2024 Online Physics Olympiad: Open Contest

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Instructions

If you wish to request a clarification, please use this form. To see all clarifications, see this document.

- Use $g = 9.8\ \text{m/s}^2$ in this contest, **unless otherwise specified**. See the constants sheet on the following page for other constants.
- This test contains 35 short answer questions. Each problem will have three possible attempts.
- The weight of each question depends on our scoring system found here. Put simply, the later questions are worth more, and the overall amount of points from a certain question decreases with the number of attempts that you take to solve a problem as well as the number of teams who solve it.
- Any team member is able to submit an attempt. Choosing to split up the work or doing each problem together is up to you. Note that after you have submitted an attempt, your teammates must refresh their page before they are able to see it.
- Answers should contain **three** significant figures, unless otherwise specified. All answers within the 1% range will be accepted.
- When submitting a response using scientific notation, please use exponential form. In other words, if your answer to a problem is $A \times 10^B$, please type $A\mathrm{e}B$ into the submission portal.
- A standard scientific or graphing handheld calculator *may* be used. Technology and computer algebra systems like Wolfram Alpha or the one in the TI Nspire will not be needed, but they may be used.
- You are *allowed* to use Wikipedia or books in this exam. Asking for help on online forums or your teachers will be considered cheating and may result in a possible ban from future competitions.
- Top scorers from this contest will qualify to compete in the Online Physics Olympiad *Invitational Contest*, which is an olympiad-style exam. More information will be provided to invitational qualifiers after the end of the *Open Contest*.
- In general, answer in SI units (meter, second, kilogram, watt, etc.) unless otherwise specified. Please input all angles in degrees unless otherwise specified.
- If the question asks to give your answer as a percent and your answer comes out to be "x%", please input the value x into the submission form.
- Do not put units in your answer on the submission portal! If your answer is "x meters", input only the value x into the submission portal.
- **Do not communicate information to anyone else apart from your team-members before August 25, 2024**.

List of Constants

- Proton mass, $m_p = 1.67 \cdot 10^{-27}$ kg
- Neutron mass, $m_n = 1.67 \cdot 10^{-27}$ kg
- Electron mass, $m_e = 9.11 \cdot 10^{-31}$ kg
- Avogadro's constant, $N_0 = 6.02 \cdot 10^{23}\ \mathrm{mol}^{-1}$
- Universal gas constant, $R = 8.31\ \mathrm{J/(mol \cdot K)}$
- Boltzmann's constant, $k_B = 1.38 \cdot 10^{-23}$ J/K
- Electron charge magnitude, $e = 1.60 \cdot 10^{-19}$ C
- 1 electron volt, $1\ \mathrm{eV} = 1.60 \cdot 10^{-19}$ J
- Speed of light, $c = 3.00 \cdot 10^{8}$ m/s
- Universal Gravitational constant,
$$G = 6.67 \cdot 10^{-11}\ (\mathrm{N \cdot m^2})/\mathrm{kg}^2$$
- Solar Mass
$$M_\odot = 1.988 \cdot 10^{30}\ \mathrm{kg}$$
- Acceleration due to gravity, $g = 9.8\ \mathrm{m/s^2}$
- 1 unified atomic mass unit,
$$1\ \mathrm{u} = 1.66 \cdot 10^{-27}\ \mathrm{kg} = 931\ \mathrm{MeV/c^2}$$
- Planck's constant,
$$h = 6.63 \cdot 10^{-34}\ \mathrm{J \cdot s} = 4.41 \cdot 10^{-15}\ \mathrm{eV \cdot s}$$
- Permittivity of free space,
$$\epsilon_0 = 8.85 \cdot 10^{-12}\ \mathrm{C^2/(N \cdot m^2)}$$
- Coulomb's law constant,
$$k = \frac{1}{4\pi\epsilon_0} = 8.99 \cdot 10^{9}\ (\mathrm{N \cdot m^2})/\mathrm{C^2}$$
- Permeability of free space,
$$\mu_0 = 4\pi \cdot 10^{-7}\ \mathrm{T \cdot m/A}$$
- Magnetic constant,
$$\frac{\mu_0}{4\pi} = 1 \cdot 10^{-7}\ (\mathrm{T \cdot m})/\mathrm{A}$$
- 1 atmospheric pressure,
$$1\ \mathrm{atm} = 1.01 \cdot 10^{5}\ \mathrm{N/m^2} = 1.01 \cdot 10^{5}\ \mathrm{Pa}$$
- Wien's displacement constant, $b = 2.9 \cdot 10^{-3}\ \mathrm{m \cdot K}$
- Stefan-Boltzmann constant,
$$\sigma = 5.67 \cdot 10^{-8}\ \mathrm{W/m^2/K^4}$$

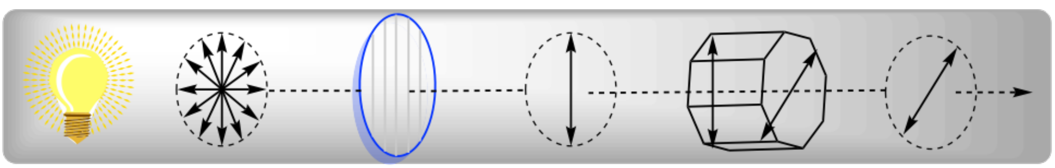
Problems

1. Jaywalking You are a jaywalker at a distance x from an intersection. At time $t = 0$ you start to cross it at a constant velocity perpendicular to the direction of the street such that it would take you time T to cross it. Incidentally, at $t = 0$, there is also a car at $X = 40.0$ m away from the intersection. It is a self-driving Tesla, and it drives toward the intersection at such a speed that if the light remained green, the car would reach the intersection at $t = T$.

At time $t = 0$, the light turns red, and since the car is driven by a computer, it immediately begins decelerating with constant acceleration a such that it comes to a stop at exactly $X = 0$.

You are $x = 8.00$ m from the intersection. Do you get hit by the car? If yes, report the speed of the car relative to the ground when it hits you, as a percent of its original speed before the light turned red. If no, report how far away the car is from you when you finish crossing the street.

2. Ambidexterity When plane-polarized light $\lambda = 500$ nm is passed through a solution with chiral molecules (eg. glucose/DNA), the exiting light is observed to have been rotated by an angle $\Delta\theta$. In chemistry, this optical rotation is measured with a polarimeter device. It is used to measure the relative abundances of left-handed and right-handed molecules, each having their own refractive index $n_L = 1.333333$ and $n_R = 1.333338$ affecting left and right circularly polarized respectively.



If the length of the solution container is $L = 0.15$ m, by which angle will the polarization of light rotate? i.e., what is the optical rotation $\Delta\theta$?

3. Ball Drop A ball with uniform density ρ_b is placed on the surface of a pool with depth d and liquid density $\rho_p < \rho_b$. Another identical ball is lifted a height h above the pool, and then both balls are released at the same time. In order for both balls to touch the bottom of the pool at the same time, the condition $d = nh$ must be met for some dimensionless n that depends on the values of ρ_p and ρ_b. If we define

$$r = \frac{\rho_b - \rho_p}{\rho_b}$$

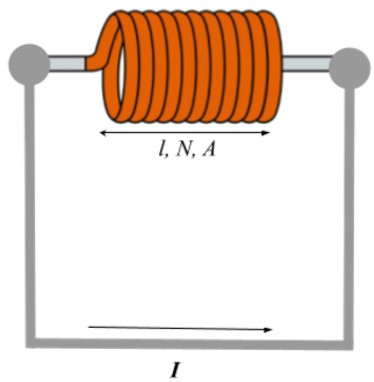
Then we can express n as

$$n = \frac{Ar^3 + Br^2 + Cr}{Dr^2 + Er + F}$$

Where A, B, C, D, E, F are all nonzero integers, $\gcd(A, B, C, D, E, F) = 1$, and $A > 0$. What is $A + B + C + D + E + F$? You may assume that the only forces present are gravity and the buoyant force from the pool. The airborne ball retains all of its energy as it enters the pool.

4. Coilfun Consider an air-cored superconducting solenoid of length $l = 1$ m with cross-sectional area $A = 0.1$ m^2 and $N = 1000$ turns. We connect the ends of the solenoid to each other with superconducting wires and run a current $I = 1600$ A through the entire setup. Assume that the solenoid behaves ideally.

Cosmonaut Carla has a core with the same dimensions as the solenoid. The core has relative permeability $\frac{\mu_i}{\mu_0} = 10000$ and mass 10 kg. The core is released at rest far from the solenoid and, due to magnetic forces, flies through the solenoid. Cosmonaut Carla can choose to quench the solenoid (instantaneously removing the current) at any time. What is the maximum attainable exit velocity of the core?



5. Into Orbit A cannon is fixed to the top of a platform of height $R = 6.00 \times 10^6$ m, which sits on a planet of mass $M = 6.00 \times 10^{24}$ kg and radius R. Both the cannon and the platform it rests on have negligible mass, and the cannon's chamber is angled horizontally relative to the planet below it. The cannon then fires a cannonball of with mass $m = 45$ kg through a chamber of length $l = 3$ m at a constant acceleration such that the cannonball is able to successfully enter an elliptical orbit around the planet. What is the minimum force that must be applied by the cannon on the cannonball in order to make this possible? You may assume that both the planet and the platform do not move during this process.

6. Roundabout 1 A ramp with length d is raised an angle θ ($0° < \theta < 90°$) above the horizontal. A block with mass m is placed at the top of the ramp, with the coefficient of friction between the block and the ramp being μ. Once the block reaches the bottom of the ramp, it retains its velocity as it is smoothly transitioned onto a frictionless circular track with radius d and bank angle θ, rotating on the track without sliding off. A *solution* is a set of values $\{d, \mu, \theta\}$ that result in the situation described above. What is the largest θ for which a solution exists?

7. Roundabout 2 While the block is going around the circular track, it is given a small push perpendicular to its current velocity and parallel to the surface of the track, causing it to oscillate with period T. What is the smallest possible value of T when $d = 5$ m?

8. Atom Smasher An alpha particle is the nucleus of a ^{4}He atom, and is composed of two protons and two neutrons bound together. A neutron is given speed v and collides with an alpha particle at rest. If all five protons and neutrons become unbound as a result of the collision, what is the minimum possible value of v/c? You may find the following values useful:

$$m_p = 938.27 \text{ MeV}/c^2, \quad m_n = 939.57 \text{ MeV}/c^2, \quad m_\alpha = 3727.4 \text{ MeV}/c^2$$

9. Dying Light Follin creates a vat of a peculiar liquid with index of refraction $n = 1 + i(1 \cdot 10^{-6})$. Just a bit complex. While working with the liquid, he accidentally drops a photodetector into it. Follin shines a red laser with wavelength $\lambda = 700$ nm and vacuum intensity $I_0 = 5 \cdot 10^6$ W/m^2 down into the liquid. How far does the photodetector sink before it detects an intensity less than $I_f = 10^{-10}$ W/m^2? Assume that the lab is completely dark and that the laser light is perfectly transmitted into the liquid.

10. Motorized Pendulum 1 A pendulum is made of a massless rod of length l and a point mass m hanging at one end. The angle between the rod and the vertical is θ. A motor attached to the pivot supplies a torque. The maximum value of this torque is angle-dependent and is given by $\tau(\theta)$.

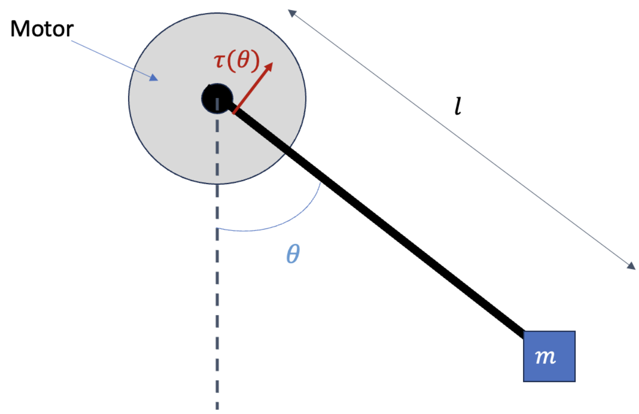


Figure 1: Motorized pendulum

Let $m = 15.00$ kg and $l = 0.5000$ m. The gravitational acceleration is $g = 9.807$ m/s^2. Also, $\tau(\theta) = \frac{1+\cos\theta}{2}\tau_0$ for $0 \leq \theta \leq 90°$.

The pendulum initially is given a small angular velocity counterclockwise and is at $\theta = 0$. The mass is extremely sensitive and cannot tolerate high speeds. Therefore, assume the motor always supplies just enough torque for the mass to move at a negligibly small constant speed. What is the minimum value of τ_0 needed so that the pendulum eventually reaches $\theta = 90°$?

11. Motorized Pendulum 2 The pendulum initially is at $\theta = 0$. This time, the mass is not so sensitive. The motor may supply its full torque for all θ. What is the minimum value of τ_0 needed so that the pendulum reaches $\theta = 90°$ in a single unidirectional swing?

12. Motorized Pendulum 3 The pendulum initially is at $\theta = 0$. The mass contains extremely sensitive electronics that cannot tolerate speeds above $v_{max} = 0.1000$ m/s. To three significant figures, what is the minimum value of τ_0 needed so that the pendulum reaches $\theta = 90°$ without exceeding this speed threshold?

13. Gravitational Oscillations 1 You are given the charge distribution on a conductive ellipsoid described by the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

If we denote its total charge by q, the surface charge density σ is given by

$$\sigma = \frac{q}{4\pi abc}\left(\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}\right)^{-1/2}$$

Now suppose we model a planet as a uniform density disk. The issue with this is that everyone at the edges would be pulled towards the center (not "downwards"). In what follows, suppose that the disk has a fixed radius R and a height $h \ll R$. This comes at the cost that different people feel different gravitational "constants" downward. Consider the density distribution of the disk $\rho = \rho(r)$ such that people living on it would only feel a gravitational pull downwards. What is the ratio $\rho(\frac{R}{3})/\rho(\frac{2R}{3})$?

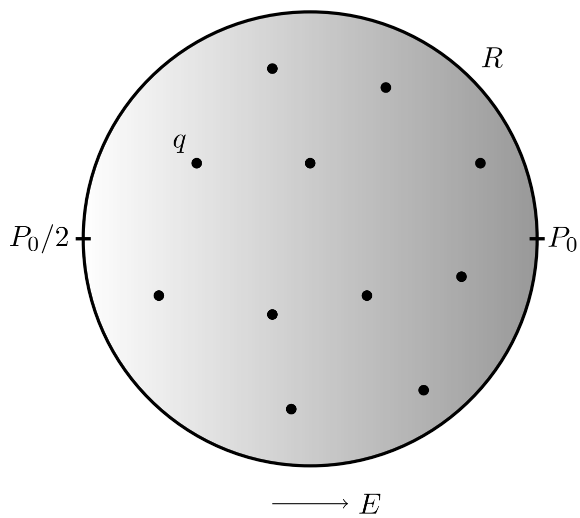
14. Gravitational Oscillations 2 Let the density right at the center of the planet be $\rho_0 = 10000$ kg/m^3, and let the height of the disk be $h = 100$ km. What is the mass enclosed in the ring with outer radius $R = 6000$ km and inner radius $R - \epsilon$, where $\epsilon = 1$ km?

15. Gravitational Oscillations 3 Now imagine that we drill a hole through the disk at a distance $r_0 = \frac{R}{3}$ and drop a ball through. What is the period of its oscillation? Neglect air friction.

16. Liquid Lenses A large cylindrical well, with a radius of 1 meter and a depth much greater than the radius ($d \gg r$), is filled with reflective liquid metal. The well and the liquid within it is then set into rotation around its central axis at an angular speed of $\omega = 5$ rad/s, causing the edge of the liquid surface to rise and touch the brim of the well. Positioned directly above the well is a circular lamp with a radius of 1 m, emitting photons vertically downward at a uniform rate and density. If the rate at which photons leave the lamp is r, then the rate at which photons collide with the liquid metal can be expressed as nr, where n is a dimensionless constant. What is the value of n?

17. Electric Slide Consider a gas of small particles, each with charge q, inside an origin-centered spherical chamber of radius R. A uniform electric field $E\hat{\mathbf{x}}$ is applied inside the chamber. The field is adjusted until point $(R, 0, 0)$ has pressure P_0 and point $(-R, 0, 0)$ has pressure $P_0/2$ (at equilibrium).

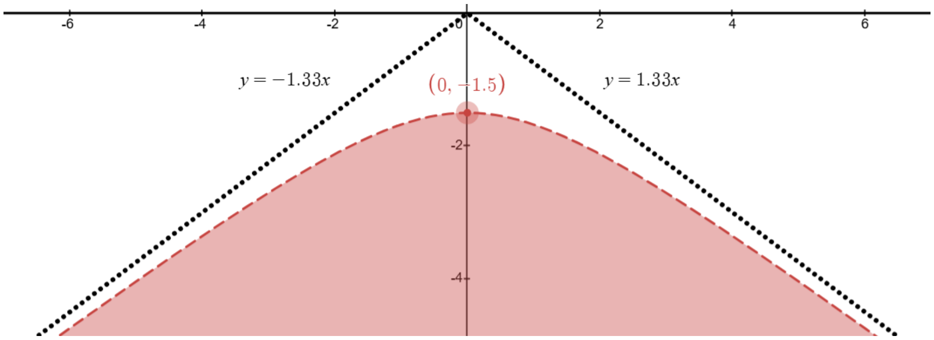
The electric field is quickly decreased to zero, and the gas comes to equilibrium again. If the final pressure in the chamber is P_1, find P_1/P_0. Neglect interactions between particles and assume that the temperature of the gas remains nearly constant.

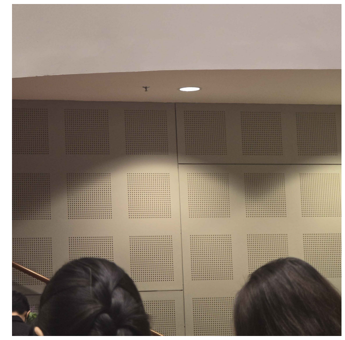


18. Ping-pong 1 A legal serve in ping-pong requires that the ball bounces on one side of the table and that the ball goes over the net. A certain world-class Olympic ping-pong player does the serve at the level of the table at a distance $d = 1.37$ m from the net of height $h = 15.25$ cm. The Olympic player can give the ball such a spin that the translational speed of the ball is conserved after a bounce but the direction of velocity can be controlled freely. What is the minimal serving speed v_1 (up to two decimal places)?

19. Ping-pong 2 We consider a serve with n bounces before going over the net. The Olympic player is so incredibly good that he can control the direction of the velocity after each bounce as he pleases. Naturally more bounces decreases the minimal serving speed v_n. However, for some N, when $n \geq N$ the minimal serving speed no longer decreases if bounces are added, i.e. $v_m = v_N$ for all $m \geq N$. Find v_{N-1}^N.

20. An Envelope of Light A point light source on the ceiling is located at the center of a cylindrical housing (with an open base) of radius R and height H. A wall is a horizontal distance D from the center of the cylinder. Now consider a coordinate system with the light source at the origin. The wall, at $x = -D$, has the following shape in the $y - z$ plane:





The vertical coordinate of the highest point of the curve observed is -1.5 m, while the gradients of the lines asymptotically tangent to the curve are $\pm 1.33... = \pm 4/3$. On the right is shown an example setup of this phenomenon. Find the horizontal distance D of the wall to the light source.

21. Under The Lamplight A large vat of the magical liquid Ophonium lies before you, with a depth of 5 meters. This liquid has a special property - its index of refraction changes variable to its depth! Its index of refraction can be expressed by the equation

$$n = 1 + 2y$$

where y is the depth of the liquid, in meters. A lamp, acting as a point source of light, is hung 3 meters above the vat. Light emanates from the lamp, casting its glow onto a circular section of the Ophonium's surface directly beneath it, covering an area of 3π square meters. As this light continues to travel downward, it enters the Ophonium, gradually penetrating its depths until it reaches the bottom of the vat. What is the area, in square meters, of the circle illuminated at the bottom of the vat? You may find the following integral useful:

$$\int \frac{1}{\sqrt{x^2 - 1}} dx = \cosh^{-1}(x) + C$$

22. Cannonball On flat ground, a cannonball is shot with a speed of $v = 13.3$ m/s over a wall of height $h = 5$ m located a distance $d = 10$ m away from the cannon. If x_{max} is the farthest away the ball can be shot over the wall and x_{min} is the shortest distance the ball can reach while going over the wall, find $x_{\text{max}} - x_{\text{min}}$.

23. Filament 1 Follin has developed a revolutionary new lightbulb filament. The filament is in the shape of a spherical shell of radius 3 cm and thickness 0.5 mm, and the ends of the filament are diametrically opposite. To attach the wires, the filament is slightly flattened on the ends, forming two circles of radius 0.01 mm, each of which is completely covered by the corresponding contact wire. If the resistivity of the filament material is $0.050\ \Omega \cdot \text{m}$, what is the resistance of Follin's lightbulb?.

24. Filament 2 Follin now modifies his filament to have six instead of two flattened circles arranged symmetrically around the sphere. He places contact wires at two flattened circles that are spaced 90° apart on their shared great circle. Find the new resistance of the lightbulb.

25. A Question of Counting A uniform beam of cold neutrons (neutrons have mass $m = 1.67 \times 10^{-27}$ m/sand speed $v = 0.66$ m/s) passes through a thin slit of width $d = 0.5$ mm. A detector, of width $w = 3$ cm, is placed a distance $r = 15$ m from the slit. Find the percentage of neutrons passing through the slit that are actually recorded by the detector.

26. Peculiar Pump A box with volume $V = 1\ \text{m}^3$ and initial temperature $T_0 = 100$ K contains monatomic ideal gas kept at a constant low pressure $p_1 = 100$ Pa. The mass of the gas molecules is $m = 7 \times 10^{-27}$ kg. The box is connected to a very large reservoir which contains monatomic ideal gas at temperature $T_2 = 400$ K and pressure $p_2 > p_1$. A small hole with area $A = 10^{-8}\ \text{m}^2$ is poked in the box at time $t = 0$ days, allowing gas to escape. For every molecule of gas that escapes through this hole, 3 molecules of gas are let into the box from the reservoir. How many days does it take for the temperature in the box to double? Assume changes in the reservoir's pressure and temperature are negligible. For reference, the rate of particles escaping out of a hole with area A is given by:

$$\Phi = \frac{pA}{\sqrt{2\pi m k_B T}}$$

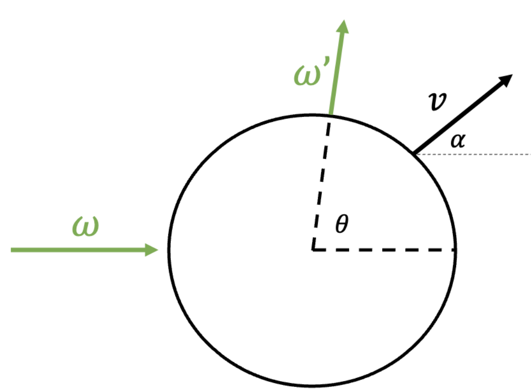
where p is the pressure of the gas, m is the mass of the gas molecules, k_B is the Boltzmann constant and T is the temperature of the gas.

27. Air Cushion An air cushion is in the shape of a cylinder with length $\ell = 10.0$ m and circular cross-sectional radius $R = 28$ cm. The ends of the cylinder lie in vertical planes, with the ground horizontal. It is filled with an incompressible gas that remains at a constant temperature. Both the surface of the air cushion and gas inside have negligible weight compared to other forces in the scenario. The surface maintains a a constant surface tension $\gamma = 5.0$ N/m whenever deformed. A flat slab of mass $m = 12.0$ kg which is wider than the cushion is balanced on top, squishing the cushion. Find the new width of the cushion. Assume that its cross-section remains symmetric about a vertical axis.

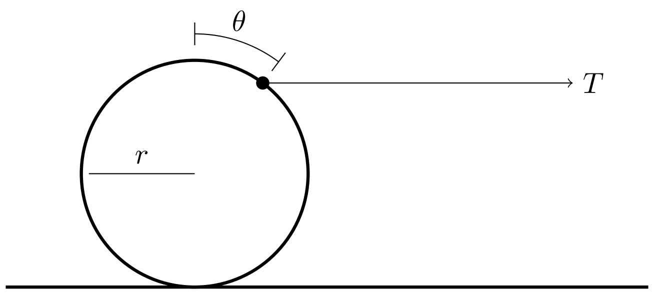
28. Soapy oscillator On a smooth table lies a square frame made of four homogeneous rods of length $\ell = 50$ cm and mass $m = 150$ g which are hinged together at the corners. Each (massless) hinge carries a charge $q = 1.5 \cdot 10^{-6}$ C. Before the frame was put on the table, it was put in a liquid soap mixture which left a soap film within the frame defined by the rods. The surface tension of said soapy water is $\sigma = 0.035$ N/m.

It turns out that it is possible to have small amplitude oscillations where opposing corners of the frame have opposite velocities (away/towards from the centre). What is the respective angular frequency of the oscillations?

29. Relativistic Scattering A small spherical particle traveling at a speed $v = 0.5c$ at an angle $\alpha = 45°$ from the horizontal is struck by an electromagnetic plane wave of angular frequency $\omega = 7.08 \times 10^{15}$ Hz propagating directly to the right. In its own reference frame, the particle scatters light in all directions with the same frequency as the frequency of incident light it perceives. Due to the relativistic Doppler effect, however, the frequency of the scattered light measured in the lab frame is generally not the same as the incident light frequency. What is the angular frequency ω' of light scattered into a scattering angle of $\theta = 89°$? Assume the radius of the particle R is small enough that $R\omega \ll c$.

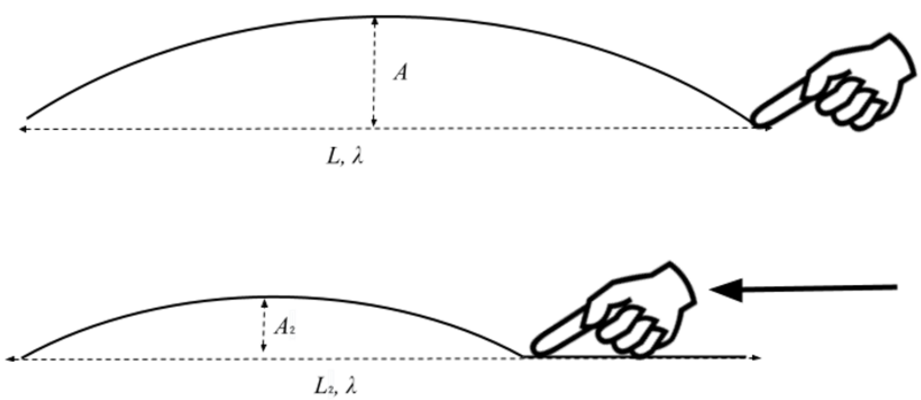


30. Reluctant Roller A hoop of mass m and radius r rests on a surface with coefficient of friction μ. At time $t = 0$, a string is attached to the hoop's highest point and a constant horizontal tension T is applied. By time t, the hoop has rotated by angle $\theta(t)$. If θ reaches but never exceeds $180°$, what is the value of $\frac{T}{\mu m g}$? You may need to solve an equation numerically.



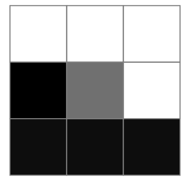
31. Solenoid Spring A solenoid of inductance $L_0 = 3.0$ mH also functions as a spring of spring constant $k = 50$ N/m. It is connected in series with a resistor of very small resistance. One end of the solenoid is firmly fixed to its wire while the other end has a conducting ring of mass $m = 0.25$ kg that is free to frictionlessly slide on its wire but always remains in contact. The length of the spring is $\ell_0 = 20$ cm. If the free end of the solenoid is slightly displaced, find the angular frequency of the resulting oscillations. The initial current through the circuit is $I = 8$ A. Assume that the thermal power lost by the resistor is negligible.

32. Devil's Trill Consider a string of length $L =$ m and linear mass density $\lambda = 1$ g/m, fixed on both ends and vibrating in the first normal mode with amplitude $A = 0.125$ cm. A frictionless, negligibly-small finger, initially at the right endpoint, slides slowly toward the left, flattening the oscillation as it goes. When the vibrating part of the string has length $L_2 = 12.5$ cm, find the new amplitude of vibrations A_2. You may assume $A_2 \ll L_2$. Diagrams are not necessarily drawn to scale.



33. Chess Imagine a 4×4 lattice with a particle in each cell. These particles move in an L-shape, similar to knights in chess. Every second, a particle moves randomly in one of the cells it can access with an L jump. Two or more particles are allowed to occupy the same cell. After some time, this system will reach an equilibrium. If the (statistical) temperature of the system and thus each cell is $T = 1$ mK, all the cells will have a definite energy level. Find the difference between the highest and lowest energy states in eV.

34. Conducting sheets Nine pieces of a thin square sheet of thickness $t = 2$ mm and side length $d = 1$ m are made: 4 with conductivity $\sigma_1 = 20 \cdot 10^5$ S/m, 4 with $\sigma_2 = 5 \cdot 10^5$ S/m and one with $\sigma_3 = 10 \cdot 10^5$ S/m. These are put together in a 3×3 grid such that the σ_3 plate (gray) is in the centre and the σ_1 (white) and σ_2 (black) plates form an L-shape as in the figure. The top and bottom side are put into contact with well-conducting electrodes with a potential difference $V = 13$ mV. What is the total current going through the sheet?



35. A Tired Flappy Bird A flappy bird can jump multiple times in the air. Each time it jumps mid-air, it can suddenly change its speed and direction. For every jump, the bird can decide when to jump and in which direction. Between jumps, the bird falls freely under gravity, which pulls it down at the acceleration g. Say, our tired flappy bird starts off the cliff of height H with the jumping velocity $V[1] = V_0$. Subsequent jumps in mid-air have decreasing velocities, i.e. the n-th jump has speed $V[n] = V_0/n$ $(n > 1)$. This majestic Vietnamese animal wants to travel as far as possible horizontally before it lands on the ground. Find the maximum horizontal distance the bird can travel (denoted as L in the figure below) in meters, given that $H = 100$m and $V_0 = 10$m/s. *Note that each jumping velocity is the total speed of the bird after the jump (rather than e.g. adding to its speed before the jump).*