732A96/TDDE15 ADVANCED MACHINE LEARNING

EXAM 2022-08-24

Teacher

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Grades

- For 732A96 (A-E means pass):
  - A=19-20 points
  - B=17-18 points
  - C=14-16 points
  - D=12-13 points
  - E=10-11 points
  - F=0-9 points
- For TDDE15 (3-5 means pass):
  - 5=18-20 points
  - 4=14-17 points
  - 3=10-13 points
  - U=0-9 points

The total number of points is rounded to the nearest integer. In each question, full points requires clear and well motivated answers and/or commented code.

Instructions

This is an individual exam. No help from others is allowed. No communication with others regarding the exam is allowed. The exam is anonymous, i.e. do not write your name anywhere. Answers to the exam questions may be sent to Urkund.

The answers to the exam should be submitted in a single PDF file using the communication client. You can make a PDF from LibreOffice (similar to Microsoft Word). You can also use Markdown from RStudio. Include important code needed to grade the exam (inline or at the end of the PDF file).

Do not ask question through the communication client. The teachers will be reachable by phone, and they will visit the room too.

Allowed help

Lecture slides, lab descriptions, course literature, your (individual and group) solutions to the labs.

1. Graphical Models (7 p)

You are asked to use a Bayesian network to solve the following problem. The probability that a healthy person remains healthy the day after is 0.9. The probability that an infected person remains infected the day after is 0.8. We never know for sure if a person is infected. Luckily, there is a lab test that gives us information about the true health status of a person. The test is not perfect, though. The sensitivity of the test (i.e., the true positive rate) is 0.6, whereas the specificity of the test (i.e., the true negative rate) is 0.7.

For a person that is healthy today with probability 0.5, what is the probability that she is healthy in three days ? What is the probability that she is healthy in three days given that she received a negative test on the second day ? And if she also got a negative test on the third day ?

2. Hidden Markov Models (3 p)

You are asked to build a hidden Markov model (HMM) to model a weather forecast system. The system is based on the following information. If it was rainy (respectively sunny) the last two days, then it will be rainy (respectively sunny) today with probability 0.75 and sunny (respectively rainy) with probability 0.25. If the last two days were rainy one and sunny the other, then it will be rainy today with probability 0.5 and sunny with probability 0.5. Moreover, the weather forecast system malfunctions with probability 0.1 when reporting the daily forecast back to the user, i.e. it reports rainy weather when the forecast is actually sunny and vice versa with probability 0.1. Implement the weather forecast system described using the `HMM` package. Sample 10 observations from the HMM built.

Hint: You may want to have hidden random variables with four states encoding the weather not only today but also yesterday. However, your observed random variables should have two states corresponding to the weather today.

3. Reinforcement Learning (5 p)

In the course, you have learned about the roles that ϵ and γ play in Q-learning. Although typically selected by the user, their values can also be selected via validation. For instance, consider the values $\epsilon = 0.1, 0.25, 0.5$ and $\gamma = 0.5, 0.75, 0.95$. For each pair of values for ϵ and γ, run Q-learning 30000 episodes. Freeze the resulting Q-table, i.e. do not update it anymore. Run Q-learning 1000 additional episodes (without updating the Q-table). Compute the average reward in these 1000 episodes. The first 30000 episodes learn the policy (training), whereas the last 1000 evaluate the policy (validation). Now, simply select the pair of values for ϵ and γ that performed best in the validation phase.

Your task is to implement the validation framework described above for the environment B in the lab. Discuss your results.

4. Gaussian Processes (5 p)

(1) (2 p) The file `KernelCode.R` distributed with the exam contains code to construct a `kernlab` function for the Matern covariance function with $\nu = 3/2$:

$$k(\mathbf{x}, \mathbf{x}') = \sigma_f^2 \left(1 + \frac{\sqrt{3}r}{\ell}\right) \exp\left(-\frac{\sqrt{3}r}{\ell}\right)$$

where $r = |\mathbf{x} - \mathbf{x}'|$. Let $f \sim \mathcal{GP}(0, k(\mathbf{x}, \mathbf{x}'))$ a priori and let $\sigma_f^2 = 1$ and $\ell = 0.5$. Plot $k(0, z)$ as a function of z. You can use the grid `zGrid = seq(0.01,1,by=0.01)` for the plotting. Interpret the plot. Connect your discussion to the smoothness of f. Finally, repeat this exercise with $\sigma_f^2 = 0.5$ and discuss the effect this change has on the distribution of f.

(2) (2 p) You are asked to extend your work on Lab 4 with the Tullinge temperatures. In the lab, you predicted the temperature as function of time with the following hyperparameter values: $\sigma_f = 20$ and $\ell = 0.2$. Now, you are asked to search for the best hyperparameter values by maximizing the log marginal likelihood. You may want to check the corresponding slides for the theoretical details. Recall that you implemented Algorithm 2.1 in the book by Rasmussen and Williams, which already returns the log marginal likelihood. Use a grid search to search for the best hyperparameter values, i.e. try different hyperparameter values while time permits.

(3) (1 p) In the previous exercise, why does it make sense to choose the hyperparameter values that maximize the log marginal likelihood ?