Homework 07 DTMC

Math 519

William Gillespie

Water Molecule Exercise

A molecule of water is hanging around the Great Lakes region,  
consisting of the 5 lakes (M,S,H,E,O), or the air above the region  
(which we'll call "A"), from which it might fall as rain or dew back  
into a lake. Suppose it follows this transition matrix.  
I'm leaving the time step size unspecified, since I'm making up  
these numbers anyway.

M S H E O A  
M .8 0 .1 0 0 .1  
S 0 .98 .01 0 0 .01  
H 0 0 .8 .15 0 .05  
E 0 0 0 .7 .2 .1  
O 0 0 0 0 .9 .1  
A .1 .3 .3 .1 .1 .1

i) Explain what is good and what is not-so-good about this model.

**A good aspect about the model is that it disallows physically impossible transitions such as Superior to Ontario. Also, it is reasonable to think that the ‘no-change’ transitions have high probabilities (the lakes are huge, and the channels are tiny).**

**One not-so-good aspect is that it does not take seasonal effects into account. For example, transitioning from a lake to air has a higher probability in summer (when air is warm and holds more water) than in winter (when air is cold and holds less water, and lakes are covered with ice).**  
ii) Now pretend the model is perfect as I specified it,  
and find the relative amounts of water in each compartment.

iii) optional/project idea: make a more accurate model, and/or  
adjust the transition probabilities.