

Maximum Likelihood Method

Example: Normal distribution

x_1, \dots, x_n iid, $x_i \sim \mathcal{N}(\mu, \sigma^2)$

$$f_{x_i}(x_i) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x_i - \mu)^2}{2\sigma^2}\right),$$

$i = 1, \dots, n$

joint density:

$$f_X(x) = f_{x_1, \dots, x_n}(x_1, \dots, x_n) = \frac{1}{(2\pi\sigma^2)^{n/2}} \exp\left(-\sum_{i=1}^n \frac{(x_i - \mu)^2}{2\sigma^2}\right)$$

→ max

same as maximization of $\log(f_X(x))$
(log is mon. increasing function)

So to maximize

$$\log(f_X(x)) = -\frac{n}{2} \log(2\pi\sigma^2) - \sum_{i=1}^n \frac{(x_i - \mu)^2}{2\sigma^2}$$

same as to minimize

$$-\log(f_X(x)).$$