Homework 07 DTMC

Math 519

William Gillespie

Water Molecule Exercise

A molecule of water is hanging around the Great Lakes region,  
consisting of the 5 lakes (M,S,H,E,O), or the air above the region  
(which we'll call "A"), from which it might fall as rain or dew back  
into a lake. Suppose it follows this transition matrix.  
I'm leaving the time step size unspecified, since I'm making up  
these numbers anyway.

M S H E O A  
M .8 0 .1 0 0 .1  
S 0 .98 .01 0 0 .01  
H 0 0 .8 .15 0 .05  
E 0 0 0 .7 .2 .1  
O 0 0 0 0 .9 .1  
A .1 .3 .3 .1 .1 .1

i) Explain what is good and what is not-so-good about this model.

**A good aspect about the model is that it disallows physically impossible transitions such as Superior to Ontario. Also, it is reasonable to think that the ‘no-change’ transitions have high probabilities (the lakes are huge, and the channels are tiny).**

**One not-so-good aspect is that it does not take seasonal effects into account. For example, transitioning from a lake to air has a higher probability in summer (when air is warm and holds more water) than in winter (when air is cold and holds less water, and lakes are covered with ice).**  
ii) Now pretend the model is perfect as I specified it,  
and find the relative amounts of water in each compartment.

**I wrote an R script that repeatedly multiplies a state vector with the transition matrix, updating the state vector each time. My code does 1,000 iterations; it converges by then (though I did not test fewer iterations). Below is a printout of the results for the initial and steady state vectors. The steady state vector has the relative amounts of water in each of the lakes (or air).**

**[1] "initial state vector:"**

**Michigan Superior Huron Eire Ontario Air**

**[1,] 0.2 0.2 0.2 0.2 0.2 0.2**

**[1] "steady state vector"**

**[1] "Michigan" "Superior" "Huron" "Eire" "Ontario" "Air"**

**[1] "0.024" "0.727" "0.121" "0.077" "0.202" "0.048"**

iii) optional/project idea: make a more accurate model, and/or  
adjust the transition probabilities.

Economic Quintile Problem

Use the graph at  
http://people.emich.edu/aross15/coursepack3419/opportunity-1.png  
to create a Markov Chain model of income mobility from one generation  
to the next. The 1st quintile is the poorest, and the 5th is the richest.  
a) What is the transition matrix? Figure out where each number goes.  
There is a subtle issue here--what is it? Figure out what it is and  
fix it in a reasonable way.

**Transition matrix:**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| From To | 1st Quintile | 2nd Quintile | 3rd Quintile | 4th Quintile | 5th Quintile |
| 1st Quintile | **0.42** | **0.23** | **0.19** | **0.11** | **0.06** |
| 2nd Quintile | **0.25** | **0.23** | **0.24** | **0.18** | **0.10** |
| 3rd Quintile | **0.17** | **0.24** | **0.23** | **0.17** | **0.19** |
| 4th Quintile | **0.08** | **0.15** | **0.19** | **0.32** | **0.26** |
| 5th Quintile | **0.09** | **0.15** | **0.14** | **0.23** | **0.39** |

**I believe the ‘subtle issue’ is that the proportion of people in the quintiles cannot really change. Even if people were to transition from one quintile to another, an equal number of people would be in each quintile. One person’s rise would be another’s fall, so in that sense it is a zero-sum game of economic mobility (quintile-wise). To make the economic transitions make more sense, I would change these quintiles to yearly income bins based on how much money the individuals on the borders of the quintiles make.**

b) If you are in the 5th quintile, what is the probability that  
your grandson will be in the 1st quintile?

**I performed this calculation in python and got an answer of 15.26%. I made an equation that shows the summation of the transition probabilities. These probabilities include all the ways that a 5th quintile individual can have a 1st quintile grandchild.**

c) What is the steady-state distribution? Compute it using methods  
learned in class.

**Still working on it. For some reason the population is exploding. This might be the ‘subtle issue’ after all.**  
d) Explain the result you got in part (c).  
e) Consider a person who is politically liberal/left-wing.  
What would they want the transition matrix in part (a) to look like?

**TODO**  
f) Consider a person who is politically conservative/right-wing.  
What would they want the transition matrix in part (a) to look like?

**TODO**

Problem 4.33 Three types of exams (also not too bad)

1. A professor continually gives exams to her students. She can give three possi- ble types of exams, and her class is graded as either having done well or badly. Let *pi* denote the probability that the class does well on a type *i* exam, and sup- pose that *p*1 = 0.3, *p*2 = 0.6, and *p*3 = 0.9. If the class does well on an exam, then the next exam is equally likely to be any of the three types. If the class does badly, then the next exam is always type 1. What proportion of exams are type *i*, *i* = 1,2,3?

**I first show the ‘broken down’ transition matrix. The 0.7, 0.4, and 0.3 in the first column represent the probability of doing poorly on test 1, 2, and 3 respectively (meaning they will have to take test 1 next time). The 0.1, 0.1, and 0.1 in the first row represents the sum the probability of doing well on test 1 (0.3). If the students do well on test 1, then there is an equally likely chance that they take test 1, 2, or 3 next time. Thus, the 0.3 is split evenly across the first row. The other rows can be filled in with the similar reasoning.**

|  |  |  |  |
| --- | --- | --- | --- |
|  | Test 1 | Test 2 | Test 3 |
| Test 1 | **0.1 + 0.7** | **0.1** | **0.1** |
| Test 2 | **0.2 + 0.4** | **0.2** | **0.2** |
| Test 3 | **0.1 + 0.3** | **0.3** | **0.3** |

**The full transition matrix is below:**

|  |  |  |  |
| --- | --- | --- | --- |
|  | Test 1 | Test 2 | Test 3 |
| Test 1 | **0.8** | **0.1** | **0.1** |
| Test 2 | **0.6** | **0.2** | **0.2** |
| Test 3 | **0.4** | **0.3** | **0.3** |