Reflection and Transmission of Electromagnetic Waves on the Sea Surface

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Abstract

Based on classical electrodynamics and statistical mechanics, this paper discusses the reflection and transmission of electromagnetic waves on the sea surface.

First, we assume that the sea level is completely at rest. The general Fresnel formula is derived through the boundary conditions of the electromagnetic wave to give the expression of reflected energy flow ratio. With the limited condition of the signal-to-noise ratio, we calculated the maximum number of electromagnetic wave reflections in this situation. Then, through analysis, we found that the laminar flow is almost the same as the rest.

Later we simulate the wave line equation and analyze changes in the position of the reflection and incident angle. After that, we use a simple model to describe the change in the dielectric constant within condition of turbulence. That is, assuming that the undulation of relative dielectric constant satisfies the logarithmic Gaussian distribution, the average value of the reflected energy flow ratio is obtained through numerical simulation. Here we come to the preliminary conclusion that the turbulence increases the reflectance slightly.

Based on this, we established the turbulence model from the displacement and orientation polarizations. In the former model, we start from the harmonic oscillator model and add a correction term to describe the influence caused by turbulence. In the latter one we use the Boltzmann distribution to calculate the average dipole moment. Finally, we obtain the expression of dielectric constant, thereby we calculate the reflected energy flow ratio, which validates the preliminary conclusion.

In addition, we also consider the effects of ion polarization and magnetization of all particles in the seawater, both of which do not affect the model.

On this basis, we give the reflection model of the electromagnetic wave on the land surface and analyze the differences between the two from three different aspects: the angle and height of the reflection surface, the dielectric constant and the loss phenomenon.

Furthermore, we make a partial spherical wave modification of the model and find the longest distance that the ship can maintain communication under the same multi-hop path.

Finally, we give two other possible models for the problem. The first one is based on the transport phenomenon, which is described by the Poisson-Boltzmann equation. The second one refers to Heisenberg model and determines the polarized electric field using Monte Carlo method.

Contents

1 Introduction

3-30mHz high frequency (defined as HF) electromagnetic waves can be used for long-distance communication on the earth's surface through multiple reflections between the ionosphere and the surface medium. For the ionosphere, there exists a known maximum frequency (MUF). Electromagnetic waves will pass through the ionosphere once the frequency of it is higher than the MUF. If the frequency of the electromagnetic wave is lower than the MUF, the electromagnetic wave can return from the ionosphere and continue reflective transmission between the earth's surface and the ionosphere.

As we know, the nature of the surface material affects the intensity of reflections and therefore determines how far the electromagnetic wave can ultimately be transmitted.

In this paper, the main background is set in the marine environment. According to the requirement, mathematical models will be established, and our work includes:

- We will discuss the different transmission efficiency and distance of electromagnetic waves in two different situations of calm sea surface and turbulent sea surface.
- At the same time in the flat terrain and the rugged mountain model will be established and therefore we can find the contrast between different surface material.
- Finally, we will discuss how to adjust the model and the maximum transmission distance that this type of communication can achieve in the model after adjustment while navigating the turbulent sea.

2 Assumptions

- The relative permittivity and relative permeability in the air are 1.
- When calculating the parameters, the temperature of the seawater is 20 ° C and the salinity is 35 ‰.
- The distance from the ground to the ionosphere is taken as 60 km.
- Without explanation, the radio wave is considered as an ideal plane wave at a frequency of 15 MHz, ignoring its loss when propagating in the air and ionosphere.
- The various ions in seawater constitute the Boltzmann system, satisfying the Maxwell-Boltzmann distribution law.

3 Parameter

In this paper we use the constant parameters in Table.1 to describe our model. Other symbols and parameters that are used only once will be described later.

Parameter	Value
dielectric constant in vacuum	$\varepsilon_0 = 8.8542 \times 10^{-12} F/m$
permeability in vacuum	$\mu_0 = 4\pi \times 10^{-7} N/A^2$
Boltzmann constant	$k_b = 1.3806 \times 10^{-23} J/K$
Avogadro constant	$N_A = 6.02 \times 10^{23}$
Elementary charge	$e = 1.6022 \times 10^{-19} C$
Electronic quality	$m = 9.1 \times 10^{-31} kg$

Table. 1 Parameters and its value.

4 The Model

4.1 Part I

4.1.1 Calm Ocean Model

4.1.1.1 Preparation of Electrodynamics

First, we start with the most basic situation. Assuming a calm sea level is a stationary model, that is, all the liquid molecules are basically maintained in their respective positions, and the effect of motion is very slight. According to the integral form of Maxwell's equations, we have the following boundary conditions on the surface of the medium:

$$B_{1n} = B_{2n}, H_{1t} = H_{2t}, D_{1n} = D_{2n}, E_{1t} = E_{2t}. \quad (1)$$

Suppose incident wave, reflected wave and refracted wave are respectively defined as follows:

$$\begin{cases} \widetilde{E_1} = E_1 e^{i(k_1 \cdot r - \omega_1 t)}, \widetilde{H_1} = H_1 e^{i(k_1 \cdot r - \omega_1 t)} \\ \widetilde{E_2} = E_2 e^{i(k_2 \cdot r - \omega_2 t)}, \widetilde{H_2} = H_2 e^{i(k_2 \cdot r - \omega_2 t)} \\ \widetilde{E_3} = E_3 e^{i(k_3 \cdot r - \omega_3 t)}, \widetilde{H_3} = H_3 e^{i(k_3 \cdot r - \omega_3 t)} \end{cases}. \quad (2)$$

We have following equation by electromagnetic field theory

$$E \perp H, E \times H \parallel k, \quad (3)$$

$$\sqrt{\varepsilon_0 \varepsilon_r} E = \sqrt{\mu_0 \mu_r} H. \quad (4)$$

At the same time, we think that the law of refraction and the law of reflection are correct, that is, equations below are satisfied:

$$i_1 = i_2 = i,$$
$$n_1 sin i_1 = n_2 sin r, \quad (5)$$
$$n = \sqrt{\varepsilon\mu}.$$

where, respectively, i_1, i_2, r represent the incident angle, reflection angle and refraction angle, n_1, n_2 represent the refractive index of the two surfaces.

Because our goal is to solve the Pointing vector, that is

$$\vec{S} = \vec{E} \times \vec{H}, \quad (6)$$

$$< \vec{S} > = \frac{1}{2} EH = \frac{1}{2} \sqrt{\frac{\varepsilon_0 \varepsilon_r}{\mu_0 \mu_r}} E^2. \quad (7)$$

Therefore, only the reflection and refraction of the electric field component are considered.

4.1.1.2 Reflected Energy Flow Ratio

First consider two independent polarization states. The first one is that the electric field component is perpendicular to the incident surface, corresponding to the boundary conditions of E and H; the second is that the electric field component is parallel to the incident surface, corresponding to the boundary conditions of D and B, which is shown in Fig. 1.

In the first condition, we have equations

$$E_0 + E_0'' - E_0' = 0, \quad (8)$$

$$\sqrt{\varepsilon/\mu}(E_0 - E_0'') cos i - \sqrt{\varepsilon'/\mu'} E_0' cos r = 0. \quad (9)$$

Solve the equations above and we can get the result here

$$r = \frac{E_0''}{E_0} = \frac{n cos i - (\mu/\mu')\sqrt{n'^2 - n^2 \sin^2 i}}{n cos i + (\mu/\mu')\sqrt{n'^2 - n^2 \sin^2 i}}. \quad (10)$$

So the reflection capacity ratio corresponding to the polarization state is as follow:

$$R = r^2. \quad (11)$$

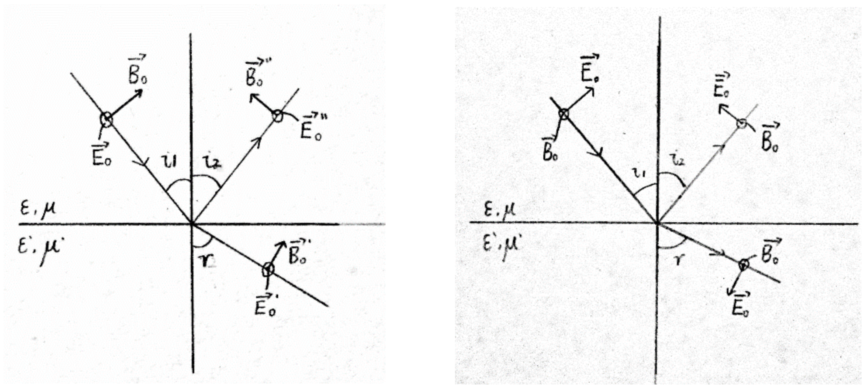

Fig. 1 Two independent polarization states: the first one is that the electric field component is perpendicular to the incident surface, and the second is that the electric field component is parallel to the incident surface

Making an image of R and angle of incidence i (assuming that magnetic field is small enough to be neglected, which will be explained later), it can be seen that as the angle of incidence increases, the reflected energy flow ratio increases.

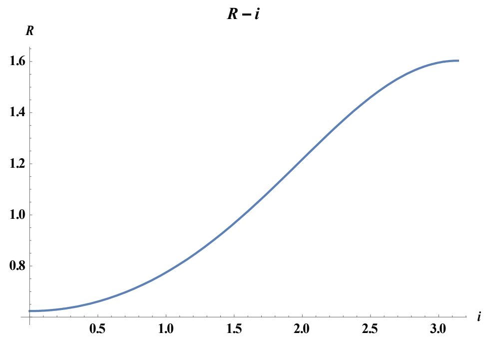

Fig. 2 The image of R and angle of incidence i in the first condition.

In the second condition, we have equations:

$$cosi(E_0 - E_0'') - cosrE_0' = 0, \tag{12}$$

$$\sqrt{\varepsilon/\mu}(E_0 + E_0'') - \sqrt{\varepsilon'/\mu'}E_0' = 0. \tag{13}$$

Solve the equations, and the result is as follow:

$$r = \frac{E_0''}{E_0} = \frac{(\mu/\mu')n'^2 cosi - n\sqrt{n'^2 - n^2 \sin^2 i}}{(\mu/\mu')n'^2 cosi + n\sqrt{n'^2 - n^2 \sin^2 i}}. \tag{14}$$

Making an image of R and angle of incidence i (assuming that magnetic field is small enough to be neglected, which will be explained later), it can be seen that as the angle of incidence increases, the reflected energy flow ratio increases.

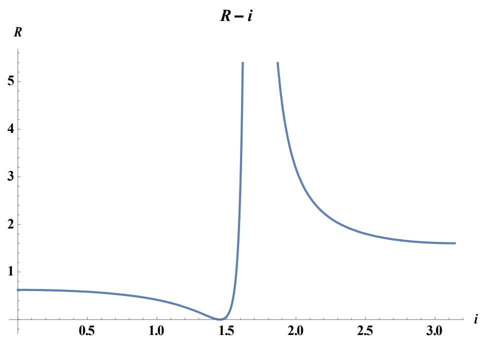

Fig. 3 The image of R and angle of incidence i in the second condition.

By using the two polarization states above as the basis, the electric field intensity at any azimuth can be expressed as a linear combination of two bases:

$$\vec{E} = \lambda E_{\perp} + (1-\lambda)E_{\parallel}, \tag{15}$$

where $\lambda = cos^2 \alpha$.

The corresponding reflectance ratio is

$$R = \lambda\left(\frac{ncosi-(\mu/\mu')\sqrt{n'^2-n^2 \sin^2 i}}{ncosi+(\mu/\mu')\sqrt{n'^2-n^2 \sin^2 i}}\right)^2 + (1-\lambda)\left(\frac{(\mu/\mu')n'^2cosi-n\sqrt{n'^2-n^2 \sin^2 i}}{(\mu/\mu')n'^2cosi+n\sqrt{n'^2-n^2 \sin^2 i}}\right)^2. \tag{16}$$

4.1.1.3 Best Angle of Incidence

Next, we determine the best angle of incidence based on the following considerations.

In the ionosphere, electromagnetic waves are reflected by the following path.

According to the scattering theory, the refractive index of the ionosphere satisfies equation below:

$$n = \sqrt{\varepsilon_r} = \sqrt{1-80.0N(z)/f^2}, \tag{17}$$

where N (z) denotes the free electron density and f denotes the frequency of the electromagnetic wave.

The ionosphere is divided into many layers and by the law of refraction we have

$$n_0 sini_0 = n_1 sini_1 = \cdots = n_n sini_n = n_n sin\frac{\pi}{2}. \tag{18}$$

Solve (17)(18) and we get the result:

$$sini_0 = \sqrt{1-80.0N(z)/f^2}. \tag{19}$$

On the one hand, we can get the value of MUF from a geometric relationship

$$f_m = \frac{\sqrt{80.0N_m}}{\sqrt{1-[1/(1+z_m/R)]^2}}, \tag{20}$$

where z_m represents the height between ionosphere and the surface of the earth. In the ionosphere, $N_m = 2\times 10^{12}/m^2$. So we can get

$$f_m = 51.9\sim 39.9MHz.$$

On the other hand, for a certain frequency of electromagnetic waves (assuming $f = 15MHz$), the incident angle has a minimum value:

$$i_m = arcsin\left(\sqrt{1-80.8N_m/f^2}\right) = 0.5596\, rad.$$

In order to improve the accuracy of the information received by the ground equipment, the distance between the two reflections should be kept as small as possible so that angle of incidence should be set to the minimum

$$i_m = 0.5596 \text{ rad}.$$

Since the path of the electromagnetic wave in the ionosphere is symmetrical, the angle of incidence is equal to the angle of exit, which is also the angle of incidence at sea level.

Here, we choose the appropriate degree of polarization, that is, to determine one so that the refractive index loss of energy flow as small as possible, which means to attain a flow of the reflected energy as large as possible:

$$R = \lambda\left(\frac{ncosi_m-\sqrt{n'^2-n^2 \sin^2 i_m}}{ncosi_m+\sqrt{n'^2-n^2 \sin^2 i_m}}\right)^2 + (1-\lambda)\left(\frac{n'^2cosi_m-n\sqrt{n'^2-n^2 \sin^2 i_m}}{n'^2cosi_m+n\sqrt{n'^2-n^2 \sin^2 i_m}}\right)^2 \tag{21}$$

Judging from deriving, $\frac{dR}{d\lambda} = 0.0971 > 0$, we have the conclusion that when we choose $\lambda = 1$, we will attain the maximum reflection energy flow ratio

$$R_m = 0.6701.$$

4.1.2 Laminar Flow Model

Furthermore, we consider the laminar flow model in which all liquid molecules move uniformly in one direction. Since the electric field is sinusoidal, the electric field generated by the polarized charge has a mean value of zero. However, though the ions in seawater laminar in the formation of a certain current, due to the same velocity, current excitation of the magnetic field does not change over time, so there is no electric field. Therefore, we can consider that the laminar flow model and static model are the same in nature.

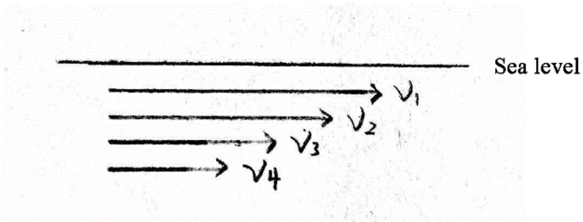

Fig. 4 Schematic of the laminar flow.

4.1.3 Turbulence Model

Next, we examine the turbulence model.

4.1.3.1 Correction of Incident Angle and Height

4.1.3.1.1 Ocean Wave Simulation

The ocean is a rapidly changing and complex phenomena of water body movement, whose most notable feature is the irregularity. Reference [1] proposes the use of multiple micro-sine wave superposition to simulate the various sea conditions. Based on the idea above, the two-dimensional ocean wave system can be expressed by a sine wave equation:

$$Z = \frac{1}{n}\sum_{i=1}^{n} A_i \sin[k_i x - 2\pi f_i(t + t_i)] + a u_i \sin(\pi/j). \tag{22}$$

On the right side of the equation above, the first half represents the superposition of n regular waves, simulating the basic effect of the waves, where the parameter n represents the total number of wave components, A_i represents the amplitude of the i^{th} component wave, k_i represents the wave number and f_i represents the wave frequency. The latter part of the formula is applied for the integration of fine wave to achieve the effect of chaos on the sea, where a represents the wave amplitude of the wavelet system, j is a rough coefficient and u is evenly distributed random number.

Now we will derive the parameters of the wave system.

Since the probability that the actual sea surface wave height appears meets the Rayleigh distribution, the probability density function is

$$P(H/\bar{H}) = 1 - e^{-\pi/4\cdot(H/\bar{H})^2}, \tag{23}$$

where H represents wave height and $\bar{H}$ represents average wave height, which is a constant.

According to Bretschneider's derivation, it is also proved that the probability of the general sea surface wavelength L and the probability of the periodic square are also Rayleigh distributions, and the probability density function thereof is as follow:

$$P(L/\bar{L}) = 1 - e^{-\pi/4\cdot(L/\bar{L})^2}, \tag{24}$$

$$P(T/\bar{T}) = 1 - e^{-0.675\cdot(T/\bar{T})^4}, \tag{25}$$

where L and T represent wavelength and period; $\bar{L}$ and $\bar{T}$ represent average wave height and average period, which are constant.

Pointed wave height is commonly used in statistical ocean as a representative wave height, and satisfies the following relationship:

$$H_s = 7.065 \times 10^{-3} v^{2.5}, \tag{26}$$

$$\bar{H} = 0.625 H_s = 4.416 \times 10^{-3} v^{2.5}, \tag{27}$$

$$\bar{A} = \bar{H}/2 = 2.208 \times 10^{-3} v^{2.5}. \tag{28}$$

From the previous formula, we can get a Rayleigh probability distribution with a ratio between 0 and 1, so we have

$$c = P(H/\bar{H}) = 1 - e^{-\pi/4\cdot(H/\bar{H})^2}, \tag{29}$$

$$H = 2\bar{H}\sqrt{ln(1/(1-c))/\pi}. \tag{30}$$

$$A = 2\bar{A}\sqrt{ln(1/(1-c))/\pi}, \tag{31}$$

where c is a random number varying from 0 to 1.

The same can be obtained:

$$L = 2\bar{L}\sqrt{ln(1/(1-c))/\pi}, \tag{32}$$

$$T = 2\bar{T}\sqrt[4]{-1.4185\, ln(1-c)}, \tag{33}$$

where $\bar{T}$ represents average wave period and $0.5\text{s} < \text{T} < 30\text{s}$.

Wave number, wave frequency, wave length and wave period has the following mathematical relationship:

$$L = 1.56T^2, \tag{34}$$

$$k = 2\pi/L, \tag{35}$$

$$f = T^{-1}. \tag{36}$$

So we have

$$k = \pi/(\sqrt{ln(1/(1-c))/\pi} \times 1.56\bar{T}^2), \tag{37}$$

$$f = \left[\sqrt[4]{-1.4185\, ln(1-c)}\right]^{-1} \bar{T}^{-1}. \tag{38}$$

Based on the formula (31) (37) (38), the synthetic wave can be constructed from the wind speed of 10 m above the sea surface and the effective period of the ocean wave, to simulate the real ocean wave system.

In theory, the sea surface is composed of innumerable different sine waves. However, taking into account the premise of the operation, we can find that the sea surface effect simulated by 6 composite waves plus the finely ground formula can be close to that obtained by 11 composite waves. So you can use a small amount of synthetic waves to simulate the sea surface and make up the real situation, meanwhile you can also effectively reduce the operating time of the entire system.

We take the measurement data of Station 51002-SOUTHWEST HAWAII (17.037 N 157.696 W) for 2016 and calculate the wind speed and effective period, and select the other parameters as follows:

Parameter	$v/m\cdot s^{-1}$	$\bar{T}/s$	n	a	j
Data	*8.135526*	*2.945485*	*6*	*1*	*12*

Table. 2 the average wind speed and effective period of data of Station 51002-SOUTHWEST HAWAII (17.037 N 157.696 W) for 2016 and other parameters for the simulation.

You can simulate the sea waves in Matlab situation as shown in Fig. 5.

By simulating the wave chart of the sea surface, it can be seen that the actual sea surface is basically imaged and the requirement of performing wave force research can also be met.

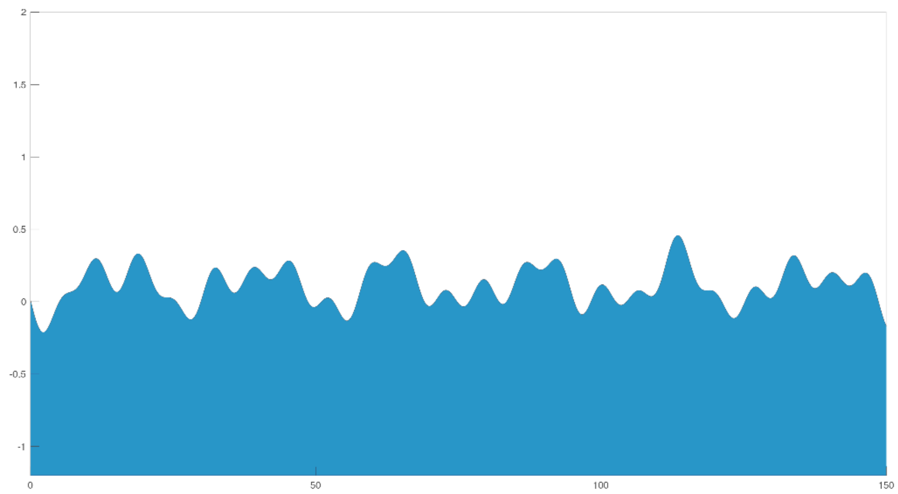

Fig. 5 Simulation of sea wave.

4.1.3.1.2 Correction of Incident Angle and Height of the Radio waves

Fluctuations in the sea surface waves can change the incident angle and height of the radio waves, thereby affecting the reflection path and the reflection angle.

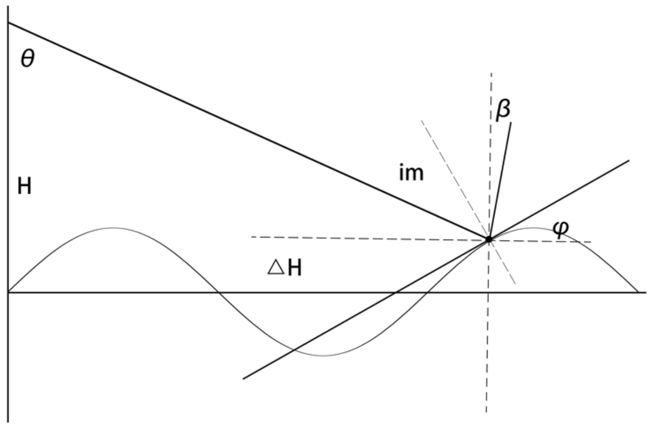

Fig. 6 Schematic of the reflection process in the sea waves.

A Height

It can be seen from the Fig. 6 that the height of the sea wave is ΔH such that the distance between the reflection point and the ionosphere becomes $H - \Delta H$. Compared with H, ΔH is extremely small, which can be ignored,

$$H - \Delta H \approx H. \tag{39}$$

Therefore, in the calculation, H is still used as the distance from the reflection point to the ionosphere.

B Angle of Incidence

Joint wave system formula and radio wave incident straight line

$$\begin{cases} Z = \frac{1}{n}\sum_{i=1}^{n} A_i \, sin[k_i x - 2\pi f_i(t + t_i)] + au_i \, sin\left(\frac{\pi}{j}\right) \\ Z = tan\left(\frac{\pi}{2} + \theta\right) * x + H \end{cases} . \tag{40}$$

And the point of intersection can be obtained as a point of incidence (as a function of time t)

$$x0 = x0(t). \tag{41}$$

Therefore, the angle of incidence of the cutting plane and the horizontal plane φ is

$$\varphi = \varphi(t) = \frac{\partial Z}{\partial x}\Big|_{x=x0=x0(t)}. \tag{42}$$

Select the time t for the radio wave incident from the Z axis incident

$$t = \frac{s}{c} = \frac{H}{tan(\theta)} * \frac{1}{c}, \tag{43}$$

where c is the speed of light and $c = 3.0 * 10^8 m/s$。

Angle φ can be included:

$$\varphi = \varphi\left(\frac{H}{tan(\theta)} * \frac{1}{c}\right) = \frac{\partial Z}{\partial x}\Big|_{x=x0=x0\left(\frac{H}{tan(\theta)} * \frac{1}{c}\right)} \tag{44}$$

A large number of simulations (100000 operations) can be performed on the above process to obtain a sequence of included angles φ, and the average of the angles can be obtained statistically:

$$\varphi = 0.098152\, rad.$$

Therefore, we can get the radio wave incident angle as

$$im = \theta - \varphi = 0.461448\, rad \tag{45}$$

4.1.3.2 Correction of Dielectric Constant

Then the effect of turbulence on the dielectric constant will be considered.

4.1.3.2.1 Rytov Perturbation Approximation Theory

Ocean turbulence can affect the electromagnetic gradient of seawater and change the permittivity of seawater. For the dielectric constant fluctuations, we use the knowledge of probability statistics to describe it.

According to the problem of propagation of electromagnetic waves in random continuous media, in the theory of weak fluctuations, the Rytov perturbation approximation theory[2] suggests that the undulation of seawater dielectric constant satisfies the logarithmic normal distribution. In general, the hypothesis of weak fluctuation theory holds true only when the logarithmic magnitude variance is less than about 0.5. At the same time, although the logarithmic fluctuation of the Rytov solution does not work well in strong fluctuations, the theory of strength and weakness has the same result in the cross-correlation problem of wave propagation in random continuous turbulent media (second-order statistics). Strong fluctuation analysis is only needed for higher-order coherence functions.[3]

Therefore, when the relative permittivity in turbulence fluctuates around the permittivity of a calm sea surface, it is expressed as the sum of the perturbations of each order and ignores the higher-order terms:

$$\varepsilon_r = \varepsilon^{(0)} + \varepsilon^{(1)} + \cdots, \tag{46}$$

where ε^0 is the dielectric constant of the calm sea surface and ε^1 satisfies the perturbation term of the logarithmic normal distribution and satisfies

$$< \varepsilon^{(1)}(\rho, z) >= 0, \tag{47}$$

$$< \varepsilon^{(1)}(\rho_1, z_1)\varepsilon^{(1)}(\rho_2, z_2) >= \delta(z_1 - z_2)A(\rho_1 - \rho_2), \tag{48}$$

where

$$A(\rho) = 2\pi \int \Phi_\varepsilon(k)\, exp(-ik \cdot \rho)\, dk. \tag{49}$$

Electromagnetic waves propagate in the Z direction (ρ as cross-sectional coordinates).

To determine the relative dielectric constant fluctuations of the variance, using Matlab for multiple simulations to obtain a random permittivity sequence, we will use the following equations to calculate reflected energy flow ratio sequence and its average value.

$$R = \left(\frac{cos i_m - \sqrt{\varepsilon_r - sin^2 i_m}}{cos i_m + \sqrt{\varepsilon_r - sin^2 i_m}}\right)^2 \tag{50}$$

σ	0	0.1	0.5	1	2	5	10
$\bar{R}$	0.6552	0.6571	0.6573	0.6582	0.6640	0.7047	0.7527

Table. 3 Different variance of the relative dielectric constant fluctuations and its corresponding average value of reflected energy flow ratio

We can see that when the degree of turbulence fluctuations become larger, so that the relative dielectric constant fluctuations become larger and ultimately will affect the average reflected energy flow ratio.

4.1.3.2.2 A model Based on Polarization

To discuss the physics behind the polarization, we'll recreate a more detailed model. Of course, here we are not going to discuss the quantum theory of multi-body problems, but only from classical electrodynamics and statistical mechanics, derive the formula of dielectric constant.

For a group of molecules, polarization mechanism has two kinds: oriented polarization and displacement polarization. The former is that the applied electric field changes the charge distribution so that each molecule produces an induced dipole moment. The latter field is a regular arrangement (orientational polarization) of the permanent dipole moment with random orientations in the outfield.

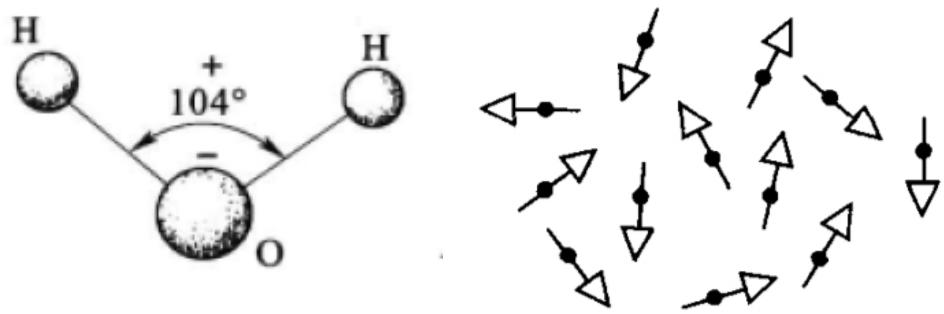

Fig. 7 The left one is the dipole moment of H_2O; the right one is schematic of irregular movement of H_2O.

Source: the left one: Electromagnetism [M]. Higher Education Press, Zhao Kaihua, 1985.

A Oriented Polarization

For oriented polarization, since the calm sea surface is an equilibrium system, for a sufficiently long time we can assume that the effect of the electric field on the orientation of the dipole moment as a whole behaves as 0, so the orientation polarization intensity is 0. If the effects of turbulence are taken into considered, it is assumed that the water molecule is intermittent. Due to the random effect, the property is in an unbalanced state and the influence of the electric field on the dipole moment is no longer mediocre. The specific analysis is as follows.

Consider the Hamiltonian of water molecules as

$$H = H_0 - \overrightarrow{p_0} \cdot \vec{E}. \tag{51}$$

At the same time, we still consider the system obeys Boltzmann distribution and the average dipole moment is

$$< \vec{p} >= \frac{\int d\Omega p_0 cos\theta \ exp(p_0 Ecos\theta/kT)}{\int d\Omega \ exp(p_0 Ecos\theta/kT)}. \tag{52}$$

Choosing the appropriate axis, apply Taylor expansion to the exponential function and ignore higher-order terms to give an approximate mean dipole moment:

$$< p >\approx \frac{1}{3}\frac{p_0^2}{kT}E. \tag{53}$$

So we have

$$\varepsilon_{op} = \frac{1}{3}\frac{Np_0^2}{kT}, \tag{54}$$

where N represents the number of molecules:

$$N = N_A \frac{\rho}{M}, \tag{55}$$

where $\mathrm{N_A}$ represents Avogadro's constant, ρ represents the density of water, and M represents the relative molecular mass of water.

With the water dipole moment $\mathrm{p_0} = 6.17 \times 10^{-30}\ C \cdot m, \mathrm{N_A} = 6.02 \times 10^{23}, \rho = 1.0 \times 10^3 kg/m^3, M = 18g/mol, T = 293K$, we can calculate and get the result as follow:

$$\varepsilon_{op} = 1.0496 \times 10^{-10}\ F/m.$$

So we have

$$\varepsilon_{op} = \begin{cases} 1.0496 \times 10^{-10}\ F/m & Turbulence \\ 0 & Laminar\ Flow \end{cases}. \tag{56}$$

B Displacement Polarization

For displacement polarization, we refer to the crystal model: the water molecules equivalent to a simple harmonic binding system (electrons in the proton-generated electric field vibration). The restoring force is

$$\vec{F} = -m\omega_0\vec{x}. \tag{57}$$

For the external electromagnetic field, we use the plane wave description, so the equation of motion is

$$m\frac{d^2\vec{x}}{dt^2} = -m{\omega_0}^2\vec{x} - e\overrightarrow{E_0}e^{-i\omega t}, \tag{58}$$

wheere ω is the electromagnetic wave frequency.

Set a special solution for the equation $\vec{x} = \vec{A}e^{-i\omega t}$, put it into the equation and we will attain the solution:

$$\vec{x} = \frac{e\overrightarrow{E_0}}{m(\omega^2 - {\omega_0}^2)}e^{-i\omega t}. \tag{59}$$

Thus, the polarization vector of the atom can be obtained:

$$\overrightarrow{P_0} = -e\vec{x}, \tag{60}$$

where is the elemental charge.

The polarization of the medium is expressed by this equation

$$\vec{P} = N\overrightarrow{P_0}, \tag{61}$$

Where N represents the number of electrons per unit volume.

Also we have electric displacement vector formula

$$\vec{D} = \varepsilon\vec{E} = \varepsilon_0\vec{E} + \vec{P} \tag{62}$$

Therefore, dielectric constant of displacement polarization is

$$\varepsilon_{dp} = \frac{10Ne^2}{m({\omega_0}^2 - \omega^2)}. \tag{63}$$

Finally, we get the expression of dielectric constant:

$$\varepsilon = \varepsilon_{op} + \varepsilon_{dp} + \varepsilon_0 = \begin{cases} \varepsilon_0 + \frac{1}{3}\frac{Np_0^2}{kT} + \frac{10Ne^2}{m({\omega_0}^2-\omega^2)} & Turbulence \\ \varepsilon_0 + \frac{10Ne^2}{m({\omega_0}^2-\omega^2)} & Laminar\ Flow \end{cases}. \tag{64}$$

As ω_0 is an unknown constant, we calibrate it using the dielectric constant of calm sea. Pay attention that $\varepsilon_{\mathrm{op}} = 0$ here.

With $\varepsilon_0 = 8.8542 \times 10^{-12}, \omega = 2\pi \times 15MHz$, we will obtain

$$\omega_0 = 3.8558 \times 10^{15}\ \mathrm{rad/s}.$$

C Correction of Turbulence

Starting from the calibrated model, we discuss turbulence correction of the model.

Considering that there is an acceleration normal to the wavy line along the sea level, we believe that the electrons in the water molecule acquire an entrainment acceleration:

$$a = y'' = \sum_i \vec{B_i} sin\,(\omega_i t - \varphi_i). \tag{65}$$

So the equation of motion is

$$m\frac{d^2\vec{x}}{dt^2} = -m\sum_i \vec{B_i}\, sin(\omega_i t - \varphi_i) - m{\omega_0}^2\vec{x} - e\vec{E_0}e^{-i\omega t}. \tag{66}$$

Further, we turn to

$$m\frac{d^2\vec{x}}{dt^2} = -\frac{m}{2i}\sum_i \vec{B_i}(e^{i(\omega_i t-\varphi_i)} - e^{-i(\omega_i t-\varphi_i)}) - m{\omega_0}^2\vec{x} - e\vec{E_0}e^{-i\omega t}. \tag{67}$$

Suppose there is a special solution to the equation

$$\vec{x} = \vec{G}e^{-i\omega t} + \sum_i \vec{C_i}e^{i\omega_i t} + \sum_i \vec{D_i}e^{-i\omega_i t}. \tag{68}$$

Substitute for calculation, and we have

$$\vec{G} = \frac{e\vec{E_0}}{m(\omega^2 - {\omega_0}^2)}, \tag{69}$$

$$\vec{C_i} = \frac{i}{2}\frac{\vec{B_i}e^{-i\varphi_i}}{\omega_0^2 - {\omega_i}^2}, \tag{70}$$

$$\vec{D_i} = \frac{i}{2}\frac{\vec{B_i}e^{i\varphi_i}}{\omega_i^2 - {\omega_0}^2}. \tag{71}$$

Calculate with the same method and we can solve

$$\tilde{\varepsilon} = \varepsilon_0 + \varepsilon_{op} + \frac{10Ne^2}{m({\omega_0}^2-\omega^2)} + \sum_i \frac{i}{2}\frac{10NeB_i e^{-i\varphi_i}}{E_0(\omega_i^2-{\omega_0}^2)}e^{i(\omega_i-\omega)t} + \sum_i \frac{i}{2}\frac{10NeB_i e^{i\varphi_i}}{E_0(\omega_0^2-{\omega_i}^2)}e^{-i(\omega_i-\omega)t} = \varepsilon_0 + \varepsilon_{op} + \frac{10Ne^2}{m({\omega_0}^2-\omega^2)} - \sum_i \frac{10NeB_i}{E_0(\omega_0^2-{\omega_i}^2)} sin\,[(\omega_i - \omega)t - \varphi_i]\ . \tag{72}$$

Before solving the result, let's make a simple analysis of the above expression (72).

The second item $\varepsilon_{\mathrm{op}}$ is that under the influence of turbulence, the orientation polarization enhances the permittivity of seawater. The fourth item $\sum_i \frac{10NeB_i}{E_0(\omega_0^2-{\omega_i}^2)} \sin\,[(\omega_i - \omega)t - \varphi_i]$ is the correction of the displacement polarization given by the macroscopic effect of turbulence. It is an oscillatory term. Judging from magnitude from the point of view, the effect of this item is very small, which can be confirmed by the following numerical calculation.

On the other hand, the entire dielectric constant changes over time due to the oscillation term. We expect to use a statistic to describe the final result. Since the time average of the trigonometric function is zero, we consider substituting the entire expression into Fresnel formula, and we can get the reflected energy flow ratio R as a function of time. Then average them and we can get the mean $<R>$.

Take the incident angle of 0.4614 rad, and using Matlab we can get

$$<R>=0.6759.$$

It shows that the correction of dielectric constant increases the reflected energy flow ratio, and on the whole, the correction of the incident angle and permittivity makes R increase, increasing the ratio of 0.87%.

Of course, the energy flux reflects here is a statistical average, because the angle of incidence and dielectric constant are a statistic.

D Influence of Ions in Seawater

Above we only consider the role of water molecules in polarization, and in the sea there are still anion and cation ions, they are also subject to the electromagnetic field, and more freedom. Before analyzing the problem, we quote without proof that the radiation in the electrodynamics of a point system as it moves in an electromagnetic field:

$$\vec{E}=\frac{e}{4\pi\varepsilon_0 c^2 r}\vec{n}\times\left(\vec{n}\times\frac{d\vec{v}}{dt}\right), \tag{73}$$

$$\vec{B}=\frac{1}{c}\vec{n}\times\vec{E}. \tag{74}$$

First, the anions and cations move in the electromagnetic field as ocean currents flow. In the laminar flow model, we consider this velocity to be uniform, so the resulting current is constant and only excites the magnetic field in space without affecting the magnetic field distribution. Also, because

$$\frac{d\vec{v}}{dt}=0,$$

ions do not radiate electromagnetic fields in space. Therefore, ions do not affect polarization during the whole motion.

Second, ions move randomly in water. Considering two-dimensional random walk model, we can simulate the trajectory of ion motion, that is, generating two random numbers r1 and r2 on [0, 1] at a time, and moving left one unit along x axis if r1> 0.5, otherwise Move a unit; r2 empathy. Perform 10000 steps to get the two-dimensional situation as shown in the Fig. 8, where the t-axis represents the virtual time.

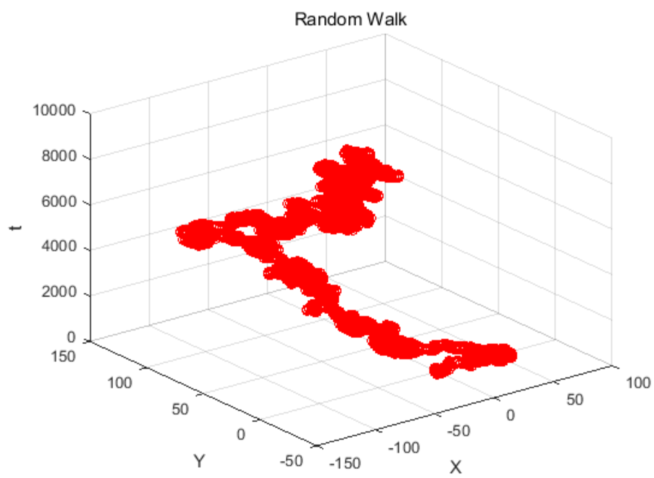

Fig. 8 Simulation of two-dimensional random walk.

Furthermore, the final position can be calculated for each random walk. After several tests, the mean of the final position tends to be zero. Take the number of N = 1000, and using Matlab we can get multiple sets of coordinate mean.

x	0.9414	-0.5982	-1.6982	1.7448
y	3.5955	-3.3425	2.3280	-2.7987

Table. 4 The coordinate mean of multiple sets in random walk.

As you can see, the final coordinates are very small, which is in line with expectations.

Due to the large degree of freedom of motion of ions, we can assume that in turbulent flow the ions always reach their equilibrium quickly, so the average displacement is zero and there is no contribution to polarization.

4.1.3.2.3 A model Based on Magnetization Mechanism

Next we consider the magnetization mechanism. In principle, the corrections brought by magnetization can be neglected. First of all, compared with the electric field, the magnetic field is very small, and the seawater change caused by it is almost negligible. Secondly, the response of the magnetization mechanism is much slower than that of the polarization. Under the HF frequency band, the medium can hardly keep up with the change of the magnetic field, effect.

Of course, we also use the same method of perturbation theory to describe the fluctuations in relative turbulence in the vicinity of the relative permeability $\mu^{(0)}$:

$$\mu_r = \mu^{(0)} + \mu^{(1)} + \cdots, \qquad (75)$$

where $\mu^{(1)} = 1$ represents the relative permeability of seawater, and μ^1 is the first-order perturbation term.

Assuming that the relative permeability fluctuation satisfies the normal distribution ($\mu^1 \sim N(0, \sigma^2)$) and ignores the higher-order terms, multiple simulations are performed using Matlab. A random sequence(μ_r) is obtained and used

$$R = \left(\frac{cos i_m - \frac{1}{\mu_r}\sqrt{\varepsilon_r \mu_r - sin^2 i_m}}{cos i_m + \frac{1}{\mu_r}\sqrt{\varepsilon_r \mu_r - sin^2 i_m}} \right)^2 \qquad (76)$$

to calculate he reflected energy flow ratio sequence and its mea. Set $\sigma^2 = 0.01$ and $\bar{R} = 0.6559$, then calculate difference with no disturbance condition when the reflected energy flow ratio is $R_0 = 0.6552$:

$$|\bar{R} - R_0| = 0.0007.$$

We can see that this effect is very small so we ignore the effect of the permeability factor.

4.1.4 Impact and Calculation of Noise

Considering the noise at one end of the transmitter, we have a power expression of noise based on Nyquist's theorem

$$P = kTB, \qquad (77)$$

where k represents the Boltzmann constant, T represents the Kelvin temperature, and B represents the frequency bandwidth of the transmitter.

Take all the parameters into the formula, and we can calculate the power $P = 1.09 \times 10^{-13} W$.

As can be seen from the previous calculations, the decay rate for each reflection is $R_m = 0.6701$, the power emitted by the emitter is $P_0 = 100\text{W}$, and the critical condition is a signal-to-noise ratio of 10 dB. So the following equation can be obtained:

$$10\,lg\left(\frac{P_0 \times {R_m}^N}{P}\right) = 10dB. \tag{78}$$

After calculating, we found that the maximum number of reflections can be N = 80 times.

4.2 Part II

4.2.1 The Model

First of all, for flat terrain, we also use the Fresnel principle to model.

From the reference [6], we can see that the relative dielectric constant of surface carbonate rocks is between 5 and 9. And only when the frequency of external electromagnetic waves is on the order of GHz, the relative dielectric of surface carbonate rocks constants will be considerably affected. In this problem, we use electromagnetic waves at frequencies between 3 and 30 MHz, two orders of magnitude smaller than GHz, so we do not need to consider the influence of the electromagnetic waves we use on the relative dielectric constant of the surface rock layers.

For the other parameters in the Fresnel equation, we select the parameter values that are consistent with those in and then set the relative permittivity of the surface rock to 7. By using the same algorithm in Part I, we can get the best angle of incidence

$$\theta = 0.5596rad.$$

Under the conditions of the incident angle, the energy ratio of each reflection can be calculated:

$$R_1 = 0.2573.$$

Then we research the modeling of the rugged mountains.

The first problem is the simulation of the terrain. Because the terrain is bumpy, we borrow the macroscopic turbulence curve in Part I to simulate the plane terrain in the rugged mountains after some stretching. The difference is that the terrain in the mountain area is stationary, so we randomly simulate a mountain curve according to the function in Part I. And by the same method we attain an incident angle as follow:

$$\theta_1 = 0.4174.$$

The second problem is that the mountainous terrain has a special way to reduce the electromagnetic wave, diffraction. By referring to the data, we get the empirical formula of diffraction attenuation of electromagnetic wave mountain:

$$|P| = |P_0| \times (1 - H_c/F_0). \tag{79}$$

The meaning of each variable in the formula are as follows:

- P represents the energy of the electromagnetic wave obtained after diffraction.
- P_0 represents that of the electromagnetic wave obtained before diffraction.
- H_c represents the distance between the highest point of the obstacle and the center line of the incident electromagnetic wave in km.
- $F_0 = 0.577F_1$, where F_1 represents the first Fresnel radius at which the electromagnetic wave collides with the obstacle.

In this problem, we take the difference between the highest point of the ejected topography and one-half the height of the ionosphere as an approximation of H_c, and for the first Fresnel radius we have the following formula:

$$F_1 = \sqrt{\frac{300 \times d_1 \times d_2}{f \times d}} \tag{80}$$

where, d_1, d_2 respectively, represent the horizontal distance between the launch point, the receiving point and the obstacles. And d represents the horizontal distance of the launch point and the receiving point. (Seen in Fig. 9) The units of all three variable mentioned above are km. Additionally, findicates the frequency of incident electromagnetic waves in GHz.

Some of the variable are indicated in the Fig. 9.

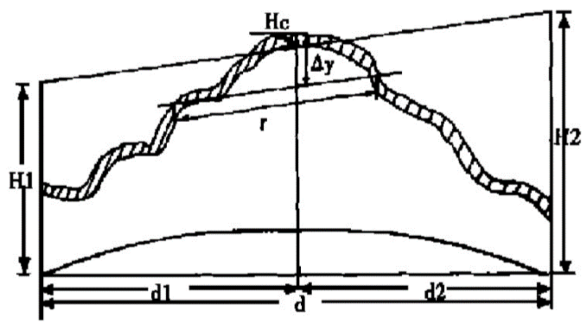

Fig. 9 Schematic of d_1, d_2, H_c

To sum up, after using the algorithm in the first question and calculating the diffraction loss, we can calculate that in the rugged mountains, the reduction rate of primary reflection is

$$R_2 = 0.2263.$$

Comparing the electromagnetic wave reflection results of flat areas and rugged mountains, we can find that the reflectivity of the rugged mountains is relatively low, with a difference of about 10%, mainly due to changes in the angle of incidence caused by topographic changes and diffraction caused by mountainous terrain to a certain extent, the loss.

4.2.2 Compare the Results with Those in 4.1 and Give an Explanation

Compared with the results in Part I, we find that the decay rate of electromagnetic waves on land is much greater than the decay rate on calm sea surface. This phenomenon can be explained by the Table. 5 indicating differences of features between land and sea.

Feature	Land	Sea
Relatively dielectric constants	5-9	About 80
Changes in dielectric constants	Slight enough to be neglected	Should be taken into considered
Curve of surface	(Mountainous or rugged terrain) Never change over time once position is selected	(Turbulence) Change over time
Height of the relection surface	Should be taken into considered	Slight enough to be neglected

Table.5 Differences of features between land and sea.

4.2.2.1 Difference in Features (Quantitative Calculation)

A Difference in Dielectric Constants and Changes in Dielectric Constants in Turbulent Condition

According to Fresnel principle which has been discussed in 4.1, the difference in dielectric constants between land and sea will greatly influence the reduction rate of reflection.

B Difference in Curve of Surface

This kind of difference in feature will influence incident angle, which is also a key factor in Fresnel principle.

C Difference in Height of the Relection Surface

This kind of difference in feature will bring a diffraction decrease which has been mentioned above.

4.2.2.2 Difference in Feature (only Qualitative Calculation)

- **Difference in Height in Relection Surface**

Since the height of the mountain is much greater than the height of the waves, the probability of the signal being blocked during transmission is far greater than the electromagnetic wave being hindered by the turbulence in the waves. In this regard, a part of the influence can be quantitatively described in the diffraction theory, while the other part of the influence can only be qualitatively described as the horizontal distance by which the electromagnetic wave can propagate becomes smaller due to the obstruction of the mountain.

4.3 Part III

Taking into account the fact that radio waves are spread in the form of spherical waves. Therefore, we modify the model as follows:

(1) Except the last time, all previous propagation processes follow the law of plane waves, that is, to ignore all the coherent phenomena in the middle, and assume the radio waves only consume when reflected on the sea surface.

(2) The last transmission follows the law of spherical waves. According to the Huygens-Fresnel principle, at the penultimate end, the light is reflected from the ionosphere, which acts as a center of a second wave and the power at any point in space at distance r is

$$P = \frac{P_{n-1}}{r^2}. \tag{81}$$

(3) The critical condition is that at the last transmission, the signal just happens to be resolved just below the center of the sub-wave.

Using SNR = 10 dB in Part I and the previous calculation, we can get the lowest power of the signal that the receiver can distinguish as

$$P_m = 1.09 \times 10^{-13} W \tag{82}$$

by

$$\begin{cases} P_m = P_{n-1}/r^2 \\ P_{n-1} = P_0 R^{n-1} \end{cases}. \tag{83}$$

Joint to get the solution n = 27.

So the hull has gone through a total of 26 half-cycles (the signal cannot be distinguished in the middle of the last one). For each period, we still select $\beta = 0.3632$ (see in Fig. 6), then the transmission distance of each periodic wave is

$$l_0 = 2H\tan\beta = 4.5607 \times 10^4 m. \tag{84}$$

Then the total propagation distance

$$L = 26 \times \frac{1}{2} l_0 = 1.2086 \times 10^6 m = 1208.6 km.$$

Besides, another idea to accommodate a shipboard receiver moving on a turbulent ocean can be seen as follows.

When the ship sails on a turbulent sea, we can use gravity sensors to record the current sea level conditions. After a period of accumulation, we use the gravity-induced data and the ocean current to predict the sea surface curve between the next ship and the coast. Then we can get a function of the angle of incidence and the distance traveled by the electromagnetic wave. After deriving the extremum of this function, we can get an incident angle that maximizes the propagation distance, so that the transmitter can be adjusted accordingly to maximize the propagation efficiency of the electromagnetic wave signal.

5 Further Work

From a more general point of view, the structure of a liquid lies between the gas and the solid, both having a structural periodicity of the solid and also a fluidity of the gas. So we can always draw on gas and solid analysis methods to give some corrections for the usual model.

5.1 Poisson- Boltzmann Equation

In a non-thermodynamically balanced system with turbulent phenomena, we can always describe the motion of dipole moment using the Boltzmann equation. However, the Boltzmann equation is hard to solve, so we consider using the average field approximation, which is from the classical field theory, and we get Poisson-Boltzmann Equation:

$$\nabla^2\varphi(\vec{r}) = -\frac{4\pi}{\varepsilon}\sum_i c_i z_i q e^{-\beta z_i q\varphi(\vec{r})}. \tag{85}$$

For $\beta z_i q\varphi(\vec{r}) \ll 1$, We do the Taylor expansion of the exponential function and discard the higher-order terms:

$$\nabla^2\varphi(\vec{r}) + \frac{4\pi z_i^2 q^2 c_i}{\varepsilon k_B T}\varphi(\vec{r}) = 0. \tag{86}$$

According to the empirical formula of turbulence, we can specify the potential distribution at the boundary, so as to solve the distribution of potential in the seawater and obtain the distribution of the electric field

$$E(\vec{r}) = -\nabla\varphi(\vec{r}). \tag{87}$$

Finally the dielectric constant is corrected according to the distribution of the electric field. At the same time, considering the uncertainty of turbulence, we can add a random term in (86), just like what Langevin have done.

$$\nabla^2\varphi(\vec{r}) + \frac{4\pi z_i^2 q^2 c_i}{\varepsilon k_B T}\varphi(\vec{r}) + \epsilon\varphi(\vec{r}) = 0. \tag{88}$$

Prescribing the distribution of random quantities, we can find the statistical average of the potentials as another correction term.

5.2 Monte Carlo Method

If we refer to the analytic method on solid, we can assume that at some time, a liquid molecule vibrates always near its initial position. As mentioned above, a water molecule is a polar molecule that has a permanent dipole moment and acts in turbulence. The orientation of the dipole moment is affected. For simplicity, it may be useful to model the Heisenberg model, assuming that its orientation can only be in six directions along the coordinate system. And the water molecules are bosons, to meet the distribution of Bose, so we can use this as a criterion, with Monte Carlo method to simulate the dipole moment orientation changes given the change of the electric field within a specific period of time, and then get the dielectric constant correction.

5.3 General Method

Of course, what we discussed here is only the classical case. The more accurate result is solving the Schrödinger equation in a special potential field, giving the wave function of the particle and then determining the charge flow and the distribution of the electric field.

6 References

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7 The Short Synopsis

Researches on the reflection of electromagnetic waves on the ocean surface

I. Introduction

Recently, researchers have found that turbulence in the ocean can affect the electromagnetic properties of the medium, especially for high frequency radio waves. If the frequency of the electromagnetic wave is lower than the MUF, the electromagnetic wave can return from the ionosphere and continue reflective transmission between the earth's surface and the ionosphere.

As we know, the nature of the surface material affects the intensity of reflections and therefore determines how far the electromagnetic wave can ultimately be transmitted.

In this paper, the main background is set in the marine environment.

- We will discuss the different transmission efficiency and distance of electromagnetic waves in two different situations of calm sea surface and turbulent sea surface.
- At the same time in the flat terrain and the rugged mountain model will be established and therefore we can find the contrast between different surface material.
- Finally, we will discuss how to adjust the model and the maximum transmission distance that this type of communication can achieve in the model after adjustment while navigating the turbulent sea.

II. Calm Ocean Model

According to the Maxwell's equation and the boundary conditions on the surface of the medium, we can calculate the reflected energy-flow ratio

$$R = \lambda\left(\frac{ncosi - (\mu/\mu')\sqrt{n'^2 - n^2\sin^2 i}}{ncosi + (\mu/\mu')\sqrt{n'^2 - n^2\sin^2 i}}\right)^2 + (1-\lambda)\left(\frac{(\mu/\mu')n'^2cosi - n\sqrt{n'^2 - n^2\sin^2 i}}{(\mu/\mu')n'^2cosi + n\sqrt{n'^2 - n^2\sin^2 i}}\right)^2$$

Then in order to improve the accuracy of the information received by the ground equipment, the distance between the two reflections should be kept as small as possible, so that angle of incidence should be set to the minimum

$$i_m = \arcsin\left(\sqrt{1 - 80.8N_m/f^2}\right) = 0.5596 \text{ rad}$$

To attain a flow of the reflected energy as large as possible, we choose $\lambda = 1$ and get the maximum reflection energy flow ratio

$$R_m = 0.6701$$

III. Turbulence Model

To discuss the physics behind the polarization, we'll recreate a more detailed model. Of course, here we are not going to discuss the quantum theory of multi-body problems, but only from classical electrodynamics and statistical mechanics, derive the formula of dielectric constant.

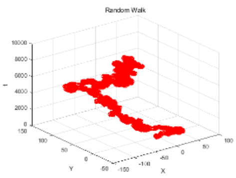
For a group of molecules, polarization mechanism has two kinds: displacement polarization and oriented polarization. In the former model, we start from the harmonic oscillator model and add a correction term to describe the influence caused by turbulence. In the latter one we use the Boltzmann distribution to calculate the average dipole moment. Finally, we obtain

$$\varepsilon_0 = \begin{cases} \varepsilon_0 + \dfrac{1}{3}\dfrac{Np_0^2}{kT} + \dfrac{10Ne^2}{m({\omega_0}^2 - \omega^2)} - \displaystyle\sum_i \dfrac{10NeB_i}{E_0(\omega_0^2 - {\omega_i}^2)} sin\,[(\omega_i - \omega)t - \varphi_i] & Turbulence \\ \varepsilon_0 + \dfrac{10Ne^2}{m({\omega_0}^2 - \omega^2)} - \displaystyle\sum_i \dfrac{10NeB_i}{E_0(\omega_0^2 - {\omega_i}^2)} sin\,[(\omega_i - \omega)t - \varphi_i] & Laminar\ Flow \end{cases}$$

Take the incident angle of 0.4614 rad, which has been fixed, and calculate using MATLAB, we get

$$< R >= 0.6759$$

In the sea, there are also anions and cations, which are subject to the electromagnetic field. As we know, ions move randomly in water. Considering two-dimensional random walk model, we can simulate the trajectory of ion motion.



Furthermore, the final position can be calculated for each random walk. After several tests, the mean of the final position tends to be zero, and there is no contribution to polarization.

III. Further

Taking into account the fact that radio waves are spread in the form of spherical waves. Therefore, we modify the model as follows:

(1) Except the last time, all previous propagation processes follow the law of plane waves, that is, to ignore all the coherent phenomena in the middle, and assume the radio waves only consume when reflected on the sea surface.

(2) The last transmission follows the law of spherical waves. According to the Huygens-Fresnel principle, at the penultimate end, the light is reflected from the ionosphere, which acts as a center of a second wave and the power at any point in space at distance r is

$$P = \frac{P_{n-1}}{r^2}$$

(3) The critical condition is that at the last transmission, the signal just happens to be resolved just below the center of the sub-wave.

Using SNR = 10 dB, we can get the lowest power of the signal that the receiver can distinguish as

$$P_m = 1.09 \times 10^{-13}W$$

by

$$\begin{cases} P_m = P_{n-1}/r^2 \\ P_{n-1} = P_0 R^{n-1} \end{cases}$$

Joint to get the solution n = 27

So the hull has gone through a total of 26 half-cycles (the signal can not be distinguished in the middle of the last one). For each period, we still select $\beta = 0.3632$, then the transmission distance of each periodic wave is $l_0 = 2Htan\beta = 4.5607 \times 10^4 m$, then the total propagation distance $L = 26 \times \frac{1}{2} l_0 = 1.2086 \times 10^6 m = 1208.6km$

Appendix

Appendix A 4.1.3.1.1 Ocean Wave Simulation

```
%%  Ocean wave simulation
    clear all;
    close all;
    clc;
    format long;
%%  [1] Input parameter
    n = 6;                  % the number of wave
    v = 8.135526;           % average speed of wind
    aveT = 2.945485;        % average period of wave
    a = 1;                  % amplitude of breaking wave
    j = 12;                 % rough coefficent of breaking wave
    theta = 0.5596;
    kk = tan(pi()/2+theta);
    b = 60000;
    aveH = 4.416*0.001*v^(2.5); % average wave high
    aveA = aveH/2;                  % average wave length
for m = 1:1000
    disp(m);
%%  [2] Parameter of composition wave
    c=0.5;
    for i=1:n
        c = rand(1);
        A(i) = 2*aveA*sqrt(-(log(1-c))/pi());
        c = rand(1);
        k(i) = pi()/(1.56*(aveT^2)*sqrt(-(log(1-c))/pi()));
        c = rand(1);
        f(i) = 1/(aveT*power(-1.4185*log(1-c),1/4));
        c = rand(1);
        t(i) = aveT*c;
        u(i) = rand(1);
      %  u(i) = 0.5;
    end
%%  [3] Wave Model
    tt=0;
    x_begin = 1;
    x_end = 300;
    Z = zeros(x_end - x_begin +1,1);
    for x=x_begin:x_end
        Z(x) = 0;
        for i=1:n
            Z(x) = Z(x)+A(i)*sin(k(i)*x-2*pi()*f(i)*(tt+t(i)))+a*u(i)*sin(pi()/j);
        end
```

```
            Z(x) = Z(x)/n;
        end

        syms xx tt;
        ZZ = 0;
        for i=1:n
            ZZ = ZZ+A(i)*sin(k(i)*xx-2*pi()*f(i)*(tt+t(i)))+a*u(i)*sin(pi()/j);
        end
        ZZ = ZZ/n;

%% [4] Newton iteration

        ff = subs(ZZ - (kk*xx+b),tt,0);
        x0 = -b/kk;
        MAX = 100;    %max number iteration
        e = 1e-5;     %accuracy
        x1 = x0 - subs(ff,xx,x0)/subs(diff(ff,xx),xx,x0);

        for i=1:MAX
            if (abs(subs(ff,xx,x0))>e | abs(x1-x0)>e )
                x0 = x1;
                x1 = x0 - subs(ff,xx,x0)/subs(diff(ff,xx),xx,x0);
            else
                break;
            end
        end

        angle(m) = subs(diff(ZZ,xx),xx,x1);

        ans_tt = vpa(subs(ZZ,xx,x1),4);
        diffans_tt = vpa(diff(ans_tt,2),4);
        for i=1:n
            w(m,i) = 2*pi()*f(i);
            fi(m,i) = -vpa(k(i)*x1+2*pi()*f(i)*t(i));
            B(m,i) = A(i)*w(i)*w(i)/n;
        end

        for i=1:n
            A_sum(m,i) = A(i);
            k_sum(m,i) = k(i);
            fi_sum(m,i) = -2*pi*f(i)*t(i);
            changshu_sum(m,i) = a*u(i)*sin(pi()/j);
        end
end
```

Appendix B 4.1.3.2.1 Rytov Perturbation Approximation Theory

```
%%  random dielectric constant
    clc;
    clear;
    close all;

%%  Parameter
    im = 0.461448;      % angle of incidence
    mu = 0;             % mean of lognormal distribution
    sigma = 0.5;        % variance of lognormal distribution
    [M,V]= lognstat(mu,sigma);
    e0 = 72.47353708527893; % dielectric constant

%%  Formula
    e1 = lognrnd(mu,sigma,1,1e6);    % random dielectric constant

    for i =1:1e6
        R(i) = ((cos(im)-sqrt((e0+e1(i))-(sin(im))^2))/(cos(im)+sqrt((e0+e1(i))-
(sin(im))^2)))^2;
    end

    R_mean = mean(R);
    R_var = var(R);

    R0 = ((cos(im)-sqrt((e0)-(sin(im))^2))/(cos(im)+sqrt((e0)-(sin(im))^2)))^2;
    disp(R_mean);
```

Appendix C 4.1.3.2.3 A model Based on Magnetization Mechanism

```
%%  Random magnetic permeability
    clc;
    clear;
    close all;

%%  Parameter
    im = 0.461448;          % angle of incidence
    mu = 0;                 % mean of lognormal distribution
    sigma = sqrt(0.01);             % variance of lognormal distribution
    er = 72.47353708527893; % dielectric constant
    u0 = 1;                         % Magnetic permeability

%%  Formula
    u1 = normrnd(mu,sigma,1,1e6);

    for i =1:1e6
        ur = u0+u1(i);
```

```
        nn = er*ur;
        R(i) = ((ur*cos(im)-sqrt((nn)-(sin(im))^2))/(ur*cos(im)+sqrt((nn)-(sin(im))^2)))^2;
    end

    R_mean = mean(R);
    R_var = var(R);
```

Appendix D 4.1.3.2.2.C Correction of Turbulence

```
% Calculate the reflected energy flow ratio considering turbulence
    clear;
    clc;
% The parameters of waves

w=[2.373368737222600,2.440442982963471,2.392607423758321,2.470416689587460,2.428156114605642,2.382875294781810];

B=[0.200223219295296,0.298032635944935,0.336896055939343,0.716851116527641,0.659298395921803,0.340319288297806];
    fai=[-0.264103798093131,-0.278130415796451,-0.251949725532695,-0.281669634093326,-0.266772261921991,-0.262854229665976];
% Other parameters
    im=0.4614
    N=10^6*6.02*10^23/18;
    w_0=3.8558*10^(15);
    E=0.001290547812215;
    w_1=15*10^6;
    T=2*pi/w_1;
% Calculation
    syms t;
    A=10.*N.*1.6.*10^(-19).*B./E./(w.^2-w_0^2).*sin(w_1.*t+fai);
    e0=72.47353708527893+(1.0496*10^(-10)+sum(A))/(8.854187817e-12);
    m=quad(matlabFunction(sqrt((e0)-(sin(im))^2)),0,T)/T;
    R0 = ((cos(im)-m)/(cos(im)+m))^2;
```

Appendix E 4.1.3.2.2.D Influence of Ions in Seawater

```
% Random walk(2 dimensions)
clear;
for k=1:1000
    x(1)=0;
    y(1)=0;
    t(1)=0;
    for j=2:10000
```

```
        t(j)=j;
        r1=rand();
        r2=rand();
        if r1>=0.5
            x(j)=x(j-1)+1;
        else
            x(j)=x(j-1)-1;
        end
        if r2>=0.5
            y(j)=y(j-1)+1;
        else
            y(j)=y(j-1)-1;
        end

    end
    %scatter3(x,y,t,'r');
    px(k)=sum(x)/10000;py(k)=sum(y)/10000;
end
xx=sum(px)/1000;yy=sum(py)/1000;
```