Time Series and Sequence Learning

Discussion seminar for Lecture 5

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lecture5a/b - Classical Decomposition

- Why do we model using

$$\Delta^k \mu_t = \zeta_t,$$

  instead of modelling with

$$\Delta^k \mu_t = 0?$$

  Why is this random variable necessary?
- Is there some difference if we talk about the seasonal component instead of the trend component?
- What is the effect of the 1's on the lower diagonal in the transition matrices?

We have modeled the trend as linear and the seasonal component with $s = 3$, giving the following models:

Trend component ($k = 2$):

$$\alpha_t = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix} \alpha_{t-1} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \zeta_t$$

$$\mu_t = \begin{bmatrix} 1 & 0 \end{bmatrix} \alpha_t$$

Seasonal component ($s = 3$):

$$\alpha_t = \begin{bmatrix} -1 & -1 \\ 1 & 0 \end{bmatrix} \alpha_{t-1} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \omega_t$$

$$\gamma_t = \begin{bmatrix} 1 & 0 \end{bmatrix} \alpha_t$$

Combine these models into one LGSS model.

What is the state of this model?

How do we find the decomposition $y_t = \mu_t + \gamma_t + \varepsilon_t$ in the model?

Kalman filter

Kalman filter: For $t = 1, 2, \dots$

- **Predict:**

  $\cdot$ Predict α_t: $$\begin{cases} \hat{\alpha}_{t|t-1} = T\hat{\alpha}_{t-1|t-1}, \\ P_{t|t-1} = TP_{t-1|t-1}T^{\mathsf{T}} + RQR^{\mathsf{T}} \end{cases} \qquad (\star)$$

  $\cdot$ Predict y_t: $$\begin{cases} \hat{y}_{t|t-1} = Z\hat{\alpha}_{t|t-1}, \\ F_{t|t-1} = ZP_{t|t-1}Z^{\mathsf{T}} + \sigma_\varepsilon^2 \end{cases}$$

- **Update:**

  $\cdot$ Kalman gain: $$K_t = P_{t|t-1}Z^{\mathsf{T}}F_{t|t-1}^{-1}$$

  $\cdot$ Update filter: $$\begin{cases} \hat{\alpha}_{t|t} = \hat{\alpha}_{t|t-1} + K_t(y_t - \hat{y}_{t|t-1}), \\ P_{t|t} = (I - K_tZ)P_{t|t-1} \end{cases} \qquad (\star\star)$$

($\star$) At time $t = 1$ we initialize $\hat{\alpha}_{1|0} = a_1$ and $P_{1|0} = P_1$.

($\star\star$) If y_t is missing we skip the update and set $\hat{\alpha}_{t|t} = \hat{\alpha}_{t|t-1}$ and $P_{t|t} = P_{t|t-1}$.

- If I run the Kalman filter and get $\hat{\alpha}_{t|t}$. How do I find my filtered seasonal component and filtered trend component?
- What would happen if $\sigma_\varepsilon^2 = 0$? Can you look at the model and guess what the filter mean would be?
- Same questions but with the state noise $Q = 0$.

lecture5d/e - AR and ARMA models / Stability

- LGSS models are often referred to as hidden Markov models.
- How can we check if a given AR(p) model is stable?
- Previously we were mainly interested in finding stable/stationary AR models. The presented models here are marginally stable. Why is this important?
- Is there ever a need for unstable models?