Chapter 8 Homework: Queues

(however, much of this homework depends on lecture notes rather than  
the textbook)  
  
  
Your homework should be typed in Word or something similar, with  
appropriate spreadsheet segments copied and pasted in.  
  
1. A ) For an M/M/1 queue with lambda=9 jobs/hour and mu=10 jobs/hour,  
compute L, W, Wq, and Lq using the formulas given in class.  While you may  
\_check\_ your work using a queueing package like QTS Plus, you should do it  
\_by hand\_ to start with.  Type up not just the answers but also the  
calculations that get you there.

**I could not find all the formulas I needed to do this problem from the queueing power point, so I used this website:** [**http://people.revoledu.com/kardi/tutorial/Queuing/MM1-Queuing-System.html**](http://people.revoledu.com/kardi/tutorial/Queuing/MM1-Queuing-System.html)**.**

**lambda = 9 jobs / hour**

**mu = 10 jobs / hour**

**L = lambda / (mu – lambda)**

**L = 9 / (10 – 9)**

**L = 9**

**W = L / lambda**

**W = 9 / 9**

**W = 1**

**------------------------------------**

**Note: p = lambda / mu -> p = 0.9**

**Lq = p^2 / (1 – p)**

**Lq = (9/10)^2 / (1 – 9/10)**

**Lq = 8.1**

**Note: W = Wq + 1 / mu**

**So,**

**Wq = W – 1 / mu**

**Wq = 1 – 1 / 10**

**Wq = 0.9**  
  
b) Repeat (a) after doubling the arrival and service rates  
(lambda = 18 jobs/hour and mu=20 jobs/hour); comment on similarities  
and differences with part (a).

**Lambda = 18 jobs / hour**

**Mu = 20 jobs / hour**

**L = lambda / (mu – lambda)**

**L = 18 / (20 – 18)**

**L = 9**

**W = L / lambda**

**W = 9 / 18**

**W = 0.5**

**------------------------------------**

**Note p = lambda / mu -> 18 / 20 = 0.9**

**Lq = p^2 / (1 – p)**

**Lq = 0.9^2 / (1 – 0.9)**

**Lq = 0.81 / 0.1**

**Lq = 8.1**

**Note: W = Wq + 1 / mu**

**Wq = W – 1 / mu**

**Wq = 0.5 – 1 / 20**

**Wq = 0.5 – 0.05**

**Wq = 0.45**

**The interesting thing is that in the second set of terms (lambda = 18, mu = 20) have the same line lengths for the queue and the total, but the wait times are much shorter than the first two terms (lambda = 9, mu = 10). When both terms are doubled, tripled, etc, the lines are the same length but people are processed faster.**  
  
2.  We said in class that you can't take a multiserver system and  
analyze it with one really fast server.  How about splitting it  
into many independent single-server systems?  
For example, consider an M/M/c system with  mu=1 and lambda=50  
with 57 servers.  
a) Compute L, Lq, W, Wq using software like QTS Plus or a web page like  
<http://www.cs.vu.nl/~koole/ccmath/ErlangC/index.php>

**I used the website** [**http://www.supositorio.com/rcalc/rcalclite.htm**](http://www.supositorio.com/rcalc/rcalclite.htm) **for my calculations.**

**L = 51.7605**

**Lq = 1.705**

**W = 1.0352**

**Wq = 0.0352**

b) Instead, consider 57 separate single-server systems, each still  
with mu=1 but with lambda=50/57; compute L, Lq, W, Wq for each and in  
total (multiply by 57 when appropriate).  Compare to part (a).

**Lambda = 50/57**

**Mu = 1**

**One\_Line\_L = lambda / (mu – lambda)**

**One\_Line\_L = (50/57) / (1 – 50/57)**

**One\_Line\_L = (50/57) / (7/57)**

**One\_Line\_L = 50 / 7**

**One\_Line\_L = 7.143**

**L = 57 \* 7.143**

**L = 407.151**

**W = L / lambda**

**W = 407.151 / ((50 / 57) \* 57)**

**W = 8.143**

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**Note W = Wq + 1 / mu**

**Wq = W – 1/ mu**

**Wq = 8.143 – 1**

**Wq = 7.143**

**Note Lq = lambda \* Wq**

**Lq = (50 / 57) \* 7.143 \* 57**

**Lq = 357.15**

c) Explain to me how I decided to have 57 servers in part (a).  Hint: I  
didn't use any software (or even a handheld calculator) to determine it.  
  
**The third law of the Applied Queueing Theory slide says:**

**# servers = p + z \* sqrt(p)**

**where z is 1 for ok service and 2 for great service.**

**Therefore, we can solve for the sqrt(p) by plugging in p, which is lambda / mu.**

**Lambda = ((50 / 57) \* 57)**

**Lambda = 50**

**And mu 1**

**So,**

**Sqrt(50) = 7.07,**

**Which can be rounded down to 7.**  
  
3. Compute W and L for the large toll bridge example done in class:  
(if we got to it this year)  
Our formulas were:  
lambda(t) = 5000\*(1 - 0.8cos(2 pi t / 24))  
for t measured in hours between 0 and 24.  
The maximum departure rate was 500 cars/hour per lane, with 15 lanes,  
for a total maximum of 7500 cars/hour.  
  
  
  
4.  For the spreadsheet we did that simulated a single-server  
First-Come-First-Serve (FCFS) queue using Lindley's Law:  
Enter the formula =-LN(rand()) for each inter-arrival time (IATn).  
Enter the formula =-0.9\*LN(rand()) for each service time (Sn)  
Simulate at least 1000 arrivals.

a) Find the mean and standard deviation of the IATs and the Sn.

|  |  |
| --- | --- |
| mean IAT | mean svc |
| 1.020405942 | 0.8815167 |
| sd IAT | sd svc |
| 1.006366024 | 0.882211182 |

b) Do a histogram of the IATs and the Sn.

**They are the same looking histogram with an exponential distribution.**

IAT histogram

Sn histogram

c) Graph the resulting Wqn values.

**The Wqn values differ greatly from run to run.**

d) Compute Wq from the simulation.

**The Wq I get from the simulation is very different from run to run.**

**Here is an example:**

|  |
| --- |
| mean Wq |
| 7.387823991 |

**The mean varies from 2 to 30.**

e) Compare Wq to Question 1, above.

**The Wq from this simulation is much larger than the small Wq value of 0.9 in question 1.**

f) Change the service times to be just =0.9  
(that is, all of them are just 0.9), and recompute Wq.

|  |
| --- |
| mean new Wq |
| 3.187512887 |

Comment on the results.

**The value for the new Wq is not as variable as the original Wq. The new Wq’s values are between 2 and 7, which is much lower than the original Wq.**  
  
You should include a copy of the first part of your spreadsheet  
in your report, but don't include all 1000 or so rows, please!  
  
5. The League of American Bicyclists reported, on June 5th 2009,  
"A 3 percent reduction in vehicle miles traveled (VMT) in 2008 resulted  
in a 30 percent reduction in congestion in metropolitan areas around  
the country, suggesting that demand management strategies such as  
increasing bicycling and walking are extremely effective in addressing  
congestion and other traffic-related issues."  
<http://www.bikeleague.org/programs/bikeadvocacy/>  
a) Using M/M/1 thinking, figure out the relative utilization  
(lambda/mu) before and after the 3 percent reduction in VMT.  
b) Is M/M/1 an appropriate approximation here?  Explain.

**No, I don’t think the M/M/1 is an appropriate approximation. We can think of the road lane as a server, and multiple lanes as multiple servers. Many roads have multiple lanes, so cars could be served my multiple servers if one lane is congested.**

**Perhaps in a parking lot that had one exit, we could think of it as an M/M/1 system.**  
c) Discuss the part of their sentence "suggesting that..."  
using what you know from queueing theory.

**The link above is broken.**  
  
6. In the government-health-care debate, according to [cnn.com](http://cnn.com/),  
<http://transcripts.cnn.com/TRANSCRIPTS/0907/21/cnr.07.html>  
SEN. JOHN BARRASSO (R), WYOMING  
said "a ten-week wait for an MRI or CAT scan in Canada.  
It's something you can get done the same week here in the United States. "  
a) Using M/M/1 thinking, figure out the relative utilization  
for MRI or CAT scan machines in Canada.  Remember to account for  
"business hours"--don't just do 10 weeks \* 7 days/week \* 24 hours/day.  
Also interesting:  
<http://secure.cihi.ca/cihiweb/en/media_08feb2006_tab3_e.html>  
b) Is M/M/1 an appropriate approximation here?  Explain.