Graphics 2008/2009, period 1

Lecture 5

Linear and affine transformations

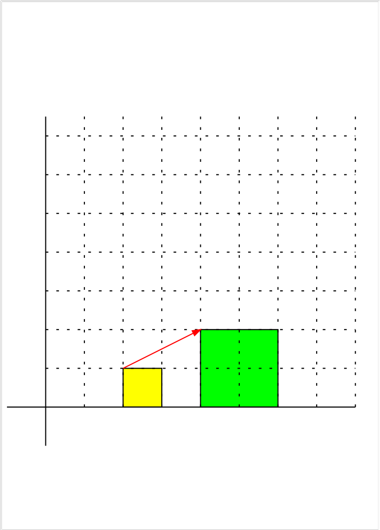
Example: scaling

To scale with a factor two with respect to the origin, we apply the matrix

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x \\ 2y \end{pmatrix} = 2\begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_{11}x + a_{12}y \\ a_{21}x + a_{22}y \end{pmatrix}$$

$m \times n \quad n \times 1 \quad m \times 1$



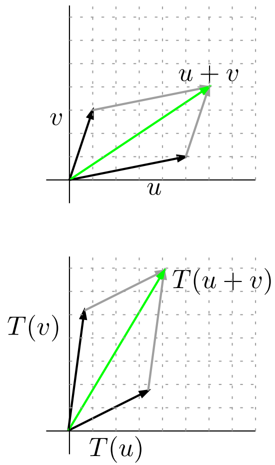
Linear transformations

A function $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is called a linear transformation if it satisfies

1. $T(u + v) = T(u) + T(v)$ for all $u, v \in \mathbb{R}^n$.
2. $T(cv) = cT(v)$ for all $v \in \mathbb{R}^n$ and all scalars c.

alt.:

$T(c_1\vec{u} + c_2\vec{v}) = c_1 \cdot T(\vec{u}) + c_2 T(\vec{v})$



Linear transformations in graphics

Many transformations that we use in graphics are linear transformations.

Linear transformations can be represented by matrices.

A sequence of linear transformations can be represented with a single matrix.

With some tricks, we can represent translations and perspective projections with matrices as well.

Example: scaling

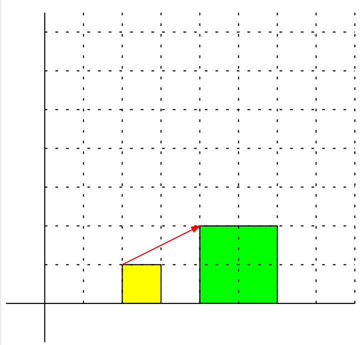
To scale with a factor two with respect to the origin, we apply the matrix

factor 3

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}, \quad \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

General case:

$$\begin{pmatrix} a_{11}x & + a_{12}y \\ a_{21}x & + a_{22}y \end{pmatrix}, \quad \begin{pmatrix} * & 0 \\ 0 & * \end{pmatrix}$$

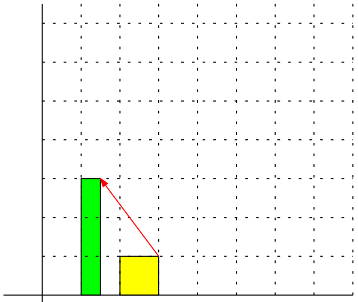


Example: scaling

Scaling doesn't have to be uniform. Here, we scale with a factor one half in x-direction, and three in y-direction:

$$\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 3 \end{pmatrix}$$

Q: what is the inverse of this matrix?



$$\left(\begin{array}{cc|cc} \frac{1}{2} & 0 & 1 & 0 \\ 0 & 3 & 0 & 1 \end{array}\right) \begin{array}{c} \xrightarrow{2} \\ \xrightarrow[1/3]{} \end{array} \begin{pmatrix} 2 & 0 \\ 0 & \frac{1}{3} \end{pmatrix}$$

$$A^{-1}(A\vec{x}) = (A^{-1}A)\vec{x} = I\vec{x} = \vec{x}$$

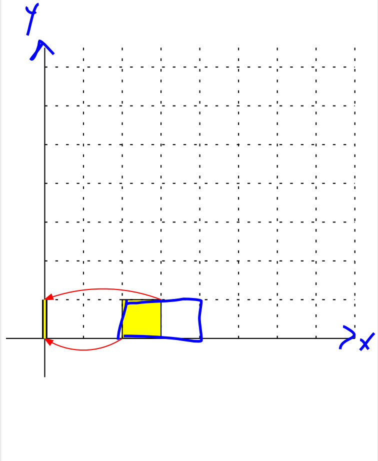
Example: projection

We can also use matrices to do orthographic projections, for instance, onto the Y-axis:

$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ y \end{pmatrix}$$

Q: what is the inverse of this matrix?

undef.



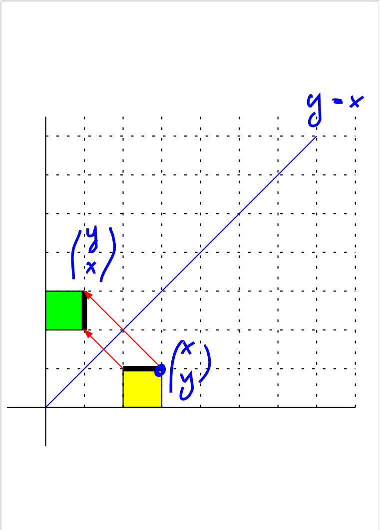
Example: reflection

Reflection in the line $y = x$ boils down to swapping x- and y-coordinates:

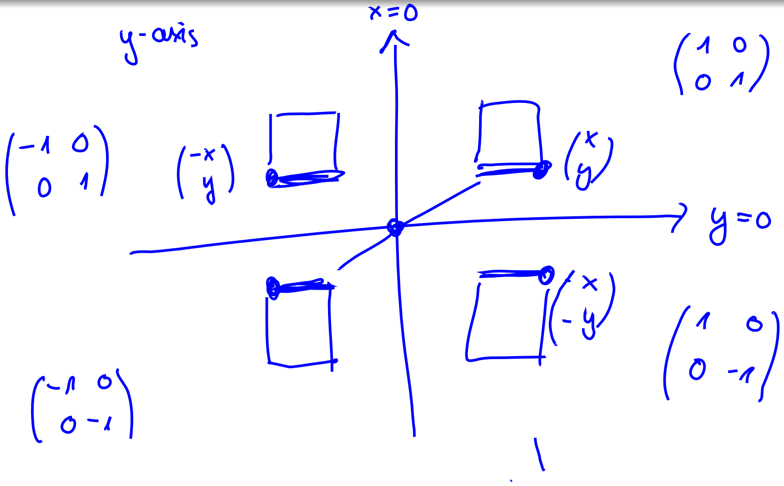
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 + y \\ x + 0 \end{pmatrix} = \begin{pmatrix} y \\ x \end{pmatrix}$$

Q: what is the inverse of this matrix?

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}^{-1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$



Example: more reflections



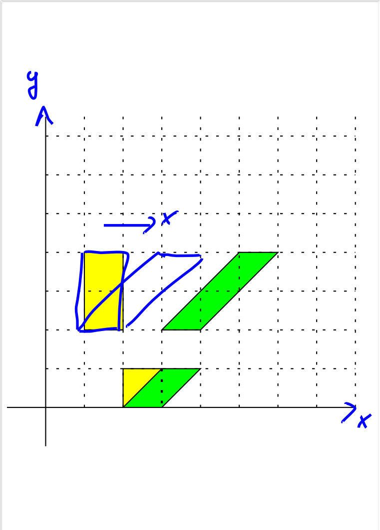
Example: shearing

Shearing in x-direction pushes things sideways:

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+y \\ y \end{pmatrix}$$

Q: What happens with the x-coordinate of points that are transformed with this matrix? And what with the y-coordinates?

y-direct.: $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$



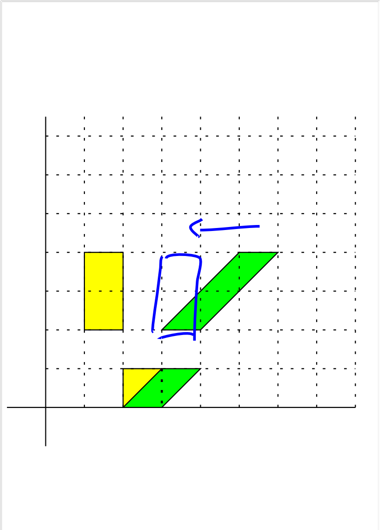
Example: shearing

Shearing in x-direction pushes things sideways:

$$\begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + sy \\ y \end{pmatrix}$$

Q: What is the inverse of this matrix?

$$\begin{pmatrix} 1 & -s \\ 0 & 1 \end{pmatrix}$$



Example: rotation

To rotate 45° about the origin, we apply the matrix

$$\begin{pmatrix} \frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} \\ \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{pmatrix}.$$

$\frac{1}{2}\sqrt{2} = \cos \frac{\pi}{4}$

$= \sin \frac{\pi}{4}$

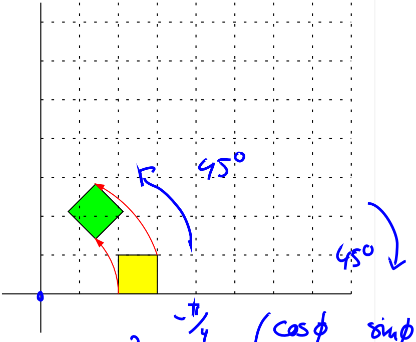
$= \cos 45^\circ$

$= \ldots$

General case:

$$\begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$$

Counterclockwise rot.



clockwise?

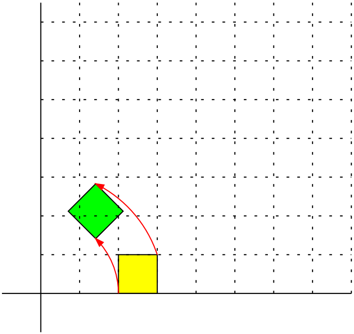
$$\begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}$$

Finding matrices

Applying matrices is pretty straightforward, but how do we find the matrix for a given linear transformation?

Let $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

Q: what is the significance of the column vectors of A?



Finding matrices

transform basis vectors $\vec{b}_1 = \begin{pmatrix}1\\0\end{pmatrix}$, $\vec{b}_2 = \begin{pmatrix}0\\1\end{pmatrix}$

Scaling (factors $a, b \neq 0$): $\begin{pmatrix}\frac{1}{2} & 0\\ 0 & 3\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}ax\\ by\end{pmatrix}$ $\curvearrowright$ $\begin{pmatrix}a\\0\end{pmatrix}, \begin{pmatrix}0\\b\end{pmatrix}$

Shearing (x-dir., factor 1): $\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}x+y\\ y\end{pmatrix}$ $\curvearrowright$ $\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}1\\1\end{pmatrix}$

Reflection (in line $y = x$): $\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}y\\ x\end{pmatrix}$ $\curvearrowright$ $\begin{pmatrix}0\\1\end{pmatrix}, \begin{pmatrix}1\\0\end{pmatrix}$

Rotation (countercl., $45°$): $\frac{1}{2}\sqrt{2}\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \frac{1}{2}\sqrt{2}\begin{pmatrix}x-y\\ x+y\end{pmatrix}$

$\curvearrowright$ $\frac{1}{2}\sqrt{2}\cdot\begin{pmatrix}1\\1\end{pmatrix}$, $\frac{1}{2}\sqrt{2}\cdot\begin{pmatrix}-1\\1\end{pmatrix}$

Finding matrices

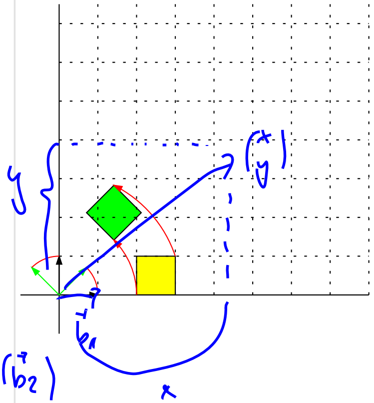
Aha! The column vectors of a transformation matrix are the images of the base vectors!

That gives us an easy method of finding the matrix for a given linear transformation.

vector $\vec{v} = \begin{pmatrix} x \\ y \end{pmatrix} = x \begin{pmatrix} 1 \\ 0 \end{pmatrix} + y \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$T(c_1 \vec{u} + c_2 \vec{v}) = c_1 T(\vec{u}) + c_2 T(\vec{v})$

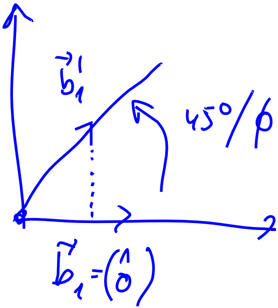
$T(\vec{v}) = \dots = x\, T(\vec{b}_1) + y\, T(\vec{b}_2)$

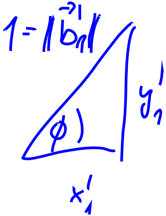


Finding matrices: example

Rotation (countercl., $45°$): $\frac{1}{2}\sqrt{2}\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \frac{1}{2}\sqrt{2}\begin{pmatrix}x-y\\ x+y\end{pmatrix}$

$$\begin{pmatrix}\cos\phi & -\sin\phi\\ \sin\phi & \cos\phi\end{pmatrix}$$





$\sin\phi = \frac{y_1'}{1}, \quad \cos\phi = \frac{x_1'}{1}$

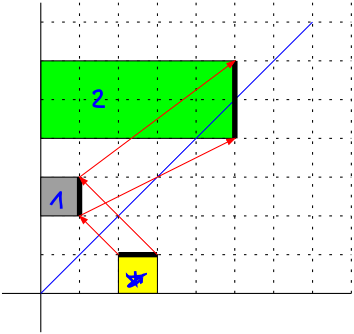
Example: reflection and scaling

Multiple transformations can be combined into one. Here, we first do a reflection in the line $y = x$, and then we scale with a factor 5 in x-direction, and a factor 2 in y-direction:

$$\underbrace{\begin{pmatrix} 5 & 0 \\ 0 & 2 \end{pmatrix}}_{\text{Scaling}} \underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}_{\text{Refl.}} = \begin{pmatrix} 0 & 5 \\ 2 & 0 \end{pmatrix}$$

associative !

$$A(B\vec{x}) = (AB)\vec{x}$$



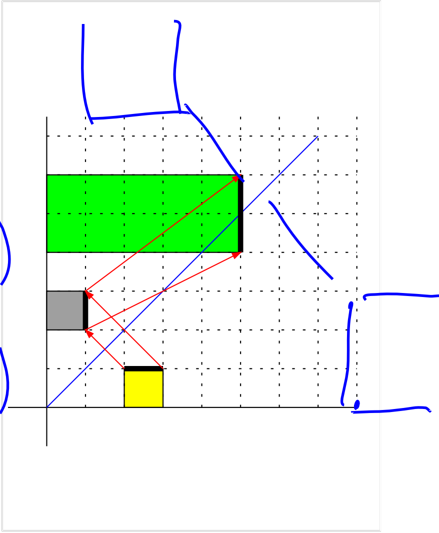
Example: reflection and scaling

Remember: Matrix multiplication is associative but not commutative. $AB \neq BA$

Q: what does this mean for multiple transformations?

$$\begin{pmatrix} 5 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 5 \\ 2 & 0 \end{pmatrix} \curvearrowright \begin{pmatrix} 5y \\ 2x \end{pmatrix}$$

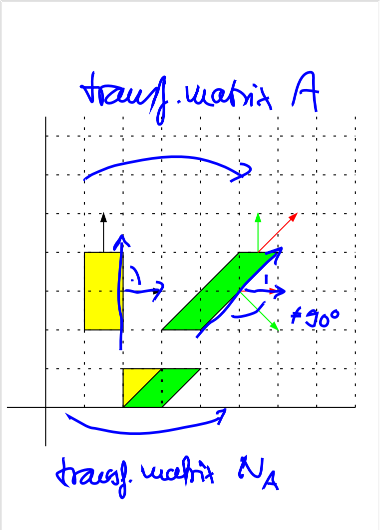
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 5 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 5 & 0 \end{pmatrix} \curvearrowright \begin{pmatrix} 2y \\ 5x \end{pmatrix}$$



Transposing normal vectors

Unfortunately, normal vectors are not always transformed properly. To transform a normal vector n under a given linear transformation A, we have to apply the matrix $(A^{-1})^T$.

Q: obviously, for shearing, normal vectors "behave funny". But what about rotations? And scalings (uniform and non-uniform)?



Transposing normal vectors

Unfortunately, normal vectors are not always transformed properly. To transform a normal vector n under a given linear transformation A, we have to apply the matrix $(A^{-1})^T$. $= N_A$

Q: obviously, for shearing, normal vectors "behave funny". But what about rotations? And scalings (uniform and non-uniform)?

tang. vect. t → $A\vec{t} = \vec{t}_A$ ✓

but → $A\vec{n} = \vec{n}_A$ not normal v.

Goal: Find N_A with $N_A \cdot \vec{n} = \vec{n}_N$ corr. normal vect.

$\vec{n}^T \cdot \vec{t} = 0$

$\vec{n}^T \cdot I \cdot \vec{t} = 0$

$\underbrace{\vec{n}^T \cdot A^{-1}} \cdot \underbrace{A \cdot \vec{t}}_{\vec{t}_A} = 0$

$\vec{n}_N^T = \vec{n}^T \cdot A^{-1}$

$\vec{n}_N = (\vec{n}^T \cdot A^{-1})^T$

$= (A^{-1})^T \cdot \vec{n}$

Transposing normal vectors

Unfortunately, normal vectors are not always transformed properly. To transform a normal vector n under a given linear transformation A, we have to apply the matrix $(A^{-1})^T$.

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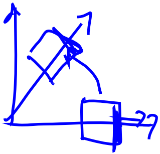
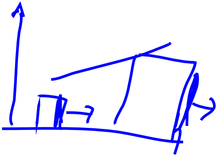




$$\begin{pmatrix} \cos & -\sin \\ \sin & \cos \end{pmatrix}^{-1} = \begin{pmatrix} \cos & \sin \\ -\sin & \cos \end{pmatrix}$$

$$\begin{pmatrix} s_1 & 0 \\ 0 & s_2 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{1}{s_1} & 0 \\ 0 & \frac{1}{s_2} \end{pmatrix}$$

$$1 = \cos^2 + \sin^2$$

Linear transformations in graphics

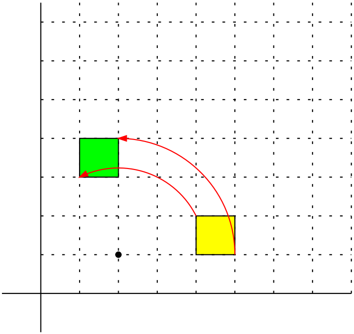
Many transformations that we use in graphics are linear transformations.

- Linear transformations can be represented by matrices.
- A sequence of linear transformations can be represented with a single matrix.
- With some tricks, we can represent translations and perspective projections with matrices as well.

More complex transformations

So now we know how to determine matrices for a given transformation. Let's try another one:

Q: what is the matrix for a rotation of $90°$ about the point $(2, 1)$?

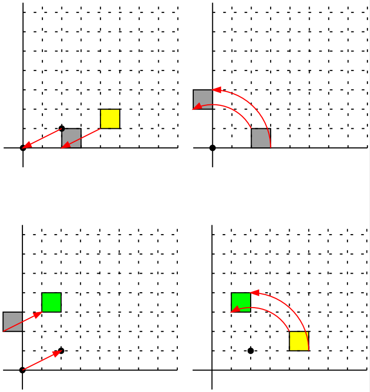


More complex transformations

We can form our transformation by composing three simpler transformations:

- Translate everything such that the center of rotation maps to the origin.
- Rotate about the origin.
- Revert the translation from the first step.

Q: but what is the matrix for a translation?



More complex transformations

Translation is not a linear transformation.

With linear translations we get:

$$\underline{\underline{A}}x = \begin{pmatrix} a_{11}x & + & a_{12}y \\ a_{21}x & + & a_{22}y \end{pmatrix}$$

But we need something like:

$$\begin{pmatrix} x & + & x_t \\ \cancel{x}\,y & + & y_t \end{pmatrix}$$

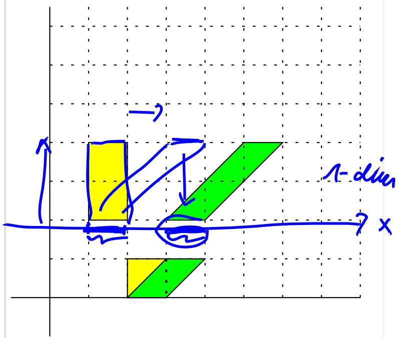
We can do this with a combination of linear transformations and translations called affine transformations.

Homogeneous coordinates

Observation: shearing in 2D smells a lot like translation in 1D

(and shearing in 3D smells like translation in 2D and . . .)

Idea: move one dimension higher by adding so called homogeneous coodinates.



Homogeneous coordinates

Shearing in 3D based on the z-coordinate is a simple generalization of 2D shearing:

$$\begin{pmatrix} 1 & 0 & x_t \\ 0 & 1 & y_t \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x & + & x_t z \\ y & + & y_t z \\ & z & \end{pmatrix}$$

Or if we only look at the plane $\underline{z = 1}$:

$$\begin{pmatrix} 1 & 0 & x_t \\ 0 & 1 & y_t \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x & + & x_t \\ y & + & y_t \\ & 1 & \end{pmatrix}$$

Homogeneous coordinates

Translations in 2D can be represented as shearing in 3D by looking at the plane $z = 1$.

The matrix for a translation over the vector $t = \begin{pmatrix} x_t \\ y_t \end{pmatrix}$ is

homogenious coord $\rightarrow$

$$\underbrace{\begin{pmatrix} s_1 & 0 & 0 \\ 0 & s_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & x_t \\ 0 & 1 & y_t \\ 0 & 0 & 1 \end{pmatrix}} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} s_1(x + x_t) \\ s_2(y + y_t) \\ 1 \end{pmatrix}$$

How should we represent points then? And vectors?

$$\begin{pmatrix} 1 & 0 & x_t \\ 0 & 1 & y_t \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} = \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$$

Homogeneous coordinates

For homogeneous coordinates (in 2D) we add

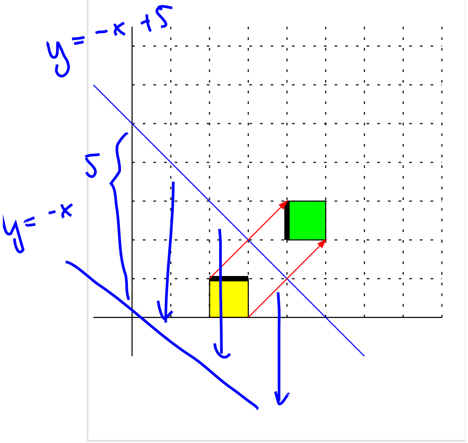
- a third coordinate $z = 1$ to each location $\begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$
- a third coordinate $z = 0$ to each vector $\begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$
- a third row $(0 \; \dots \; 0 \; 1)$ to each matrix $\begin{pmatrix} \boxed{} & \vdots \\ 0 \; 0 \cdots 0 & 1 \end{pmatrix}$

Affine transformations (i.e. linear transformations and translations) can then be done with simple matrix multiplication.

Affine transformations: example

Q: What is the matrix for reflection in the line $y = -x + 5$?

Hint: move ~~the line~~ everything to the origin, reflect, and move the line back.



Affine transformations: example

$$\begin{pmatrix} 1 & 0 & x_t \\ 0 & 1 & y_t \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x + x_t \\ y + y_t \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -5 \\ 0 & 0 & 1 \end{pmatrix} = T_1$$

$$x_t = 0, \quad y_t = -5$$

$$R = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 & 5 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad y = x \quad \Big| \quad \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \quad y = -x$$

$$x_t = 0, \quad y_t = 5$$

$$T_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 & 5 \\ -1 & 0 & 5 \\ 0 & 0 & 1 \end{pmatrix}$$

$$(T_2 R T_1) x$$

Affine transformations

Q: The matrix for reflection in the line $y = -x + 5$ is

Reflec.

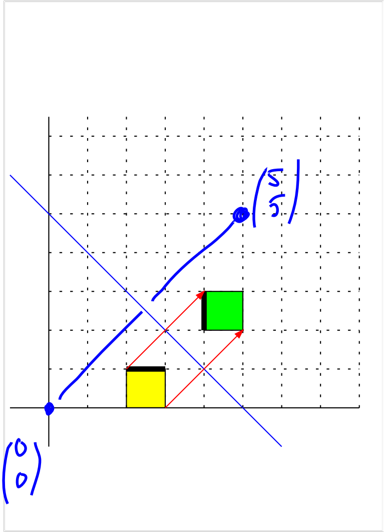
$$\begin{pmatrix} 0 & -1 & 5 \\ -1 & 0 & 5 \\ 0 & 0 & 1 \end{pmatrix}$$

translation

homog. coord.

Q: what is the significance of the columns of the matrix?

Does that give us a faster way to find matrices?



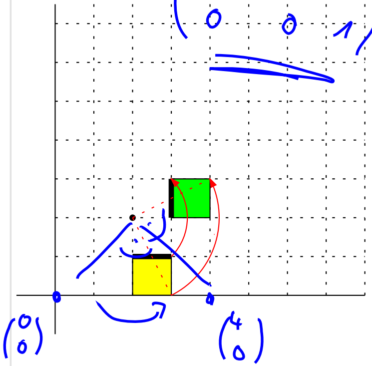
Affine transformations: example

Q: What is the matrix for rotation about the point $(2, 2)$?

$$\begin{pmatrix} \cos\alpha & \sin\alpha & ? \\ \sin\alpha & \cos\alpha & ? \\ 0 & 0 & 1 \end{pmatrix}$$

$$\alpha \overset{!}{=} 90^\circ$$

$$\begin{pmatrix} 0 & -1 & 4 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



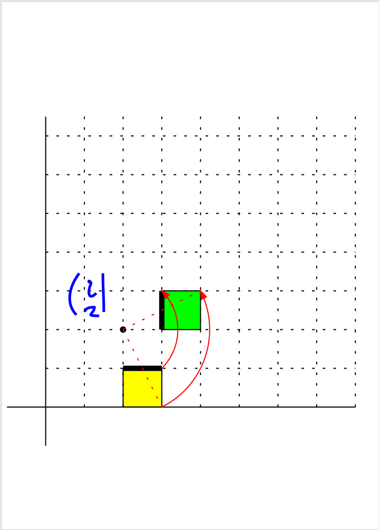
Affine transformations: example

Q: What is the matrix for rotation about the point $(2, 2)$?

$$\begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}$$



Transformations in 3D

Transformations in 3D are very similar to those in 2D:

$$\begin{pmatrix} S_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & S_z \end{pmatrix}$$

- For scaling, we have three scaling factors on the diagonal of the matrix.
- Shearing can be done in either x-, y-, or z-direction (or a combination thereof).

x-direction:

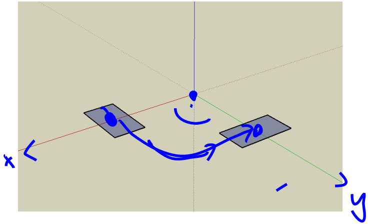
$$\begin{pmatrix} 1 & y_t & z_t & \cancel{0} \\ 0 & 1 & 0 & \cancel{0} \\ 0 & 0 & 1 & \cancel{0} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

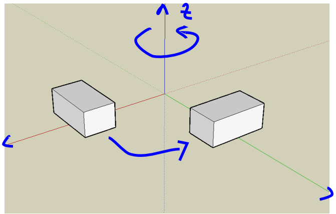
- Reflection is done with respect to *planes*
- Rotation is done about *directed lines*.

- For translations (and affine transformations in general), we use 4×4 matrices.

Transformations in 3D: rotations

Q: What is the matrix for a rotation of angle ϕ about the z-axis?





y-axis:

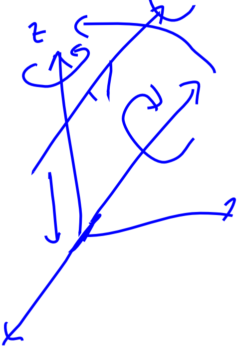
$$\begin{pmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{pmatrix}$$

$$\begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Transformations in 3D: rotations

Q: What is the matrix for a rotation of angle ϕ about the z-axis? ✓

Q: What is the matrix for a rotation of angle ϕ about the vector $(3, 0, 4)$?



Transformations in 3D: rotations

Q: What is the matrix for a rotation of angle ϕ about the z-axis?

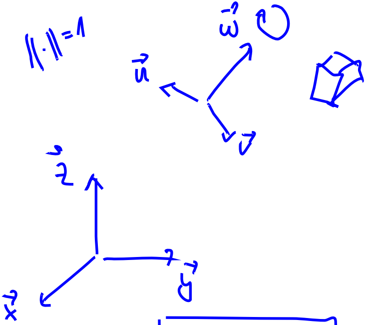
Q: What is the matrix for a rotation of angle ϕ about the vector $(3, 0, 4)$?

Q: What is the matrix for a rotation of angle ϕ about the directed line $(0, 1, 2) + t(3, 0, 4) \quad t > 0$?

Transformations in 3D: rotations

We need a 3D transformation that rotates an arbitrary vector $\vec{w}$ to the Z-axis. How do we do that?

$\|\cdot\| = 1$



$$\boxed{R_2 \cdot R_z \cdot R_1 \vec{w}}$$

1st.: Rotate $\vec{w}$ (and $\vec{u}, \vec{v}$) to $\vec{z}$ (and $\vec{x}, \vec{y}$, resp.)

2nd: Rot. around $\vec{z}$

3rd: Rotate everything back to original position

Transformations in 3D: rotations

$$\underbrace{\begin{pmatrix} x_u & y_u & z_u \\ x_v & y_v & z_v \\ x_w & y_w & z_w \end{pmatrix}}_{R_1} \underbrace{\begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{R_z} \underbrace{\begin{pmatrix} x_u & x_v & x_w \\ y_u & y_v & y_w \\ z_u & z_v & z_w \end{pmatrix}}_{R_2}$$

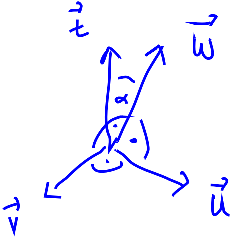
$$R_2 \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \vec{u}$$

Transformations in 3D: rotations

We need a 3D transformation that rotates an arbitrary vector to the Z-axis. How do we do that?

Q: Is such a rotation unique? Does it need to be?

$* \ \frac{\wedge}{\|\cdot\|}$



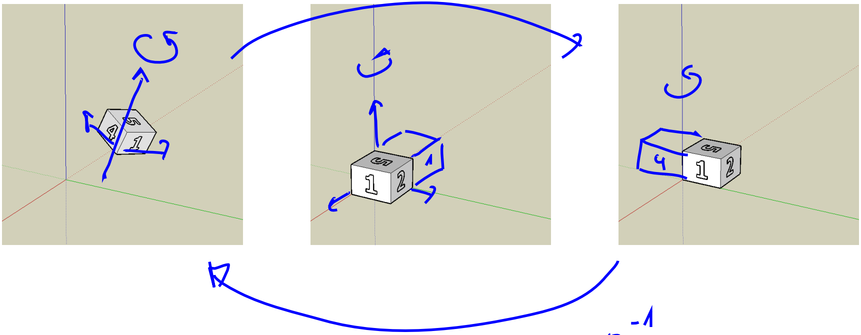
$\vec{t} \times \vec{w} = \vec{u}$

$\vec{w} \times \vec{u} = \vec{v}$

Transformations in 3D: rotations

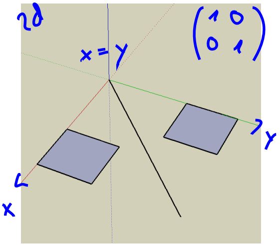
We need a 3D transformation that rotates an arbitrary vector to the Z-axis. How do we do that? R_1

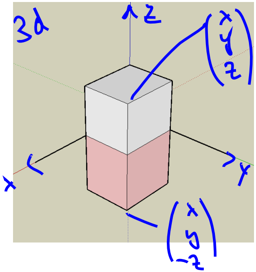
Q: Is such a rotation unique? Does it need to be?



$R_2 = R_1^{-1}$

Transformations in 3D: reflections

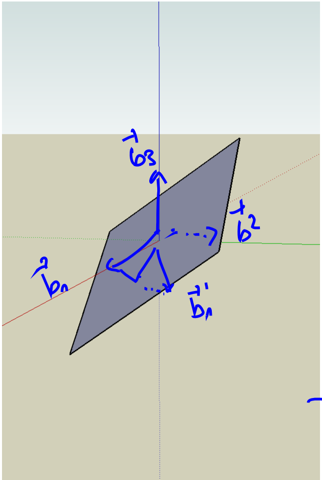




Q: What is the matrix for reflection in XY-plane?

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

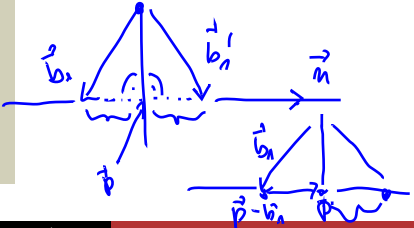
Transformations in 3D: reflections



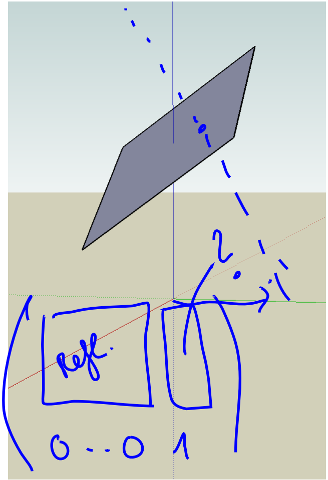
Q: What is the matrix for reflection in XY-plane?

Q: What is the matrix for reflection in the plane $3x + 4y - 12z = 0$?

$\vec{n} = \begin{pmatrix} 3 \\ 4 \\ 12 \end{pmatrix}$



Transformations in 3D: reflections



Q: What is the matrix for reflection in XY-plane?

Q: What is the matrix for reflection in the plane $3x + 4y - 12z = 0$?

Q: What is the matrix for reflection in the plane $\rightarrow 3x + 4y - 12z = 11$?

1st. : transl. to orig.
2nd : refl.
3rd : transl. back