# Bispectrum analysis for fourth generation CMB experiments



## Wu Hyun Sohn

Department of Applied Mathematics and Theoretical Physics University of Cambridge

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# Chapter 1

## Introduction

## 1.1 From background to foreground

Radio astronomers Arno Penzias and Robert Wilson were calibrating their 50-feet-long horn antenna when they found a mysterious background noise. The measurements were independent of time and location in the sky, and persisted after the removal of various potential contaminants. After the theoretical work of Robert Dicke, Jim Peebles, and David Wilkinson was brought forward [1], Penzias and Wilson identified the noise as cosmic microwave background radiation (CMB): ancient light from the early universe reaching us after billions of years [2]. The discovery provided us with one of the most valuable probes of the physical universe, leading to major development in observational cosmology.

On the theoretical side, modern mathematical formulation of cosmology owes to Einstein's work on general relativity in 1915. Using his framework, Friedmann, Lemaître, Robertson, and Walker contributed to writing down the unique metric for spatially homogeneous and isotropic universe. The FLRW metric dictates growth of the universe from the Big Bang to present day. Such expansion of the universe was supported by Edwin Hubble's measurements of Cepheid variables and redshift (add year), as well as the aforementioned discovery of CMB. What is widely accepted to be the standard model of modern cosmology, the ACDM model, appeared only in the late 1990s. The six-parameter model assumes presence of cold dark matter and dark energy, in addition to baryons and radiation, as main contributors to the total energy density of the universe.

The recent Planck satellite mission has played a major role in estimating various cosmological parameters. Planck

CMB measures parameters. Seeded by initial conditions, feels gravity - backlight of the universe's aquarium.

2 Introduction

Numerous CMB experiments including Planck. Planck specifications, LCDM successfully explains measured anisotropy. Tight constraints on cosmological parameters, as well as inflationary scenarios.

(Primordial non-Gaussianity and bispectrum?)

Upcoming CMB experiments. Simons and CMB-stage 4.

The need for development of bispectrum routines suitable for new specifications...

## 1.2 The homogenous universe

## **General Relativity**

The revolutionary idea of Einstein was that spacetime is be bent by its contents. Physical distance in the curved spacetime is represented using a metric tensor g;

$$ds^2 = g_{\mu\nu} \, dx^{\mu} dx^{\nu}, \tag{1.1}$$

where the Greek letters  $\mu, \nu = 0, 1, 2, 3$  denote time (0) and spatial (1-3) indices. Flat spacetime has metric  $g_{\mu\nu} = \eta_{\mu\nu} = \text{diag}\{-1, 1, 1, 1\}$ , known as the Minkowski metric. Throughout this thesis we adopt the sign convention (-, +, +, +) and work in units where c = 1. Unless specified otherwise, the Einstein summation convention is assumed.

In curved spacetime, a comoving (free-falling) object may follow a non-trivial trajectory given by the geodesic equation;

$$\frac{d^2x^{\mu}}{ds^2} + \Gamma^{\mu}_{\nu\rho} \frac{dx^{\nu}}{ds} \frac{dx^{\rho}}{ds} = 0, \tag{1.2}$$

with s an affine parameter parametrising the trajectory, and  $\Gamma^{\mu}_{\nu\rho}$  the Christoffel symbol representing metric connection. Its values are given in terms of the metric tensor by

$$\Gamma^{\mu}_{\nu\rho} = \frac{1}{2} g^{\mu\sigma} \left( g_{\nu\sigma,\rho} + g_{\rho\sigma,\nu} - g_{\nu\rho,\sigma} \right). \tag{1.3}$$

Here and for rest of this thesis, commas in subscripts  $(\cdot)_{,\mu}$  denote partial derivatives with respect to given index  $\mu$ .  $g^{\nu\sigma}$  is the inverse metric satisfying  $g^{\mu\nu}g_{\nu\rho}=\delta^{\nu}_{\rho}$ .

# Chapter 2

# My second chapter

## 2.1 Reasonably long section title

I'm going to randomly include a picture Figure 2.1.

If you have trouble viewing this document contact Krishna at: kks32@cam.ac.uk or raise an issue at https://github.com/kks32/phd-thesis-template/

#### **Enumeration**

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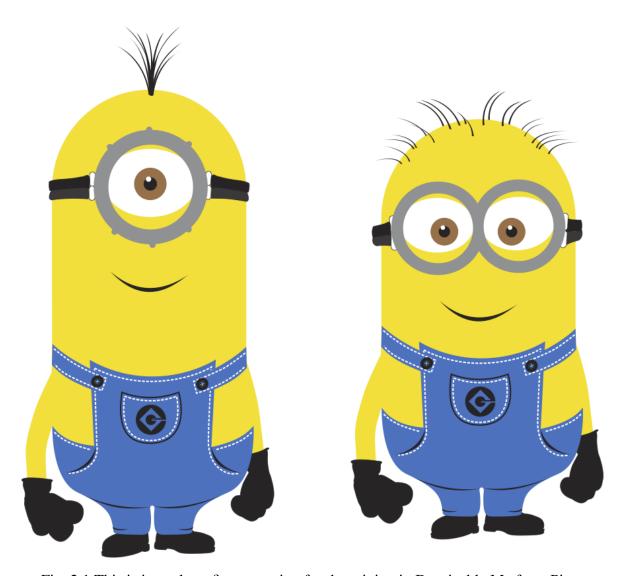


Fig. 2.1 This is just a long figure caption for the minion in Despicable Me from Pixar

2.1 Short title 5

porttitor, quam sem luctus massa, eu fermentum arcu diam ac massa. Praesent ut quam id leo molestie rhoncus. Praesent nec odio eget turpis bibendum eleifend non sit amet mi. Curabitur placerat finibus velit, eu ultricies risus imperdiet ut. Suspendisse lorem orci, luctus porta eros a, commodo maximus nisi.

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- 1. The first topic is dull
- 2. The second topic is duller
  - (a) The first subtopic is silly
  - (b) The second subtopic is stupid
- 3. The third topic is the dullest

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## **Itemize**

- The first topic is dull
- The second topic is duller
  - The first subtopic is silly
  - The second subtopic is stupid
- The third topic is the dullest

## **Description**

The first topic is dull

The second topic is duller

**The first subtopic** is silly

The second subtopic is stupid

The third topic is the dullest

2.2 Hidden section 7

#### 2.2 Hidden section

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Etiam elementum tristique lacus, sit amet eleifend nibh eleifend sed <sup>1</sup>. Maecenas dapibu augue ut urna malesuada, non tempor nibh mollis. Donec sed sem sollicitudin, convallis velit aliquam, tincidunt diam. In eu venenatis lorem. Aliquam non augue porttitor tellus faucibus porta et nec ante. Proin sodales, libero vitae commodo sodales, dolor nisi cursus magna, non tincidunt ipsum nibh eget purus. Nam rutrum tincidunt arcu, tincidunt vulputate mi sagittis id. Proin et nisi nec orci tincidunt auctor et porta elit. Praesent eu dolor ac magna cursus euismod. Integer non dictum nunc.

<sup>&</sup>lt;sup>1</sup>My footnote goes blah blah blah! ...



Fig. 2.2 Best Animations

I can cite Wall-E (see Fig. 2.2b) and Minions in despicable me (Fig. 2.2c) or I can cite the whole figure as Fig. 2.2

# Chapter 3

# My third chapter

## 3.1 First section of the third chapter

And now I begin my third chapter here ...

And now to cite some more people??

#### 3.1.1 First subsection in the first section

... and some more

#### 3.1.2 Second subsection in the first section

... and some more ...

#### First subsub section in the second subsection

... and some more in the first subsub section otherwise it all looks the same doesn't it? well we can add some text to it ...

#### 3.1.3 Third subsection in the first section

... and some more ...

#### First subsub section in the third subsection

... and some more in the first subsub section otherwise it all looks the same doesn't it? well we can add some text to it and some more and some more...

My third chapter

#### Second subsub section in the third subsection

... and some more in the first subsub section otherwise it all looks the same doesn't it? well we can add some text to it ...

## 3.2 Second section of the third chapter

and here I write more ...

## 3.3 The layout of formal tables

This section has been modified from "Publication quality tables in LATEX\*" by Simon Fear.

The layout of a table has been established over centuries of experience and should only be altered in extraordinary circumstances.

When formatting a table, remember two simple guidelines at all times:

- 1. Never, ever use vertical rules (lines).
- 2. Never use double rules.

These guidelines may seem extreme but I have never found a good argument in favour of breaking them. For example, if you feel that the information in the left half of a table is so different from that on the right that it needs to be separated by a vertical line, then you should use two tables instead. Not everyone follows the second guideline:

There are three further guidelines worth mentioning here as they are generally not known outside the circle of professional typesetters and subeditors:

- 3. Put the units in the column heading (not in the body of the table).
- 4. Always precede a decimal point by a digit; thus 0.1 not just .1.
- 5. Do not use 'ditto' signs or any other such convention to repeat a previous value. In many circumstances a blank will serve just as well. If it won't, then repeat the value.

A frequently seen mistake is to use '\begin{center}' ... '\end{center}' inside a figure or table environment. This center environment can cause additional vertical space. If you want to avoid that just use '\centering'

Table 3.1 A badly formatted table

	Species I		Species II	
Dental measurement	mean	SD	mean	SD
I1MD	6.23	0.91	5.2	0.7
I1LL	7.48	0.56	8.7	0.71
I2MD	3.99	0.63	4.22	0.54
I2LL	6.81	0.02	6.66	0.01
CMD	13.47	0.09	10.55	0.05
CBL	11.88	0.05	13.11	0.04

Table 3.2 A nice looking table

Dental measurement	Species I		Species II	
Dentai measurement	mean	SD	mean	SD
I1MD	6.23	0.91	5.2	0.7
I1LL	7.48	0.56	8.7	0.71
I2MD	3.99	0.63	4.22	0.54
I2LL	6.81	0.02	6.66	0.01
CMD	13.47	0.09	10.55	0.05
CBL	11.88	0.05	13.11	0.04

Table 3.3 Even better looking table using booktabs

Dental measurement	Species I		Species II	
	mean	SD	mean	SD
I1MD	6.23	0.91	5.2	0.7
I1LL	7.48	0.56	8.7	0.71
I2MD	3.99	0.63	4.22	0.54
I2LL	6.81	0.02	6.66	0.01
CMD	13.47	0.09	10.55	0.05
CBL	11.88	0.05	13.11	0.04

# References

- [1] Dicke, R. H., Peebles, P. J. E., Roll, P. G., and Wilkinson, D. T. (1965). Cosmic black-body radiation. *The Astrophysical Journal*, 142:414–419.
- [2] Penzias, A. A. and Wilson, R. W. (1965). A measurement of excess antenna temperature at 4080 mc/s. *The Astrophysical Journal*, 142:419–421.

# **Appendix A**

# How to install LATEX

## Windows OS

## **TeXLive package - full version**

- 1. Download the TeXLive ISO (2.2GB) from https://www.tug.org/texlive/
- 2. Download WinCDEmu (if you don't have a virtual drive) from http://wincdemu.sysprogs.org/download/
- 3. To install Windows CD Emulator follow the instructions at http://wincdemu.sysprogs.org/tutorials/install/
- 4. Right click the iso and mount it using the WinCDEmu as shown in http://wincdemu.sysprogs.org/tutorials/mount/
- 5. Open your virtual drive and run setup.pl

or

## Basic MikTeX - TEX distribution

- Download Basic-MiKTEX(32bit or 64bit) from http://miktex.org/download
- 2. Run the installer
- 3. To add a new package go to Start » All Programs » MikTex » Maintenance (Admin) and choose Package Manager

4. Select or search for packages to install

## TexStudio - TeX editor

- Download TexStudio from http://texstudio.sourceforge.net/#downloads
- 2. Run the installer

## Mac OS X

## MacTeX - TEX distribution

- Download the file from https://www.tug.org/mactex/
- 2. Extract and double click to run the installer. It does the entire configuration, sit back and relax.

## TexStudio - TEX editor

- Download TexStudio from http://texstudio.sourceforge.net/#downloads
- 2. Extract and Start

## **Unix/Linux**

## TeXLive - T<sub>E</sub>X distribution

#### **Getting the distribution:**

- 1. TexLive can be downloaded from http://www.tug.org/texlive/acquire-netinstall.html.
- 2. TexLive is provided by most operating system you can use (rpm,apt-get or yum) to get TexLive distributions

#### **Installation**

1. Mount the ISO file in the mnt directory

```
mount -t iso9660 -o ro, loop, noauto /your/texlive###.iso /mnt
```

- 2. Install wget on your OS (use rpm, apt-get or yum install)
- 3. Run the installer script install-tl.

```
cd /your/download/directory
./install-tl
```

- 4. Enter command 'i' for installation
- 5. Post-Installation configuration: http://www.tug.org/texlive/doc/texlive-en/texlive-en.html#x1-320003.4.1
- 6. Set the path for the directory of TexLive binaries in your .bashrc file

#### For 32bit OS

For Bourne-compatible shells such as bash, and using Intel x86 GNU/Linux and a default directory setup as an example, the file to edit might be

```
edit $~/.bashrc file and add following lines
PATH=/usr/local/texlive/2011/bin/i386-linux:$PATH;
export PATH
MANPATH=/usr/local/texlive/2011/texmf/doc/man:$MANPATH;
export MANPATH
INFOPATH=/usr/local/texlive/2011/texmf/doc/info:$INFOPATH;
export INFOPATH
```

#### For 64bit OS

```
edit $~/.bashrc file and add following lines
PATH=/usr/local/texlive/2011/bin/x86_64-linux:$PATH;
export PATH
MANPATH=/usr/local/texlive/2011/texmf/doc/man:$MANPATH;
export MANPATH
```

INFOPATH=/usr/local/texlive/2011/texmf/doc/info:\$INFOPATH;
export INFOPATH

#### Fedora/RedHat/CentOS:

```
sudo yum install texlive
sudo yum install psutils
```

#### **SUSE:**

sudo zypper install texlive

#### **Debian/Ubuntu:**

```
sudo apt-get install texlive texlive-latex-extra
sudo apt-get install psutils
```

# Appendix B

# Installing the CUED class file

LATEX.cls files can be accessed system-wide when they are placed in the <texmf>/tex/latex directory, where <texmf> is the root directory of the user's TeXinstallation. On systems that have a local texmf tree (<texmflocal>), which may be named "texmf-local" or "localtexmf", it may be advisable to install packages in <texmflocal>, rather than <texmf> as the contents of the former, unlike that of the latter, are preserved after the LATeXsystem is reinstalled and/or upgraded.

It is recommended that the user create a subdirectory <texmf>/tex/latex/CUED for all CUED related LATeXclass and package files. On some LATeXsystems, the directory look-up tables will need to be refreshed after making additions or deletions to the system files. For TeXLive systems this is accomplished via executing "texhash" as root. MIKTeXusers can run "initexmf -u" to accomplish the same thing.

Users not willing or able to install the files system-wide can install them in their personal directories, but will then have to provide the path (full or relative) in addition to the filename when referring to them in LATEX.