Atividade de Cálculo Numérico
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1,2)

03/07/23

1º) Pivô = 1

$$A = \begin{bmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1/3 & 1/4 \\ 1/3 & 1/4 & 1/5 \end{bmatrix} , \quad B = \begin{bmatrix} 11/6 \\ 13/12 \\ 47/60 \end{bmatrix}$$

$L_2 - (1/2) \cdot L_1 \to L_2$

$$A = \begin{bmatrix} 1 & 1/2 & 1/3 \\ 0 & 1/12 & 1/12 \\ 1/3 & 1/4 & 1/5 \end{bmatrix} \times (-1/3) \; ; \; L_3 - (1/3) \cdot L_1 \to L_3$$

$$A = \begin{bmatrix} 1 & 1/2 & 1/3 \\ 0 & 1/12 & 1/12 \\ 0 & 1/12 & 4/45 \end{bmatrix} \times (-1) \; ; \; L_3 - 1 \cdot L_2 \to L_3$$

$$A = \begin{bmatrix} 1 & 1/2 & 1/3 \\ 0 & 1/12 & 1/12 \\ 0 & 0 & 1/180 \end{bmatrix}$$

Determinante = $1/2160$

2º Generalizando o vetor para uma dimensão arbitrária

$$\begin{cases} x_1 + 1/2\, x_2 + 1/3\, x_3 = 11/6 \\ 1/2\, x_1 + 1/3\, x_2 + 1/4\, x_3 = 13/12 \\ 1/3\, x_2 + 1/4\, x_2 + 1/5\, x_3 = 47/60 \end{cases}$$

$$D = \left[\begin{array}{ccc|cc} 1 & 1/2 & 1/3 & 1 & 1/2 \\ 1/2 & 1/3 & 1/4 & 1/2 & 1/3 \\ 1/3 & 1/4 & 1/5 & 1/3 & 1/4 \end{array}\right] = 3/20 - 323/2160 = 1/2160$$

$1/27 + 1/16 + 1/20 \qquad 1/15 + 1/24 + 1/24$

$\dfrac{323}{2160} \qquad \dfrac{3}{20}$

tilibra

03/07/23

$$x_1 = \begin{bmatrix} 11/6 & 1/2 & 1/3 \\ 13/12 & 1/3 & 1/4 \\ 47/60 & 1/4 & 1/5 \end{bmatrix} = 1/2160 \quad \hookrightarrow x_1 = \frac{1/2160}{1/2160} = 1$$

$$x_2 = \begin{bmatrix} 1 & 11/6 & 1/3 \\ 1/2 & 13/12 & 1/4 \\ 1/3 & 47/60 & 1/5 \end{bmatrix} = 1/2100 \quad \hookrightarrow x_2 = \frac{1/2160}{1/2160} = 1$$

$x_1, x_2 = 1$

$\hookrightarrow 1 \cdot 1 + 1/2 \cdot 1 + 1/3 \cdot x_3 = 11/6$

$1 + 1/2 + x_3/3 = 11/6$

$3/2 + x_3/3 = 11/6$

$x_3/3 = 11/6 - 3/2$

$x_3/3 = 1/3$

$x_3 = 1/3 \cdot 3$

$x_3 = 1$

$$\text{Logo} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

2) Para todo tamanho de matriz de Hilbert, o vetor solução vai ser uma matriz identidade.

3) https://github.com/Wuel1/calculonumerico/blob/main/3%20Atividade/Hilbert.ipynb

N = 3

```
A, b = Hilbert(3)
A_t,b_t= gauss(A,b);
print(A_t)
print(20*'__')
print(b_t)
print(20*'__')

x = subs_reversa(A_t, b_t)
print(x)
```

```
[[ 1.00000000e+00  5.00000000e-01  3.33333333e-01]
 [ 0.00000000e+00  8.33333333e-02  8.33333333e-02]
 [ 0.00000000e+00 -1.38777878e-17  5.55555556e-03]]
________________________________________
[1.8333333333333333, 0.16666666666666663, 0.0055555555555554526]
________________________________________
[0.999999999999998, 1.0000000000000122, 0.9999999999999875]
```

N = 10

```
print(A_t)
print(20*'__')
print(b_t)
print(20*'__')

x = subs_reversa(A_t, b_t)
print(x)
```

```
[[ 1.00000000e+00  5.00000000e-01  3.33333333e-01  2.50000000e-01
   2.00000000e-01  1.66666667e-01  1.42857143e-01  1.25000000e-01
   1.11111111e-01  1.00000000e-01]
 [ 0.00000000e+00  8.33333333e-02  8.33333333e-02  7.50000000e-02
   6.66666667e-02  5.95238095e-02  5.35714286e-02  4.86111111e-02
   4.44444444e-02  4.09090909e-02]
 [ 0.00000000e+00 -1.38777878e-17  5.55555556e-03  8.33333333e-03
   9.52380952e-03  9.92063492e-03  9.92063492e-03  9.72222222e-03
   9.42760943e-03  9.09090909e-03]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  3.57142857e-04
   7.14285714e-04  9.92063492e-04  1.19047619e-03  1.32575758e-03
   1.41414141e-03  1.46853147e-03]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
   2.26757370e-05  5.66893424e-05  9.27643785e-05  1.26262626e-04
   1.55400155e-04  1.79820180e-04]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00  1.43154905e-06  4.29464715e-06  8.09375809e-06
   1.23333457e-05  1.66500166e-05]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00  9.00974923e-08  3.15341224e-07
   6.72727944e-07  1.13522841e-06]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00  5.65997065e-09
   2.26398821e-08  5.39361899e-08]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00 -1.69406589e-21  0.00000000e+00  0.00000000e+00
   3.55136713e-10  1.59811304e-09]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  2.16840434e-19
   0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00  2.22677000e-11]]
________________________________________
[2.9289682539682538, 0.5553932178932182, 0.07149470899470844, 0.007462398712398968, 0.0006336124193268584, 4.2803316612260826e
-05, 2.2133950653494808e-06, 8.223604342646523e-08, 1.9532488241232643e-09, 2.2268779976630608e-11]
________________________________________
[0.9999999987548343, 1.0000001067849729, 0.9999977378614759, 1.0000204794185104, 0.9999026418473482, 1.0002669070133323, 0.999
56308847027, 1.0004214011575996, 0.9997791407962119, 1.0000484987218523]
```

N = 100

```
[[ 1.00000000e+00  5.00000000e-01  3.33333333e-01 ...  1.02040816e-02
   1.01010101e-02  1.00000000e-02]
 [ 0.00000000e+00  8.33333333e-02  8.33333333e-02 ...  4.99896928e-03
   4.94949495e-03  4.90099010e-03]
 [ 0.00000000e+00 -1.38777878e-17  5.55555556e-03 ...  1.59967017e-03
   1.58449178e-03  1.56959814e-03]
 ...
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00 ...  4.62813499e-17
   6.74962477e-17  1.00505310e-16]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00 ...  0.00000000e+00
   6.64307320e-18  1.13263604e-17]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00 ...  0.00000000e+00
   0.00000000e+00  1.71363344e-17]]
________________________________________
[5.187377517639621, 1.6035897489188193, 0.3743668411752281, 0.08183241863831447, 0.017289350784816943, 0.003564991801153633,
0.0007201852940913779, 0.00014279034151084505, 2.7808977867119352e-05, 5.322054420119169e-06, 1.0016464019637201e-06, 1.880318
2561231787e-07, 4.489657534058364e-08, 1.2059565213777802e-08, 6.532447322905381e-08, 5.266956962776098e-09, 2.2474014751771e-
08, 2.442213813187901e-09, 1.389153429726968e-08, 4.3131918716543405e-10, 1.4662293304461112e-09, 2.813876186884756e-09, 5.524
098989961126e-10, 7.28798359961232e-10, 4.072519635541038e-10, 1.824248880517929e-09, -1.903054661031038e-09, 2.32589394230603
96e-10, 7.026156356590356e-09, 1.3082984812382782e-10, 1.3755377102951866e-10, 1.0659759023068862e-10, -1.2967716886749918e-1
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346396681e-12, -1.8683568362718477e-12, -1.203487089977045e-11, -5.266123539834543e-12, -8.979995401425836e-12, 2.445948136570
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703e-13, -9.167516191383732e-16, -4.59050391302992e-15, -1.2182694246872388e-15, -9.503050262802125e-14, -5.787824054636134e-1
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[1.0000000578334154, 1.00000346033331, 0.9993023186253296, 1.022841121766834 5, 0.6754244947508906, 3.5050504679527545, -10.564
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2, 67.25687920743059, -139.33749886172887, 216.16064482849418, 2.023637560906201, -98.08079978011025, 114.96278697899218, -3.8
426750392276814, -55.488058191167056, -100.47025634762863, 98.01317515481836, -7.241710801993191]
```

- Como a matriz de Hilbert é mal condicionada, quanto maior seja o tamanho da matriz mais ela tende ao seu determinante convergir a zero.

4)

Analisando os resultados, percebemos que os determinantes das matrizes de Hilbert com n=3,n=10 e n=100 são todos próximos ou exatamente zero. Isso acontece pois quando escalonamos a matriz, o produto das suas diagonais sempre tenderá a zero.