetry in form factors:

$$A_i F_{ij} = A_j F_{ji} \quad \rightarrow \quad \bar{F}_{ji} = \bar{F}_{ij} \cdot \frac{A_i}{A_j}$$

Q: What if  $A_i$  and  $A_j$  are not (completely) mutually visible?

$$F_{ij} = 0$$



## Solving the radiosity equations

Recall that the radiosities of the patches are given by

$$B_i = E_i + \rho_i \sum_j B_j F_{ji} \frac{A_j}{A_i}$$

Because of the symmetry relation we can rewrite this to

$$\rightarrow B_i = E_i + \rho_i \sum_j B_j F_{ij}$$

And so

$$\leadsto B_i - \rho_i \sum_j B_j F_{ij} = E_i$$

(Note that we have  $n$  of equations in  $n$  variables.)



## Solving the radiosity equations

$$B_i - \rho_i \sum_j B_j F_{ij} = E_i$$

If we know the emissions  $E_i$  of all patches, and all form factors  $F_{ij}$ , we can solve for the  $B_i$ 's:

$$\begin{bmatrix} 1 - \rho_1 F_{11} & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1n} \\ -\rho_2 F_{21} & 1 - \rho_2 F_{22} & \cdots & -\rho_2 F_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ -\rho_n F_{n1} & -\rho_n F_{n2} & \cdots & 1 - \rho_n F_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$$

## Solving the radiosity equations

So, we just have to:

- Compute all the form factors

$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos \theta_i \cos \theta_j}{\pi r^2} dA_j dA_i.$$

- Solve the (very large) system of linear equations on the previous slide

Hmm, “just”?

7

## Computing form factors analytically

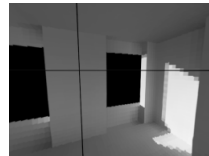
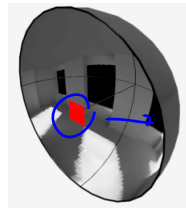
For any pair of patches  $A_i$  and  $A_j$  we have to compute the form factors

$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos \theta_i \cos \theta_j}{\pi r^2} dA_j dA_i$$

In theory this can be done **analytically**, but in practice this is too complicated, especially if we have to take care of **partial occlusion** of patches.

# The hemisphere method

Nusselt showed that computing  $F_{dij}$  for a differential patch  $dA_i$  is equivalent to projecting  $A_j$  onto a unit hemisphere centered about  $dA_i$ , projecting the projected area orthographically onto the hemisphere's unit base circle, and dividing by the area of the circle.

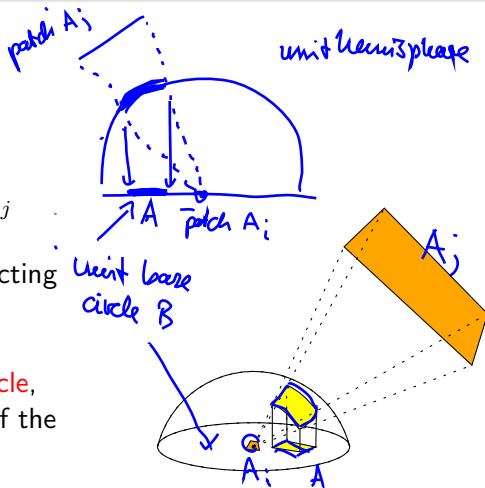


# The hemisphere method

Nusselt showed that computing  $F_{dij}$  for a differential patch  $dA_i$  is

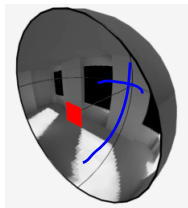
- equivalent to projecting  $A_j$  onto a unit hemisphere
- centered about  $dA_i$ , projecting the projected area orthographically onto the hemisphere's unit base circle,
- and dividing by the area of the circle.

form factor  $F_{dij} = \frac{A}{B}$

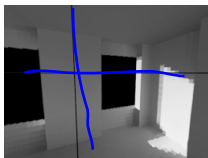
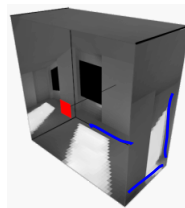


# The hemisphere method

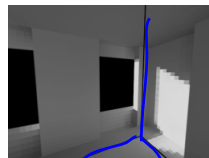
Hmm, but that still seems like a lot of work.



$\approx$



$\longleftrightarrow$



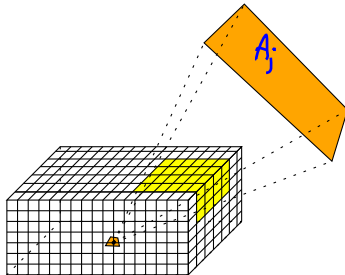
# The hemicube method

Instead of projecting  
analytically onto a hemisphere,  
we can also project onto a

- **hemicube** that is subdivided  
into cells, each cell having a  
weight that represents its  
contribution to the form  
factor. By summing the  
weights of the cells onto which  
 $A_j$  is projected, we  
approximate the form factor.

- This can be implemented on  
standard graphics hardware.

→ fast ! ☺



## Number of form factors

Note that the number of form factors is **quadratic** in the number of patches. This means that in practice, especially with large models, storing all form factors is impossible.

Therefore, form factors are (re)computed on the fly when they are needed.



## Progressive refinement

Now that we know how to compute **form factors**, we are done halfway. What is left is the computation of the radiosities. Theoretically, we could solve the system

$$\begin{bmatrix} 1 - \rho_1 F_{11} & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1n} \\ -\rho_2 F_{21} & 1 - \rho_2 F_{22} & \cdots & -\rho_2 F_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ -\rho_n F_{n1} & -\rho_n F_{n2} & \cdots & 1 - \rho_n F_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$$

In practice, this is too expensive, and we have to resort to approximation methods.

## Progressive refinement

For each patch, we have to compute  $B_i = E_i + \rho_i \sum_j B_j F_{ji}$ . We **approximate** this **iteratively**:

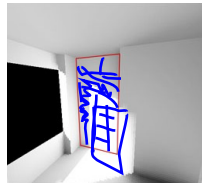
- Initially, set  $B_i = E_i$  for every patch.
- For every patch  $A_i$  compute the energy reaching it from all other patches and add this (multiplied by  $\rho_i$ ) to the radiosity of patch  $A_i$  that has been computed so far.
- Repeat the previous step, but only account for the **unshot radiosity** that was added to each patch  $A_j$  in the previous iteration.
- Repeat until the added radiosity per iteration is less than a threshold for each patch.

# Meshing

We subdivide polygons into patches because in general there is a smooth variation of the radiosity along a polygon. The finer the subdivision, the more accurate the results.

On the other hand, a uniform subdivision into very small patches leads to very long computation times.

A fine subdivision is not necessary everywhere: if the distance between patches  $A_i$  and  $A_j$  is large, the variation of the contribution of  $A_i$  to the radiosity of  $A_j$  over the surface of  $A_j$  will be small.



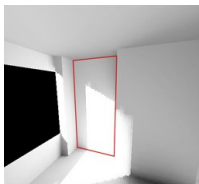
# Adaptive subdivision

Adaptive subdivision is an iterative process:

- ● Make an initial (coarse) subdivision of polygons into patches, and compute the radiosities.
- ● Check neighboring patches. If their radiosities differ more than a threshold, subdivide the patches into sub-patches (also called **elements**).
- Compute the radiosities of the **elements** by computing the influence of other **patches** (as opposed to elements).
- Repeat until the radiosities of neighboring patches differs less than a threshold, or element sizes have reached a pre-determined minimum.

# Adaptive subdivision

Example for adaptive subdivision:



## From radiositities to image

- Once the radiositities have been computed, we have to generate an
- image. This can be done by computing radiositities for the **vertices of the mesh** (e.g., as a weighted average of the surrounding elements), and then applying **Gouraud shading**.
  - If the scene is **static**, we can do walk-throughs in **real-time**.
  - Radiosity can also be used as a more realistic **replacement** of **ambient light** and **local diffuse reflections** in ray tracing.

2-pass ~~selection~~ solution

## Graphics 2008/2009, period 1

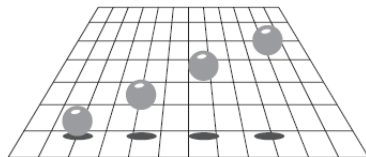
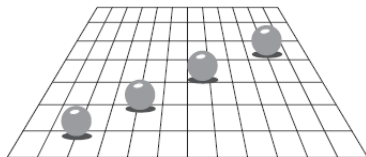
### Lecture 11

### *Radiosity and shadows*

## The need for shadows

**Shadows** come “for free” in ray tracing and radiosity methods, but in projective methods they require some extra effort.

However, these efforts are well-spent: shadows add **realism** and **depth**, as well as give a better **understanding** of the images.



Ch. 21

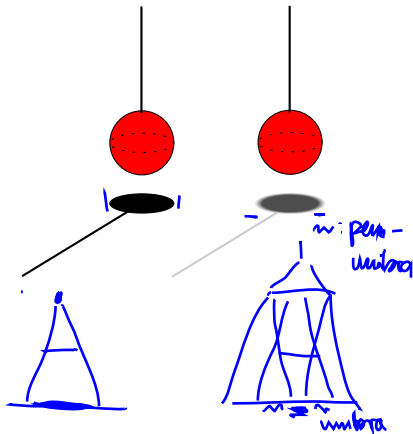


## Hard and soft shadows

Point light sources give **hard shadows**, whereas area lights give **soft shadows**.

For soft shadows, the region that receives no light at all is called the **umbra**, and the region that receives partial light is called the **penumbra**.

The penumbra gives additional cues with respect to the size of the light source and the distance from the **occluder** to the **receiver**.



# Shadow maps

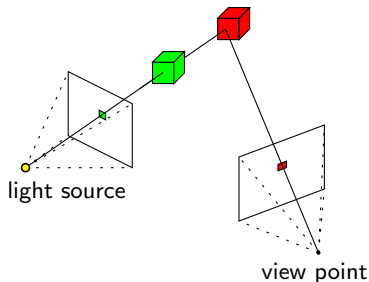
1st approach ↗

fast!

A popular **hardware supported** shadow rendering algorithm uses **shadow maps**.

The scene is first rendered from the **light source** point-of-view to fill a **depth buffer**, stored with the light.

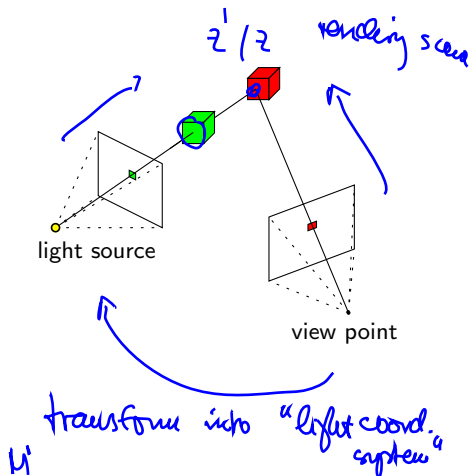
z-buffer



# Shadow maps

Next, the scene is rendered from the **camera** point-of-view. The  $z$ -coordinate rendered polygon for a pixel is converted with a **transformation matrix** to a  $z$ -coordinate in the **light source coordinate system**.

If the light source  $z$ -buffer has a **smaller  $z$ -value** stored, then the part of the polygon projected to the current pixel lies in **shadow**.



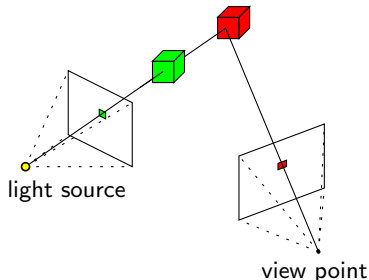
# Shadow maps

## Advantages:

- hardware-supported ( $z$ -buffer and matrix ops), so very fast.

## Disadvantages:

- Suited for **spot lights**, not really for **omni-directional lights**.
- **Precision problems** due to use of **floating points** and **discrete sampling**.

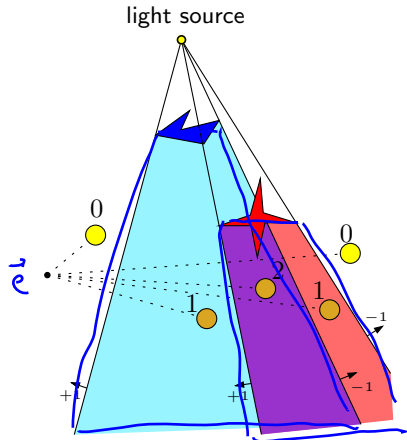


# Shadow volumes

2nd appr.

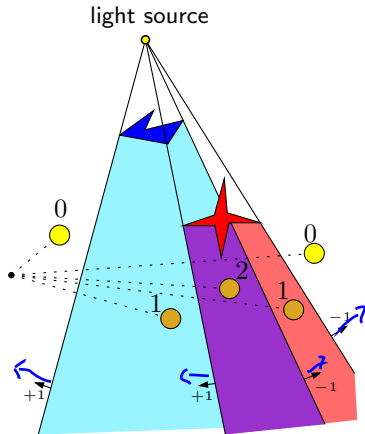
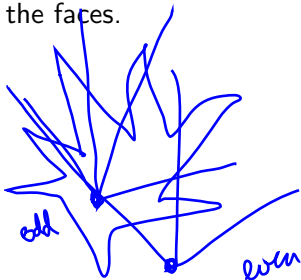
An alternative method for shadow computation uses **shadow volumes**.

For a **light source** and an **occluder**, the 3D shadow volume is determined by the **silhouette edges** of the occluder w.r.t the light source.



# Shadow volumes

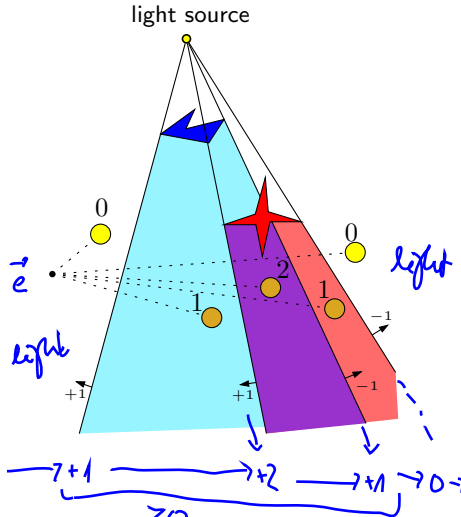
The **faces** of the shadow volumes are either **front facing** or **back facing** w.r.t the camera. This is easily determined with the normals of the faces.



# Shadow volumes

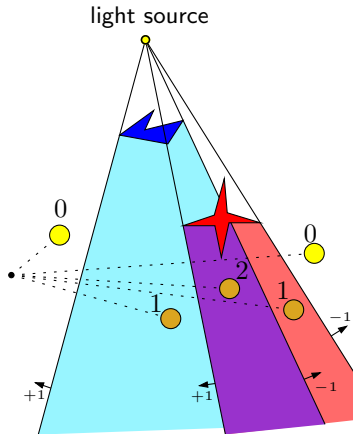
The idea: when a pixel is rendered, we follow the ray from the **viewpoint** to the **object** for the pixel.

Whenever the ray crosses a shadow volume face, we **increment** or **decrement** a counter, depending on whether the face is front-facing or back-facing.



# Shadow volumes

Objects for which the counter is **zero** are **lit**; objects for which the counter is **positive** are in **shadow**.





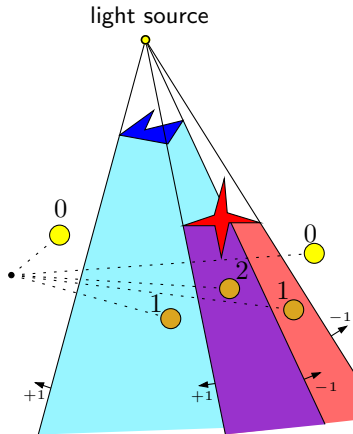
## Hardware support: stencil buffer

Obviously, we don't want to do  
**ray tracing** to determine  
shadows in **real-time rendering**  
**methods**.

*slow :-*

This is where the **stencil buffer**  
comes in.

*HW ☺*



## Hardware support: stencil buffer

The **stencil buffer** is comparable to the **z-buffer** (i.e., there is a one-to-one correspondence of pixels in the frame buffer and entries in the stencil buffer), but here every entry is a **counter**.

It supports  $\neq 0$

$\neq \wedge$

$\neq \wedge$

→ ● resetting, incrementing and decrementing the counters.

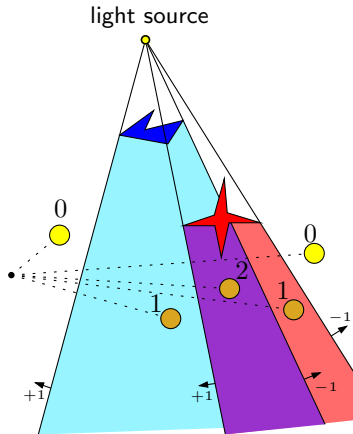
- idem, but **conditionally**, depending on a test against the **z-buffer**.

- conditional drawing in the **frame buffer**, i.e., only draw a pixel if the corresponding counter is zero/non-zero.

## Hardware support: stencil buffer

Computing shadows using the stencil buffer:

- Draw the scene with **ambient lighting** only (i.e., everything is drawn as if it is in shadow).
- Compute the shadow volumes
- “draw” the shadow volume faces in the **stencil buffer**.

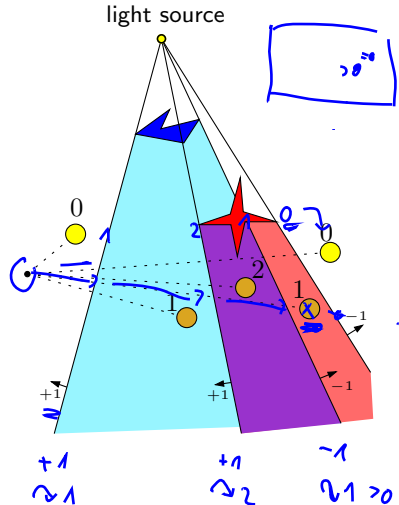


## Hardware support: stencil buffer

Drawing a pixel in the stencil buffer means: Test the shadow face “pixel” againsts the  $z$ -buffer.

IF its  $z$  value is smaller than that of the drawn (real) object, THEN

- IF it is a **front-facing** face, THEN **increment** the counter.
- IF it is a **back-facing** face, THEN **decrement** the counter.

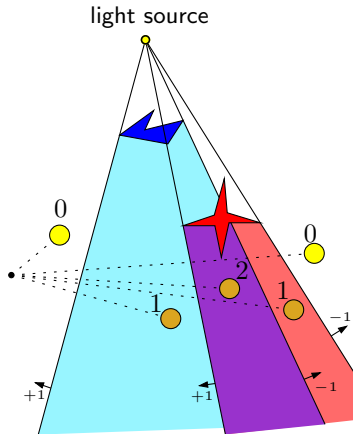


## Hardware support: stencil buffer

Once all shadow volume face have been drawn, the stencil buffer contains **zeros** for pixels that are **not in shadow**.

Next, draw the entire scene again (only the real objects), this time including lighting calculations, but tell the graphics card to ignore pixels that have a non-zero stencil buffer entry.

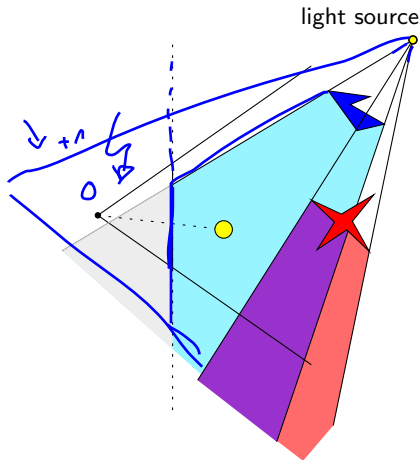
## Conditional drawing



## Shadow volumes: problems

Problems with the shadow volume approach:

- The view point may lie in one or more shadow volumes.
- The **near clipping plane** may slice open shadow volumes.

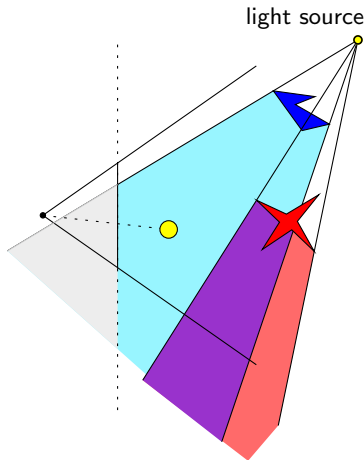


## Shadow volumes: problems

The described problems can be solved by capping the shadow volumes: the near clipping plane (partially) becomes one of the faces of each volume.

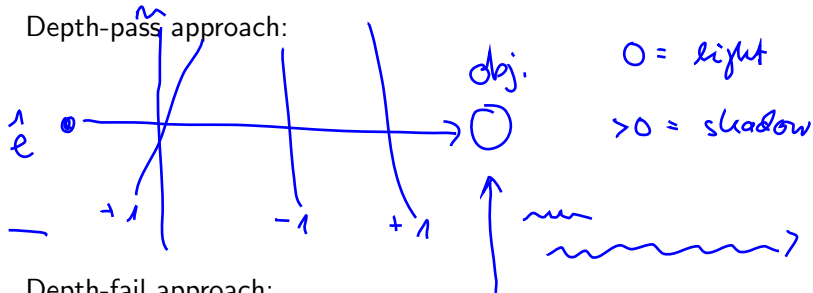
→ Drawback: **precision problems** close to near clipping plane.

The described algorithm uses a **depth-pass** approach: only shadow volume faces in front of objects are drawn in the stencil buffer.



## Shadow volumes: depth-pass vs. depth-fail

Depth-pass approach:



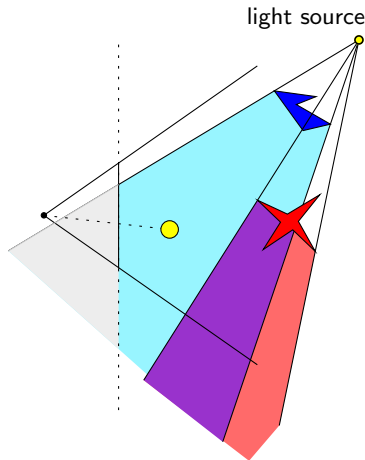
Depth-fail approach:





## Shadow volumes: problems

Alternative: **depth-fail**  
approach: only draw/count  
shadow volume faces **behind**  
object. Problems with near  
clipping plane are traded for  
problems with far clipping  
plane.



## Fake shadows

3rd app.

A fast and easy (but not extremely realistic) way of creating shadows: **fake shadows**. Only few (important) occluders are selected, and shadows are projected onto **supporting planes** of only few (important) surfaces (e.g.: table has shadows, but chairs around it haven't).

- Draw the scene, including lighting.
- Project occluders onto designated planes. Improvement: don't render projections black, but **blend** with original colors.



# Fake shadows

Problems with fake shadows:

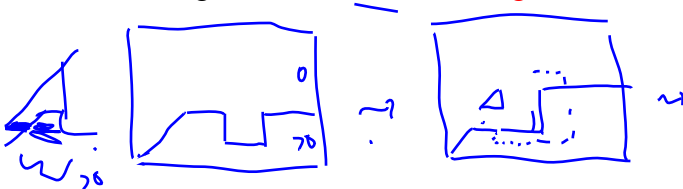
- projected shadows have **same depth** as shadow receivers.
- Shadows may **stick out** beyond shadow receivers (since we project on planes instead of objects).
- Multiple occluders may give rise to **double blending**.



# Fake shadows

Solution to the problems with fake shadows: use stencil buffer.

- ● Reset stencil buffer counters. = 0
- Draw scene. When **shadow receivers** are drawn, increment corresponding stencil buffer counters.
- Project shadow polygons, but only draw/blend for pixel that have a **non-zero** stencil value. **Reset** stencil entry after drawing, to **avoid double blending**.



## Soft Shadows

**Soft shadows** are expensive. Usual approach: approximate by **sampling area lights**, treating every sample as a point light source. Compute individual shadows, and average.

Drawback: many samples are needed to get smooth penumbras, and this hurts performance.

