2023 Online Physics Olympiad: Open Contest

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Instructions

If you wish to request a clarification, please use this form. To see all clarifications, see this document.

- Use $g = 9.8\ \mathrm{m/s^2}$ in this contest, **unless otherwise specified**. See the constants sheet on the following page for other constants.
- This test contains 35 short answer questions. Each problem will have three possible attempts.
- The weight of each question depends on our scoring system found here. Put simply, the later questions are worth more, and the overall amount of points from a certain question decreases with the number of attempts that you take to solve a problem as well as the number of teams who solve it.
- Any team member is able to submit an attempt. Choosing to split up the work or doing each problem together is up to you. Note that after you have submitted an attempt, your teammates must refresh their page before they are able to see it.
- Answers should contain **three** significant figures, unless otherwise specified. All answers within the 1% range will be accepted.
- When submitting a response using scientific notation, please use exponential form. In other words, if your answer to a problem is $A \times 10^B$, please type AeB into the submission portal.
- A standard scientific or graphing handheld calculator *may* be used. Technology and computer algebra systems like Wolfram Alpha or the one in the TI nSpire will not be needed or allowed. Attempts to use these tools will be classified as cheating.
- You are *allowed* to use Wikipedia or books in this exam. Asking for help on online forums or your teachers will be considered cheating and may result in a possible ban from future competitions.
- Top scorers from this contest will qualify to compete in the Online Physics Olympiad *Invitational Contest*, which is an olympiad-style exam. More information will be provided to invitational qualifiers after the end of the *Open Contest*.
- In general, answer in SI units (meter, second, kilogram, watt, etc.) unless otherwise specified. Please input all angles in degrees unless otherwise specified.
- If the question asks to give your answer as a percent and your answer comes out to be "x%", please input the value x into the submission form.
- Do not put units in your answer on the submission portal! If your answer is "x meters", input only the value x into the submission portal.
- **Do not communicate information to anyone else apart from your team-members before July 25, 2023**.

List of Constants

- Proton mass, $m_p = 1.67 \cdot 10^{-27}$ kg
- Neutron mass, $m_n = 1.67 \cdot 10^{-27}$ kg
- Electron mass, $m_e = 9.11 \cdot 10^{-31}$ kg
- Avogadro's constant, $N_0 = 6.02 \cdot 10^{23}$ mol^{-1}
- Universal gas constant, $R = 8.31$ J/(mol $\cdot$ K)
- Boltzmann's constant, $k_B = 1.38 \cdot 10^{-23}$ J/K
- Electron charge magnitude, $e = 1.60 \cdot 10^{-19}$ C
- 1 electron volt, 1 eV $= 1.60 \cdot 10^{-19}$ J
- Speed of light, $c = 3.00 \cdot 10^8$ m/s
- Universal Gravitational constant,
$$G = 6.67 \cdot 10^{-11} \text{ (N} \cdot \text{m}^2\text{)/kg}^2$$
- Solar Mass
$$M_\odot = 1.988 \cdot 10^{30} \text{ kg}$$
- Acceleration due to gravity, $g = 9.8$ m/s^2
- 1 unified atomic mass unit,
$$1 \text{ u} = 1.66 \cdot 10^{-27} \text{ kg} = 931 \text{ MeV/c}^2$$
- Planck's constant,
$$h = 6.63 \cdot 10^{-34} \text{ J} \cdot \text{s} = 4.41 \cdot 10^{-15} \text{ eV} \cdot \text{s}$$
- Permittivity of free space,
$$\epsilon_0 = 8.85 \cdot 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)$$
- Coulomb's law constant,
$$k = \frac{1}{4\pi\epsilon_0} = 8.99 \cdot 10^9 \text{ (N} \cdot \text{m}^2\text{)/C}^2$$
- Permeability of free space,
$$\mu_0 = 4\pi \cdot 10^{-7} \text{ T} \cdot \text{m/A}$$
- Magnetic constant,
$$\frac{\mu_0}{4\pi} = 1 \cdot 10^{-7} \text{ (T} \cdot \text{m)/A}$$
- 1 atmospheric pressure,
$$1 \text{ atm} = 1.01 \cdot 10^5 \text{ N/m}^2 = 1.01 \cdot 10^5 \text{ Pa}$$
- Wien's displacement constant, $b = 2.9 \cdot 10^{-3}$ m $\cdot$ K
- Stefan-Boltzmann constant,
$$\sigma = 5.67 \cdot 10^{-8} \text{ W/m}^2\text{/K}^4$$

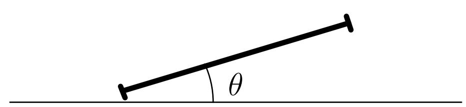
Problems

1. Coin Flip 1 The coin flip has long been recognized as a simple and unbiased method to randomly determine the outcome of an event. In the case of an ideal coin, it is well-established that each flip has an equal 50% chance of landing as either heads or tails.

However, coin flips are not entirely random. They appear random to us because we lack sufficient information about the coin's initial conditions. If we possessed this information, we would always be able to predict the outcome without needing to flip the coin. For an intriguing discussion on why this observation is significant, watch this video by Vsauce.

Now, consider a scenario where a coin with uniform density and negligible width is tossed directly upward from a height of $h = 0.75$ m above the ground. The coin starts with its heads facing upward and is given an initial vertical velocity of $v_y = 49$ m/s and a positive angular velocity of $\omega = \pi$ rad/s. What face does the coin display upon hitting the ground? **Submit 0 for heads and 1 for tails.** You only have one attempt for this problem. Assume the floor is padded and it absorbs all of the coin's energy upon contact.

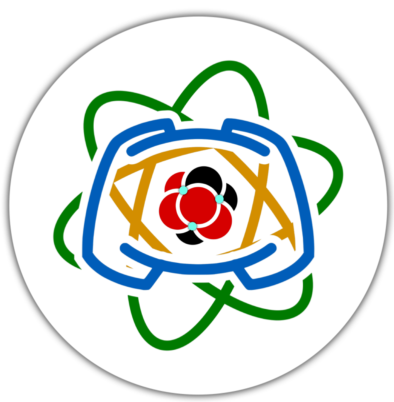
2. Coin Flip 2 A coin with a radius of $r = 1$ cm is initially at rest and is released from a slight tilt of $\theta = 8^\circ$ onto a horizontal surface with an infinite coefficient of static friction. The coin has a thicker rim, allowing it to drop and rotate on one point. With every collision, the coin switches pivot points on the rim, and energy is dissipated through heat so that $k = 0.9$ of the coin's prior total energy is conserved. How long will it take for the coin to come to a complete stop?



A cross-sectional view of the coin before release. The rim can be seen on the edges of the coin.

3. Highway Suppose all cars on a (single-lane) highway are identical. Their length is $l = 4$m, their wheels have coefficients of friction $\mu = 0.7$, and they all travel at speed v_0. Find the v_0 which maximizes the flow rate of cars (i.e. how many cars travel across an imaginary line per minute). Assume that they need to be able to stop in time if the car in front instantaneuously stops.

4. Spinning Around Here is a Physoly round button badge, in which the logo is printed on the flat and rigid surface of this badge. Toss it in the air and track the motions of three points (indicated by cyan circles in the figure) separated $L = 5$ mm apart. At a particular moment, we find that these both have the same speed $V = 4$ cm/s but are heading to different directions which form an angle of $\theta = 30^\circ$ between each pair. Determine the then angular velocity of the badge (in rad/s).



5. Born To Try In a resource-limited ecological system, a population of organisms cannot keep growing forever (such as lab bacteria growing inside culture tube). The effective growth rate g (including contributions from births and deaths) depends on the instantaneous abundance of resource $R(t)$, which in this problem we will consider the simple case of linear-dependency:

$$\frac{\mathrm{d}}{\mathrm{d}t}N = g(R)N = \alpha RN \ ,$$

where $N(t)$ is the population size at time t. The resources is consumed at a constant rate β by each organism:

$$\frac{\mathrm{d}}{\mathrm{d}t}R = -\beta N \ .$$

Initially, the total amount of resources is R_0 and the population size is N_0. Given that $\alpha = 10^{-9}$ resource-unit^{-1}s^{-1}, $\beta = 1$ resource-unit/s, $R_0 = 10^6$ resource-units and $N_0 = 1$ cell, find the total time it takes from the beginning to when all resources are depleted (in hours).

6. Lightbulb An incandescent lightbulb is connected to a circuit which delivers a maximum power of 10 Watts. The filament of the lightbulb is made of Tungsten and conducts electricity to produce light. The specific heat of Tungsten is $c = 235$ J/(K $\cdot$ kg). If the circuit is alternating such that the temperature inside the lightbulb fluctuates between $T_0 = 3000°$ C and $T_1 = 3200°$ C at a frequency of $\omega = 0.02$ s^{-1}, estimate the mass of the filament.

7. Hyperdrive In hyperdrive, Spaceship-0 is relativistically moving at the velocity $\frac{1}{3}c$ with respect to reference frame R_1, as measured by Spaceship-1. Spaceship-1 is moving at $\frac{1}{2}c$ with respect to reference frame R_2, as measured by Spaceship-2. Spaceship-k is moving at speed $v_k = \frac{k+1}{k+3}c$ with respect to reference frame R_{k+1}. The speed of Spaceship-0 with respect to reference frame R_{20} can be expressed as a decimal fraction of the speed of light which has only x number of 9s following the decimal point (i.e., in the form of $0.\underbrace{99\ldots9}_{x \text{ times}}c$). Find the value of x.

8. Asteroid The path of an asteroid that comes close to the Earth can be modeled as follows: neglect gravitational effects due to other bodies, and assume the asteroid comes in from far away with some speed v and lever arm distance r to Earth's center. On January 26, 2023, a small asteroid called 2023 BU came to a close distance of 3541 km to Earth's surface with a speed of 9300 m/s. Although BU had a very small mass estimated to be about 300,000 kg, if it was much more massive, it could have hit the Earth. How massive would BU have had to have been to make contact with the Earth? Express your answer in scientific notation with 3 significant digits.

9. Spaceship IK Pegasi and Betelgeuse are two star systems that can undergo a supernova. Betelgeuse is 548 light-years away from Earth and IK Pegasi is 154 light-years away from Earth. Assume that the two star systems are 500 light-years away from each other.

Astronomers on Earth observe that the two star systems undergo a supernova explosion 300 years apart. A spaceship, the *OPhO Galaxia Explorer* which left Earth in an unknown direction before the first supernova observes both explosions occur simultaneously. Assume that this spaceship travels in a straight line at a constant speed v. How far are the two star systems according to the *OPhO Galaxia Explorer* at the moment of the simultaneous supernovae? Answer in light-years.

Note: Like standard relativity problems, we are assuming intelligent observers that know the finite speed of light and correct for it.

10. Drag 1 A ball of mass 1 kg is thrown vertically upwards and it faces a quadratic drag with a terminal velocity of 20 m/s. It reaches a maximum height of 30 m and falls back to the ground. Calculate the energy dissipated until the point of impact (in J).

11. Drag 2 In general, we can describe the quadratic drag on an object by the following force law:

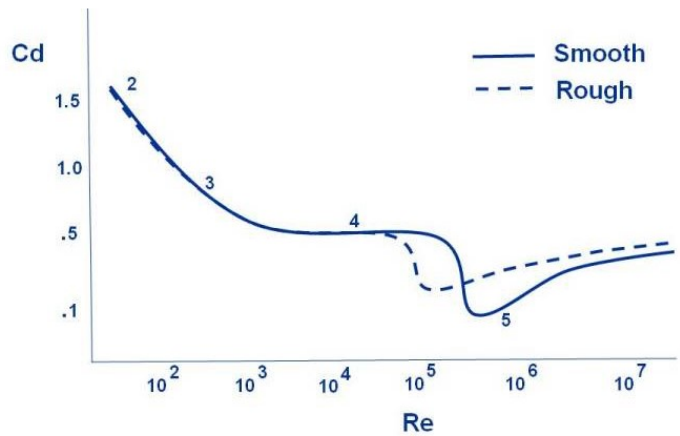
$$F_D = \frac{1}{2} C_D \rho A v^2$$

where A is the cross-sectional area of the object exposed to the airflow, v is the speed of the object in a fluid, and C_D is the drag coefficient, a dimensionless quantity that varies based on shape.

Another useful quantity to know is the Reynold's number, a dimensionless quantity that helps predict fluid flow patterns. It is given by the formula:

$$\text{Re} = \frac{\rho v L}{\mu}$$

where ρ is the density of the surrounding fluid, μ is the dynamic viscosity of the fluid, and L is a reference length parameter that varies based on each object. For a smooth [1] sphere traveling in a fluid, its diameter serves as the reference length parameter.



A logarithmic graph of C_D vs Re of a sphere from the NASA Glenn Research Center.

The relationship between the drag coefficient and the Reynold's number holds significant importance. Due to the complexity of fluid dynamics, empirical data is commonly used, as depicted in the figure provided above. Notably, the figure indicates a significant decrease in the drag coefficient around $\text{Re} \approx 4 \times 10^5$. This phenomenon, known as the drag crisis, occurs when a sphere transitions from laminar to turbulent flow, resulting in a broad wake and high drag. The table in the link below presents a range of C_d versus Re values of a smooth sphere.

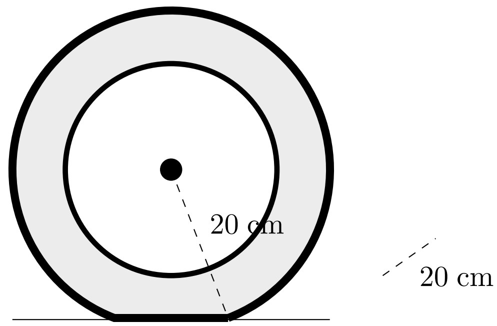
Desmos table: `https://www.desmos.com/calculator/wnpkg5wnt0`

Let's consider a smooth ball with a radius of 0.2 m and a mass of 0.1 kg dropped in air with a constant density of $\rho = 1.255$ kg/m^3. It is found that at velocity 5 m/s, the Reynold's number of the ball is $3.41 \cdot 10^5$. If the ball is dropped from rest, it approaches a *stable* terminal velocity v_1. If the ball is thrown downwards with enough velocity, it will experience turbulence, and approach a *stable* terminal velocity v_2. Find $\Delta v = v_2 - v_1$. Ignore any terminal velocities found for Reynold numbers less than an order of magnitude 10^{-1}.

Note: This problem is highly idealized as it assumes the atmosphere has air of constant density and temperature. In reality, this is not true!

[1] meaning a smooth surface.

The following information applies for the next two problems. Pictured is a wheel from a 4-wheeled car of weight 1200kg. The absolute pressure inside the tire is 3.0×10^5Pa. Atmospheric pressure is 1.0×10^5Pa. Assume the rubber has negligible "stiffness" (i.e. a negligibly low Sheer modulus compared to its Young's modulus).



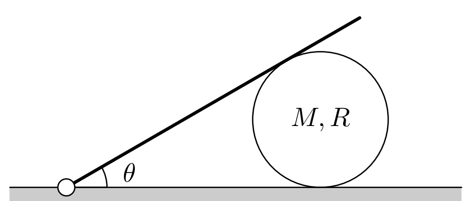
12. Hysteresis 1 The rubber on the bottom of the wheel is completely unstretched. The rubber has a thickness of 7 mm. Based on this information, find the Young's Modulus of the rubber. Is this answer reasonable?

13. Hysteresis 2 The rubber experiences a phenomena known as *hysteresis* – it takes more force to stretch the rubber than allow it to return to equilibrium. Specifically, assume that the Young's Modulus when the rubber is stretched is the answer to #2, and is 1/2 of that when the rubber returns to equilibrium. Compute the power the car's engines has to deliver to overcome the hysteresis losses, if the car moves at 20 m/s. Remember that there are 4 tires!

14. Marbles Two identical spherical marbles of radius 3 cm are placed in a spherical bowl of radius 10 cm. The coefficient of static friction between the two surfaces of the marble is 0.31 and the coefficient of static friction between the surfaces of the marbles and the bowl is 0.13. Find the maximum elevation from the bottom of the bowl that the center of one of the marbles can achieve in equilibrium.

15. Fringe effect approximation Two parallel square plates of side length 1 m are placed a distance 30 cm apart whose centers are at $(-15\text{ cm}, 0, 0)$ and $(15\text{ cm}, 0, 0)$ have uniform charge densities -10^{-6} C/m^2 and 10^{-6} C/m^2 respectively. Find the magnitude of the component of the electric field perpendicular to axis passing through the centers of the two plates at $(10\text{ cm}, 1\text{ mm}, 0)$.

16. Sliding Along A hollow sphere of mass M and radius R is placed under a plank of mass $3M$ and length $2R$. The plank is hinged to the floor, and it initially makes an angle $\theta = \frac{\pi}{3}$ rad to the horizontal. Under the weight of the plank, the cylinder starts rolling without slipping across the floor. What is the cylinder's initial translational acceleration? Assume the plank is frictionless.



A not-to-scale picture of the sphere-plank setup.

The following information applies for the next two problems. A space elevator consists of a heavy counterweight placed near geostationary orbit, a thread that connects it to the ground (assume this is massless), and elevators that run on the threads (also massless). The mass of the counterweight is 10^7 kg. Mass is continuously delivered to the counterweight at a rate of 0.001 kg/s. The elevators move upwards at a rate of 20 m/s. Assume there are many elevators, so their discreteness can be neglected.

17. Space Elevator 1 Find the minimum possible displacement radially of the counterweight. Specify the sign.

18. Space Elevator 2 Assuming a radial displacement that is 10 times what you found in the previous part, find the displacement tangentially of the counterweight. Does it lag or lead Earth's motion?

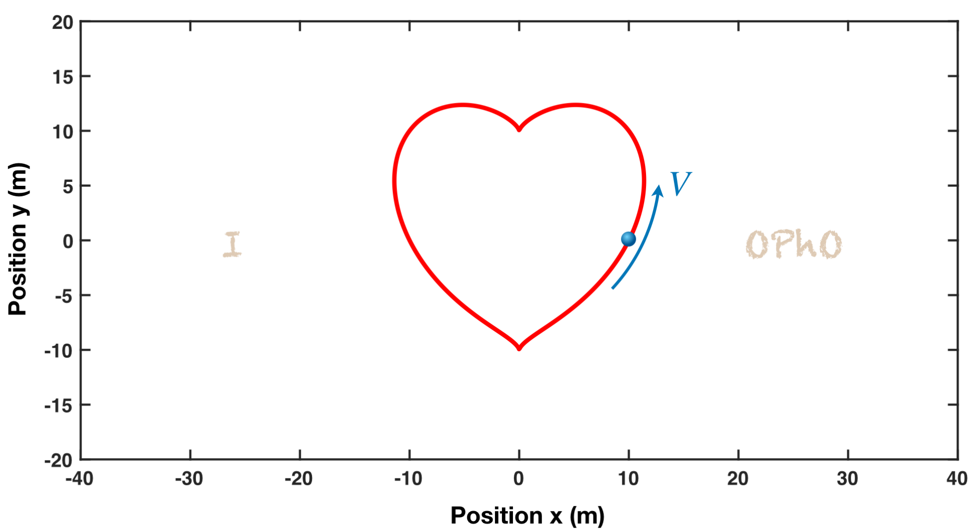
19. Laser Power Consider a spherical shell of thickness $\delta = 0.5$cm and radius $R = 5$cm made of an Ohmic material with resistivity $\rho = 10^{-7}\Omega$m. A spherical laser source with a tuned frequency of $f_0 = 3 \times 10^{12}$Hz and intensity $I_0 = 10^5$ W/m^2 is placed at the center of the shell and is turned on. Working in the limit $\delta \ll R \ll \frac{c}{f_0}$, approximate the initial average power dissipated by the shell. Neglect inductance.

20. Twinkle Twinkle. A stable star of radius R has a mass density profile $\rho(r) = \alpha(1 - r/R)$. Here, "stable" means that the star doesn't collapse under its own gravity. If the internal pressure at the core is provided solely by the radiation of photons, calculate the temperature at the core. Assume the star is a perfect black body and treat photons as a classical ideal gas. Use $R = 7 \times 10^5$km and $\alpha = 3$g/cm. Round your answer to the nearest kilokelvin.

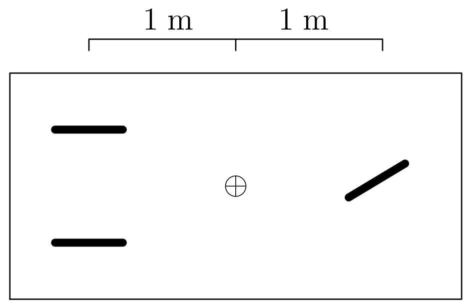
21. My Heart Will Go On On a flat playground, choose a Cartesian Oxy coordinate system (in unit of meters). A child running at a constant velocity $V = 1$m/s around a heart-shaped path satisfies the following order-6 algebraic equation:

$$\left(x^2 + y^2 - L^2\right)^3 - Lx^2y^3 = 0 \ , \ L = 10 \ .$$

When the child is at the position $(x, y) = (L, 0)$, what is the magnitude of their acceleration?



22. Tricycle A boy is riding a tricycle across along a sidewalk that is parallel to the x-axis. This tricycle contains three identical wheels with radius 0.5 m. The front wheel is free to rotate while the last two wheels are parallel to each other and to the main body of the tricycle. See the diagram.



The front wheel is rotating at a constant angular speed of $\omega = 3$ rad/s. The child is controlling the tricycle such that the front wheel is making an angle of $\theta(t) = 0.15\sin((0.1 \text{ rad/s})t)$ with the main body of the tricycle. Determine the maximum lateral acceleration in m/s^2.

23. Sonic Fryer In this problem, we consider a simple model for a thermoacoustic device. The device uses heavily amplified sound to provide work for a pump that can then extract heat. Sound waves form standing waves in a tube of radius 0.25 mm that is closed on both sides, and a two-plate stack is inserted in the tube. A temperature gradient forms between the plates of the stack, and the parcel of gas trapped between the plates oscillates sinusoidally between a maximum pressure of 1.03 MPa and a minimum of 0.997 MPa. The gas is argon, with density 1.78 kg/m^3 and adiabatic constant 5/3. The speed of sound is 323 m/s. The heat pump itself operates as follows:

The parcel of gas starts at minimum pressure. The stack plates adiabatically compress the parcel of gas to its maximum pressure, heating the gas to a temperature higher than that of the hotter stack plate. Then, the gas is allowed to isobarically cool to the temperature of the hotter stack plate. Next, the plates adiabatically expand the gas back to its minimum pressure, cooling it to a temperature lower than that of the colder plate. Finally, the gas is allowed to isobarically heat up to the temperature of the colder stack plate.

Find the power at which the thermoacoustic heat pump emits heat.

The following information applies for the next two problems. For your mass spectroscopy practical you are using an apparatus consisting of a solenoid enclosed by a uniformly charged hollow cylinder of charge density $\sigma = 50\ \mu\text{C/m}^2$ and radius $r_0 = 7$ cm. There exists an infinitesimal slit of insular material between the cylinder and solenoid to stop any charge transfer. Also, assume that there is no interaction between the solenoid and the cylinder, and that the magnetic field produced by the solenoid can be easily controlled to a value of B_0.

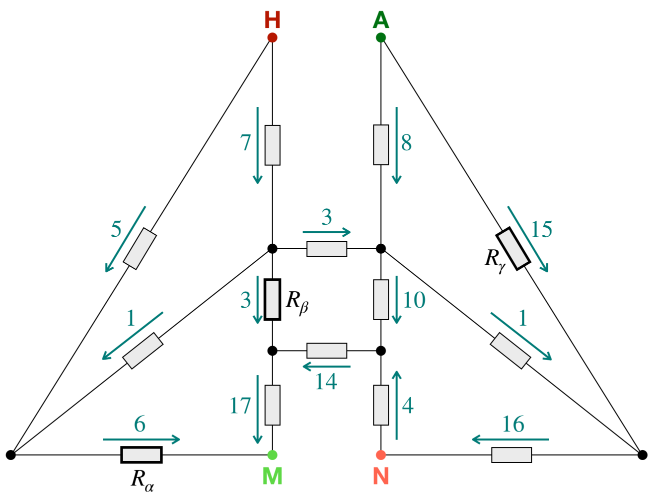
An electron is released from rest at a distance of $R = 10$ cm from the axis. Assume that it is small enough to pass through the cylinder in both directions without exchanging charge. It is observed that the electron reaches a distance R at different points from the axis 7 times before returning to the original position.

24. Isotope Separator 1 Calculate B_0.

25. Isotope Separator 2 Calculate the time it took for the particle to return to original position. Answer in milliseconds.

> **Hint:** You may find interest in the Gaussian error function:
>
> $$\text{erf}(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-t^2}\text{d}t.$$
>
> Specific values of the error function can be calculated on desmos.

26. Somos El Barco For any circuit network made of batteries and resistors, if we know the voltages of all the batteries and the resistance values of all the resistors, we can calculate all the electrical currents. However, if we know the voltages of all the batteries and all the currents, it is still not enough to uniquely determine the resistance values of all the resistors. Consider a sail-shape circuit network, in which we connect points H and N with a $\mathcal{E}_{\mathrm{HN}} = 10\mathrm{V}$ battery, points A and M with a $\mathcal{E}_{\mathrm{AM}} = 20\mathrm{V}$ battery. The electrical currents in this network have directions and magnitudes (in mA) as shown the figure. The possible resistance values of resistors R_α, R_β, R_γ is not a single point (corresponds to an unique solution) but a confined region in the three-dimensional $(R_\alpha, R_\beta, R_\gamma)$-space. Determine the volume of this region (in Ω^3).



The title of this problem means "we are the boat".

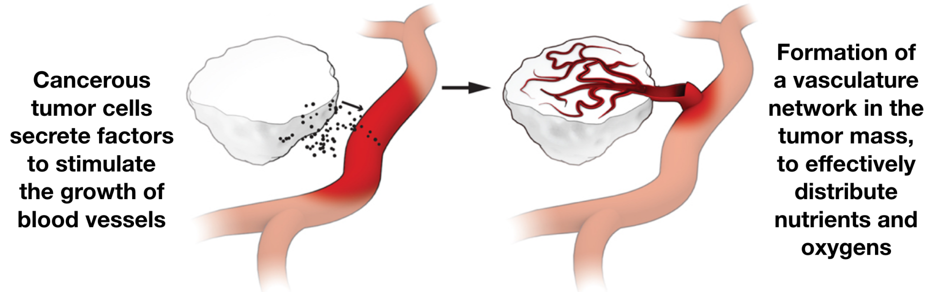
27. The Final Countdown A model of cancer tumor dynamics under a low-dose chemotherapy consists of three non-negative variables (P, Q, R), in which P represents the cancer tumor size, Q represents the (normalized) carrying capability of the tumor vasculature network, and R represents the local (normalized) activity of the immunology system:

$$\frac{\mathrm{d}}{\mathrm{d}t}P = \xi P \ln\frac{Q}{P} - \theta PR - \varphi_1 PC \quad ,$$
$$\frac{\mathrm{d}}{\mathrm{d}t}Q = bP - \left(\mu + dP^{2/3}\right) Q - \varphi_2 QC \quad ,$$
$$\frac{\mathrm{d}}{\mathrm{d}t}R = \alpha\left(P - \beta P^2\right) R + \gamma - \delta R + \varphi_3 RC \quad .$$

Here, C is the local concentration of chemotherapeutic agent at the tumor site, which we can assume to follow by a simple pharmacokinetics model:

$$\frac{\mathrm{d}}{\mathrm{d}t}C = -\frac{1}{\tau}C + U \quad ,$$

where U is the rate of chemotherapy drug administrated to the patient body. Let us assume an unchanging rate U, and treat it as another parameter of the model. All other unmentioned symbols are positive constant parameters, which values can be measured and should depends on the particular kinds of cancers and treatments. In total, there are 14 parameters – such a high-degree of complexity is very common in biophysical models. For each set of parameters, there can be many possible stationary states, which can be associated with various levels of malignancy. What is the maximum number of stationary non-zero tumor sizes (including both stable and unstable ones) for a set of parameters in this model? **For this problem, you can only submit your answer once.**



28. Magnetic Carts Two carts, each with a mass of 300 g, are fixed to move on a horizontal track. As shown in the figure, the first cart has a strong, tiny permanent magnet of dipole moment $0.5\ \mathrm{A}\cdot\mathrm{m}^2$ attached to it, which is aligned along the axis of the track pointing toward the other cart. On the second cart, a copper tube of radius 7 mm, thickness 0.5 mm, resistivity $1.73\cdot 10^{-8}\Omega$, and length 30 cm is attached. The masses of the magnet and coil are negligible compared to the mass of the carts. At the moment its magnet enters through the right end of the copper tube, the velocity of the first cart is 0.3 m/s and the distance between the two ends of each cart is 50 cm, find the minimum distance achieved between the two ends of the carts in centimeters. While on the track, the carts experience an effective coefficient of static friction (i.e., what it would be as if they did not have wheels) of 0.01. Neglect the self-inductance of the copper tube.

A picture of the two cart setup. The black rectangle represents the magnet while the gold rectangle represents the copper tube.

Hint 1: The magnetic field due to a dipole of moment $\vec{\mu}$, at a position $\vec{r}$ away from the dipole can be written as

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{3\hat{r}(\vec{r}\cdot\vec{\mu} - \vec{\mu})}{r^3}\hat{r}$$

where $\hat{r}$ is the unit vector in the direction of $\vec{r}$.

Hint 2: The following mathematical identity may be useful:

$$\int_{-\infty}^{\infty}\frac{u^2 du}{(1+u^2)^5} = \frac{5\pi}{128}.$$

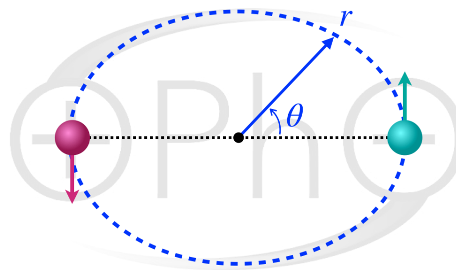
29. De-Terraforming In the far future, the Earth received an enormous amount of charge as a result of Mad Scientist ecilA's nefarious experiments. Specifically, the total charge on Earth is $Q = 1.0\times 10^{11}$C. (compare this with the current 5×10^5C).

Estimate the maximum height of a "mountain" on Earth that has a circular base with diameter $w = 1.0$ km, if it has the shape of a spherical sector. You may assume that $h_{\max} \ll w$. The tensile strength of rock is 10 MPa.

30. All Around The World The logo of OPhO describes two objects travelling around their center-of-mass, following the same oval-shape trajectory. For simplicity, we assume these objects are point-like, have identical mass, and interacts via an interacting potential $U(d)$ depends on the distance d between them. Choose the polar coordinates (r,θ) as shown in the figure, where the origin is located at the center of the logo, then the shared trajectory obeys the equation:

$$r(\theta) = \frac{L}{2}\left[1-\epsilon\cos(2\theta)\right]^{-(1+\gamma)}\ ,$$

in which we consider $\epsilon = 0.12$ and $\gamma = 0.05$. Here the smallest and largest separation between the objects are d_{min} and d_{max}. Since the interacting potential $U(d)$ is defined up to a constant, let us pick $U(L) = 0$. Find the ratio $U(d_{\text{min}})/U(d_{\text{max}})$.

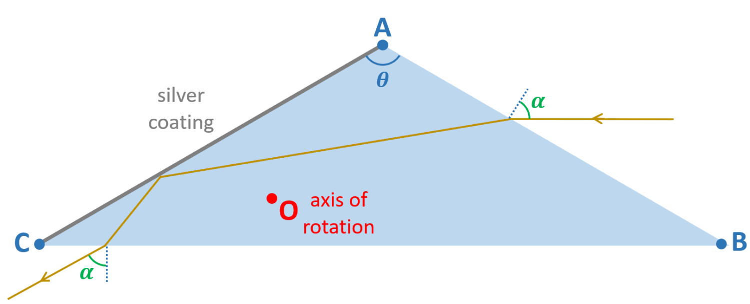


31. Electrostatic Pendulum 1 Follin is investigating the electrostatic pendulum. His apparatus consists of an insulating Styrofoam ball with a mass of $14mg$ and radius $r = 0.5$ cm suspended on a uniform electrically-insulating string of length 1 m and mass per unit length density of $1.1 \cdot 10^{-5}$ kg/m between two large metal plates separated by a distance 17 cm with a voltage drop of $10kV$ between them, such that when the ball is in equilibrium, its center of mass is exactly equidistant to the two plates. Neglect the possibility of electrical discharge throughout the next two problems.

Follin then gives the ball a charge 0.15 nC. Assuming that the charge is distributed evenly across the surface of the ball, find the subsequent horizontal deflection of the pendulum bob's center of mass from its hanging point.

32. Electrostatic Pendulum 2 Hoping to get a larger deflection, Follin replaces the insulating Styrofoam ball with a conducting pith ball of mass $250mg$ and 2 cm and daisy chains 4 additional 10 kV High Voltage Power Supplies to increase the voltage drop across the plates to 50 kV. Leaving the plate separation and the string unchanged, he repeats the same experiment as before, but forgets to measure the charge on the ball. Nonetheless, once the ball reaches equilibrium, he measures the deflection from the hanging point to be 5.6 cm. Find the charge on the ball.

The following information applies for the next two problems. Consider a uniform isosceles triangle prism ABC, with the apex angle θ at vertex A. One of the sides, AC, is coated with silver, allowing it to function as a mirror. When a ray of light approaches the midpoint of side AB at an angle of incidence α, it first refracts, then reaches side AC, reflects, and continues to base BC. After another refraction, the ray eventually exits the prism at the angle of emergence α. Given that $\theta = 110°$ and $\alpha = 70°$.



33. Man In The Mirror 1 What is the relative refractive index of the prism with respect to the outside environment.

34. Man In The Mirror 2 For every light-ray follows the above description, there exists an axis of rotation O where the emergent ray remains fixed while the prism rotates around it as the incident ray remains unchanged. Find the maximum possible length-ratio between the distance from the axis to the one of the vertices max(OA, OB, OC) and the base BC.

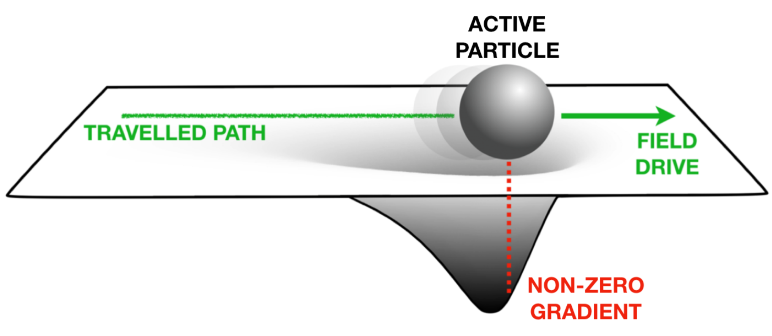
35. Funiculì, Funiculà Field-drive is a locomotion mechanism that is analogous to general relativistic warp-drive. In this mechanism, an active particle continuously climbs up the field-gradient generated by its own influence on the environment so that the particle can bootstrap itself into a constant non-zero velocity motion. Consider a field-drive in one-dimensional (the Ox axis) environment, where the position of the particle at time t is given by $X(t)$ and its instantaneous velocity follows from:

$$\frac{\mathrm{d}}{\mathrm{d}t}X(t) = \kappa \frac{\partial}{\partial x} R(x,t)\Big|_{x=X(t)} ,$$

in which κ is called the guiding coefficient and $R(x,t)$ is the field-value in this space. Note that, the operation $...|_{x=X(t)}$ means you have to calculate the part in ... first, then replace x with $X(t)$. For a biological example, the active particle can be a cell, the field can be the nutrient concentration, and the strategy of climbing up the gradient can be chemotaxis. The cell consumes the nutrient and also responses to the local nutrient concentration, biasing its movement toward the direction where the concentration increases the most. If the nutrient is not diffusive and always recovers locally (e.g. a surface secretion) to the value which we defined to be 0, then its dynamics can usually be approximated by:

$$\frac{\partial}{\partial t}R(x,t) = -\frac{1}{\tau}R(x,t) - \gamma \exp\left\{-\frac{[x - X(t)]^2}{2\lambda^2}\right\} ,$$

where τ is the timescale of recovery, γ is the consumption, and λ is the characteristic radius of influence. Before we inoculate the cell into the environment, $R = 0$ everywhere at any time. What is the smallest guiding coefficient κ (in μm^2/s) for field-drive to emerge, if the parameters are $\tau = 50$s, $\gamma = 1$s^{-1}, and $\lambda = 10\mu$m.



The title of this problem means "funicular up, funicular down".