$$??$$
$_design rafaely_fundamentals_2019$??
$R(\theta_s, \phi_s)(\theta_q, \phi_q)_fourier_2000, rafaely_fundamentals_2019$
$Q(\theta_q, \phi_q)_i ntegral \int_0^{2\pi} \int_0^{\pi} g(\theta, \phi) \sin\theta d\theta d\phi \approx$
$\sum_{q=1}^{Q} \alpha_q g(\theta_q, \phi_q), (1) \alpha_q angle_s amplingeq.Gaussian_s amplingeq.uniform_s ampling \alpha_q =$
$\frac{2\pi}{(N+1)^2} \sin(\theta_q) \sum_{q'=0}^{N} \frac{1}{2q'+1} \sin([2q'+1]\theta_q). (2)$

$$\alpha_q = \frac{\pi}{N+1} \frac{2\left(1-\cos^2\theta_q\right)}{(N+2)^2\left[P_{N+2}\left(\cos\theta_q\right)\right]^2}. \tag{3}$$

$$\alpha_q = \frac{4\pi}{Q}. \tag{4}$$

$$S_n^m(r,f) = \sum_{q=1}^{Q} \alpha_q S(r, \theta_q, \phi_q, f)[Y_n^m(\theta_q, \phi_q)]^*. \tag{5}$$

q

$$S(r, \theta_q, \phi_q, f) = \sum_{n=0}^{N_s} \sum_{m=-n}^{n} S_n^m(r,f) Y_n^m(\theta, \phi), \tag{6}$$

$$S = Y S_{nm}, \tag{7}$$

$S S_{nm} Q (N_s + 1)^2 Y Q \times (N_s + 1)^2$

$$S = [S(r, \theta_1, \phi_1, f), S(r, \theta_2, \phi_2, f), \cdots, S(r, \theta_Q, \phi_Q, f)]^T, \tag{8}$$

$$S_{nm} = [S_0^0, \cdots, S_n^m, \cdots, S_{N_s}^{N_s}]^T, \tag{9}$$

$$Y = \begin{bmatrix} Y_0^0(\theta_1, \phi_1) \cdots Y_n^m(\theta_1, \phi_1) \cdots Y_{N_s}^{N_s}(\theta_1, \phi_1) \\ Y_0^0(\theta_2, \phi_2) \cdots Y_n^m(\theta_2, \phi_2) \cdots Y_{N_s}^{N_s}(\theta_2, \phi_2) \\ \\ Y_0^0(\theta_Q, \phi_Q) \cdots Y_n^m(\theta_Q, \phi_Q) \cdots Y_{N_s}^{N_s}(\theta_Q, \phi_Q) \end{bmatrix}. \tag{10}$$

$Q = (N+1)^2 {}_r andom_s ampling S_{nm} = Y^{-1} S, (11) Q > (N+1)^2 {}_r andom_s ampling: S_{nm} = Y^{\dagger} S = (Y^H Y)^{-1} Y^H S. (12) Q < (N+1)^2 {}_r andom_s ampling$
$_fourier_2000$
$(\theta_s, \phi_s)(r, \theta_q, \phi_q)$

$$S_{\mathrm{dir}}(r, \theta_q, \phi_q, f) = \sum_{n=0}^{\infty} 4\pi i^n j_n(kr) \sum_{m=-n}^{n} [Y_n^m(\theta_s, \phi_s)]^* Y_n^m(\theta_q, \phi_q). \tag{13}$$

$$S_{\mathrm{scat}}(r, \theta_q, \phi_q, f) = -\sum_{n=0}^{\infty} 4\pi i^n \frac{j_n'(kr)}{(h_n^{(1)})'(kr)} h_n^{(1)}(kr) \sum_{m=-n}^{n} [Y_n^m(\theta_s, \phi_s)]^* Y_n^m(\theta_q, \phi_q), \tag{14}$$

$j_n'(kr)(h_n^{(1)})'(kr) n j_n(kr) n h_n^{(1)}(kr)$
$\theta_q, \phi_q, f) = S_{\mathrm{dir}}(r, \theta_q, \phi_q, f) + S_{\mathrm{scat}}(r, \theta_q, \phi_q, f)$
$= \sum_{n=0}^{\infty} 4\pi i^n \left(j_n(kr) - \frac{j_n'(kr)}{(h_n^{(1)})'(kr)} h_n^{(1)}(kr)\right) \sum_{m=-n}^{n} [Y_n^m(\theta_s, \phi_s)]^* Y_n^m(\theta_q, \phi_q),$

$1:$
$49)[-80-65-55-45:5:45556580]$
$80°??0°??$
$_HRIR[][width=0.48]figure/chapter3/80du_HRTFCIPIC°$
$_HRIR[][width=0.48]figure/chapter3/0du_HRTFCIPIC°$
$??\theta$
$??\theta z\theta=$
$0°,90°,180°\phi\phi=$
$0°,90°,180°,270°\phi=$
$0°,90°,180°,270°??$
θ

$-90° \le \theta \le 90°$	$0 \le \overset{x}{\theta} \le 180°$	$0 \le \overset{x}{\theta} \le 180°$
$\theta = 90°$	$\theta = 0°$	$\theta = 0°$
$\theta = 0°$	$\theta = 90°$	$\theta = 90°$
$\theta = -90°$	$\theta = 180°$	$\theta = 180°$
$\phi = 0°$	$\phi = 0°$	$\phi = 0°$
$\phi = 270°$	$\phi = 270°$	$\phi = 90°$
$\phi = 180°$	$\phi = 180°$	$\phi = 180°$
$\phi = 90°$	$\phi = 90°$	$\phi = 270°$

$\phi=$
$mod(360°-\phi_{SOFA},360°)$
$\theta=$
$90°-$
$\theta_{SOFA},$
mod
$??P$

$$H_{L,R}(\theta_p,\phi_p,f)=\sum_{n=0}^{N_h}\sum_{m=-n}^{n}\beta_{nm}^{L,R}(f)Y_n^m(\theta_p,\phi_p)$$
(17)

$\beta_{nm}^{L,R}(f)_random_samplingH_{L,R}=$
$Y\beta_{nm}{}^{L,R},(18)YP\times$
$(N_h+$
$1)^2H_{L,R}\beta_{nm}{}^{L,R}P(N_h+$
$1)^2$

$$Y=\begin{bmatrix} Y_0^0(\theta_1,\phi_1)\cdots Y_n^m(\theta_1,\phi_1)\cdots Y_{N_h}^{N_h}(\theta_1,\phi_1)\\ Y_0^0(\theta_2,\phi_2)\cdots Y_n^m(\theta_2,\phi_2)\cdots Y_{N_h}^{N_h}(\theta_2,\phi_2)\\ \\ Y_0^0(\theta_P,\phi_P)\cdots Y_n^m(\theta_P,\phi_P)\cdots Y_{N_h}^{N_h}(\theta_P,\phi_P)\end{bmatrix},$$
(19)

$$H_{L,R}=[H_{L,R}(\theta_1,\phi_1,f),H_{L,R}(\theta_2,\phi_2,f),\cdots,H_{L,R}(\theta_P,\phi_P,f)]^T,$$
(20)

$$\beta_{nm}{}^{L,R}=[\beta_{0,0}^{L,R},\cdots,\beta_{n,m}^{L,R},\cdots,\beta_{N_h,N_h}^{L,R}]^T.$$
(21)

$P>$
$(N_h+$
$1)^2{}_Pinv\beta_{nm}{}^{L,R}=$
$Y^\dagger H_{L,R}=$
$(Y^HY)^{-1}Y^HH_{L,R}.(22)$
$\beta_{nm}{}^{L,R}(\theta,\phi)$

$$\hat{H}_{L,R}(\theta,\phi,f)=y\beta_{nm}{}^{L,R}.$$
(23)

$y=$
$\left[Y_0^0(\theta,\phi),\cdots,Y_n^m(\theta,\phi),\cdots,Y_{N_h}^{N_h}(\theta,\phi)\right](\theta,\phi)$
$??$
$_decompositionHRTF(interauralphasedifference,IPD)2015EfficientHRTFHRTFIPD2018Binaural2018comp$
$H_{L,R}^a(\theta,\phi,f)$

$$H_{L,R}^a(\theta,\phi,f)=\{H_{L,R}(\theta,\phi,f),f<f_cH_{L,R}(\theta,\phi,f)e^{i\zeta},f\ge f_c,$$
(24)

$f_c\zeta\zeta=$
$\angle([H_{L,R}(\theta,\phi,f)]^*H_{L,R}(\theta,\phi,f_c))$
$=$
$\angle(H_{L,R}(\theta,\phi,f_c))-$
$\angle(H_{L,R}(\theta,\phi,f)),$
$\angle(\cdot)$

$$\zeta=2\pi(f_c-f)\tau_p^{L,R},$$
(25)

$\tau_p^{L,R}(\theta_p,\phi_p)$
$\tau_p^{L,R}$

$$\tau_p^r=\cos(\theta_p)\sin(\phi_p)a/c,\tau_p^l=-\tau_p^r$$