Brady Bulls

Question 1

$$U_{(c_1,c_2)} = \ln(C_t) + \beta\ln(C_{t+1})$$

i) $P_1\ Y_t = C_t + A_t$, $P_2: A_{t+1} = C_{t+1}$

$A_t(1+r) = C_{t+1})$

$A_t = \frac{C_{t+1}}{(1+r)}$

Intertemporal

$$\boxed{Y_t = C_t + \frac{C_{t+1}}{(1+r)}}$$

ii) $$\underset{C_t, C_{t+1}, \lambda}{\mathcal{L}} = \ln C_t + \beta(\ln C_{t+1}) - \lambda\left(C_t + \frac{C_{t+1}}{(1+r)} - Y_t\right)$$

$$\frac{d\mathcal{L}}{dC_t} = \frac{1}{C_t} - \lambda = 0, \quad \frac{1}{C_t} = \lambda$$

$$\frac{d\mathcal{L}}{dC_{t+1}} = \frac{\beta}{C_{t+1}} - \frac{\lambda}{(1+r)} = 0, \quad \frac{\beta}{C_{t+1}} = \frac{\lambda}{(1+r)}, \quad \lambda = \frac{\beta(1+r)}{C_{t+1}}$$

$$\frac{d\mathcal{L}}{d\lambda} = -C_t - \frac{C_{t+1}}{(1+r)} + Y_t = 0$$

$$\frac{\beta(1+r)}{C_{t+1}} = \frac{1}{C_t}, \quad C_t \cdot \beta(1+r) = C_{t+1}, \quad \boxed{\frac{C_{t+1}}{C_t} = \beta(1+r)}$$

Question 2

$$U(c_1, c_2) = \frac{c_1^{1-\sigma}}{1-\sigma} + \beta \frac{c_2^{1-\sigma}}{1-\sigma}$$

Period 1: $\hat{w}_t = c_t + s_t$, Period 2: $\hat{w}_{t+1} + s_t(1+r) = c_{t+1}$

$$s_t = \frac{c_{t+1}}{(1+r)} - \frac{\hat{w}_{t+1}}{(1+r)}$$

Intertemporal:

$$\hat{w}_t = c_t + \frac{c_{t+1}}{(1+r)} - \frac{\hat{w}_{t+1}}{(1+r)},$$

$$c_t + \frac{c_{t+1}}{(1+r)} = \hat{w}_t + \frac{\hat{w}_{t+1}}{(1+r)}$$

$$\mathcal{L}_{c_t, c_{t+1}, \lambda} = \frac{c_1^{1-\sigma}}{1-\sigma} + \beta \frac{c_{t+1}^{1-\sigma}}{1-\sigma} - \lambda\left(c_t + \frac{c_{t+1}}{(1+r)} - \hat{w}_t - \frac{w_{t+1}}{(1+r)}\right)$$

$$\frac{d\mathcal{L}}{dc_t} = c_t^{-\sigma} - \lambda = 0, \quad c_t^{-\sigma} = \lambda$$

$$\frac{d\mathcal{L}}{dc_{t+1}} = \beta \cdot c_{t+1}^{-\sigma} - \frac{\lambda}{(1+r)} = 0, \quad \beta \cdot c_{t+1}^{-\sigma} = \frac{\lambda}{(1+r)}$$

$$\frac{d\mathcal{L}}{d\lambda} = -c_t - \frac{c_{t+1}}{(1+r)} + \hat{w}_t + \frac{w_{t+1}}{(1+r)} = 0$$

$$\beta \cdot c_{t+1}^{-\sigma} = \frac{c_t^{-\sigma}}{(1+r)}$$

$$\frac{c_t^{-\sigma}}{c_{t+1}^{-\sigma}} = \beta \cdot (1+r)$$

$$\left(\frac{c_t}{c_{t+1}}\right)^{-\sigma} = \beta \cdot (1+r)$$

$$\left(\frac{c_t}{c_{t+1}}\right)^{\sigma} = \frac{1}{\beta \cdot (1+r)}$$

$$\boxed{1 = \beta \cdot \left(\frac{c_t}{c_{t+1}}\right)^{\sigma} \cdot (1+r)}$$

Question 3:

$$U_{(c_1, c_2)} = \log(c_1) + \log(c_2)$$

Period 1:
$$C_1 + S_1 = W_1 - \tau_1$$

Period 2:
$$C_2 = S_1(1+r) + W_2 - \tau_2$$

$$(W_1, W_2) = (100, 25)$$
$$(\tau_1, \tau_2) = (40, 5)$$
$$(g_1, g_2) = (40, 5)$$

government ITC:
$$g_t + (1-r_{t-1})b_{t-1} = \tau_t + b_1$$
$$b_0 = 0, \quad b_2 = 0$$

a) The price that agents optimize & markets clear,

markets clear when $b_1 = S_1$

goods market: $C_t + g_t = W_t$

b) $C_1 + S_1 = 100 - 40$, $C_2 = S_1(1+r) + 25 - 5$

$C_1 + S_1 = 60$, $C_2 - S_1(1+r) = 20$

$$\frac{C_2}{(1+r)} - \frac{20}{(1+r)} = S_1$$

$$C_1 + \frac{C_2}{(1+r)} - \frac{20}{(1+r)} = 60, \quad C_1 + \frac{C_2}{(1+r)} = 60 + \frac{20}{(1+r)}$$

$$\begin{bmatrix} C_1 = 60 \\ C_2 = 20 \end{bmatrix}$$

$$40 + (1-r)0 = 40 + b_1$$
$$b_1 = 0, \quad S_1 = 0$$

$$\mathcal{L}_{C_1, C_2, \lambda} = \log C_1 + \log C_2 - \lambda\left(C_1 + \frac{C_2}{(1+r)} - 60 - \frac{20}{(1+r)}\right)$$

$$\frac{d\mathcal{L}}{dC_1} = \frac{1}{C_1} = \lambda$$

$$\frac{d\mathcal{L}}{dC_2} = \frac{1}{C_2} = \frac{\lambda}{(1+r)}$$

$$\frac{(1+r)}{C_2} = \frac{1}{C_1} \quad C_1(1+r) = C_2$$

$$\frac{d\mathcal{L}}{d\lambda} = -C_1 - \frac{C_2}{(1+r)} + 60 - \frac{20}{(1+r)} = 0, \quad 60 - \frac{20}{(1+r)} = C_1 + \frac{C_1(1+r)}{(1+r)}$$

$$60 - \frac{20}{(1+r)} = 2C_1, \quad C_1 = 30 - \frac{10}{(1+r)}$$

$$C_2 = (1+r)\left(30 - \frac{10}{(1+r)}\right)$$

$$C_2 = 30(1+r) - 10$$

$C_1 = 30 + \frac{10}{(1+r)}$

$C_2 = 30(1+r) + 10$

$S_1 = W_1 - \tau_1 - C_1$

$S_1 = 100 - 40 - C_1$

$S_1 = 60 - 30 - \frac{10}{(1+r)}$

$0 = 60 - 30 - \frac{10}{(1+r)}$

$30 = \frac{10}{(1+r)}$

$1+r = 1/3,\ 0.333$

$C_1 = 60$

$C_2 = 20$

$S_1 = 0$

$$\boxed{\begin{aligned}(C_1, C_2) &= (60, 20)\\ (W_1, W_2) &= (100, 25)\\ (\tau_1, \tau_2) &= (40, 5)\\ (g_1, g_2) &= (40, 5)\\ (1+r) &= 0.333\end{aligned}}$$

c) new:

$(\tau_1, \tau_2) = (20,\ 5 + 20(0.333))$

$(20,\ 11.666)$

$C_1 + S_1 = W_1 - \tau_1,\quad C_2 = S_1(1+r) + W_2 - \tau_2$

$C_1 + S_1 = 100 - 20,\quad C_2 = S_1(1+r) + 25 + 11.666$

$\frac{C_2}{(1+r)} - \frac{13.334}{(1+r)} = S_1$

$C_1 + \frac{C_2}{(1+r)} + \frac{13.334}{(1+r)}) = 80$

$\mathcal{L}_{C_1, C_2, \lambda} = \log C_1 + \log C_2 - \lambda\left(C_1 + \frac{C_2}{(1+r)} - 80 - \frac{13.334}{(1+r)}\right)$

$C_1 = 40 + \frac{6.667}{(1+r)},\quad C_2 = 40(1+r) + 6.667$

$S_1 = 40 - \frac{6.667}{(1+r)}$

$20 = 40 - \frac{6.667}{(1+r)}$

$20 = \frac{6.667}{(1+r)}$

$\boxed{(1+r) = 1/3}$

$g_1 + (1+r_0)b_0 = \tau_1 + b_1$

$40 = 20 + b_1$

$b_1 = 20$

Plus, $(C_1, C_2) = (60, 20)$

$(W_1, W_2) = (100, 25)$

$(\tau_1, \tau_2) = (20, 11.666)$

$(g_1, g_2) = (40, 5)$

$(b_1, b_2) = (20, 0)$

d) The government changed its tax policy w/o changing its spending policy, so no actual changes happen. The government borrowed money, to cover the lowered taxes, so agents saved more to account for those changes in the future.

e) The budgent constraints don't change. They are still limited by the same wage & tax parameters, and since those stay the same, the whole system stays the same.