Homework #5 Brady Bulls

a) Period 1

$$C_1 + S_1 = w_1(1-\tau_1)\ell_1$$

Period 2

$$C_2 = \left(S_1(1+r_1) + w_2(1-\tau_2)\ell_2\right)\frac{1}{1+r}$$

$$\frac{1}{1+r}C_2 = S_1 + \frac{1}{1+r}\left(w_2(1-\tau_2)\ell_2\right)$$

combine

$$\frac{C_2}{1+r} - \frac{w_2(1-\tau_2)\ell_2}{1+r} = S_1$$

$$C_1 + \frac{C_2}{1+r} - \frac{w_2(1-\tau_2)\ell_2}{1+r} = w_1(1-\tau_1)\ell_1$$

$$\boxed{C_1 + \frac{1}{1+r}C_2 = w_1(1-\tau_2)\ell_1 + \frac{1}{1+r}\cdot w_2(1-\tau_2)\ell_2}$$

b) $$\mathcal{L}_{C_1,C_2,\ell_1,\ell_2,\lambda} = \ln C_1 + b\frac{(1-\ell_1)^{1-\gamma}}{1-\gamma} + e^{-\rho}\left[\ln C_2 + \frac{(1-\ell_2)^{1-\gamma}}{1-\gamma}\right] + \lambda\left[w_1(1-\ell_1) + \frac{1}{1+r}w_2(1-\tau_2)\ell_2 - C_1 - \frac{1}{1+r}C\right.$$

$$\frac{d\mathcal{L}}{dC_1} = \frac{1}{C_1} - \lambda = 0, \quad \frac{1}{C_1} = \lambda$$

$$\frac{d\mathcal{L}}{dC_2} = \frac{e^{-\rho}}{C_2} - \frac{\lambda}{1+r} = 0$$

$$\frac{d\mathcal{L}}{d\ell_1} \quad u(\ell_1) = 1-\ell_1, \; u'(\ell_1) = -1$$

$$v(\ell_1) = u(\ell_1)^{1-\gamma} = (1-\gamma)u(\ell_1)^{-\gamma}\cdot\frac{du}{d\ell_1} = (1-\gamma)\cdot(1-\ell_1)^{-\gamma}\cdot(-1)$$

$$= -(1-\gamma)\cdot(1-\ell_1)^{-\gamma} \to b\cdot\left[\frac{-\cancel{(1-\gamma)}(1-\ell_1)^{-\gamma}}{\cancel{(1-\gamma)}}\right] =$$

$$\to -b(1-\ell_1)^{-\gamma} + \lambda w_1(1-\tau_1) = 0$$

$$\frac{d\mathcal{L}}{d\ell_2} = -e^{-\rho}(1-\ell_2)^{-\gamma} + \lambda w_2(1-\tau_2)\cdot\frac{1}{1+r} = 0$$

$$\frac{d\mathcal{L}}{d\lambda} = w_1(1-\ell_1) + \frac{1}{1+r}w_2(1-\tau_2)\ell_2 - C_1 - \frac{1}{1+r}C_2$$

(continued on next page)

List of part b functions

$$\frac{1}{c_1} = \lambda, \quad \frac{e^{-\rho}}{c_2} = \lambda \cdot \frac{1}{1+r}, \quad b(1-l_1)^{-\gamma} = \lambda(w_1(1-\tau_1)), \quad e^{-\rho}(1-l_2)^{-\gamma} = \frac{\lambda w_2(1-\tau_2)}{1+r}$$

b cont.)

$$\frac{b(1-l_1)^{-\gamma}}{w_1(1-\tau_1)} = \lambda, \quad e^{-\rho}(1-l_2)^{-\gamma} = \lambda \cdot \frac{w_2(1-\tau_2)}{1+r}$$

$$e^{-\rho}(1-l_2)^{-\gamma} = \frac{b(1-l_1)^{-\gamma}}{w_1(1-\tau_1)} \cdot \frac{w_2(1-\tau_2)}{1+r}$$

$$(1-l_2)^{-\gamma} = \frac{1}{e^{-\rho}} \cdot \frac{b}{1} \cdot \frac{(1-l_1)^{-\gamma}}{1} \cdot \frac{1}{w_1(1-\tau_1)} \cdot \frac{w_2(1-\tau_2)}{1} \cdot \frac{1}{1+r}$$

$$\boxed{\frac{(1-l_2)^{-\gamma}}{(1-l_1)^{-\gamma}} = \frac{w_2(1-\tau_2)}{w_1(1-\tau_1)} \cdot \frac{b}{e^{-\rho}(1+r)}}$$

• Holding all others constant, an increase in the relative wage in one period leads to a corresponding increase in leisure for that same period.

c) Returning to the equation in part b, an increase in tax in one period would lead to a reduction in leisure for that same period. It appears, based on this utility function, that the consumer has a balanced preference towards work and leisure. If wages go up in one period, they're happy to work less to maintain the same consumption, but if taxes go up, they will work more, again to maintain steady consumption.

d) Referring to the equation derived in part b, gamma controls the elasticity of leisure. A high gamma would lead to a highly elastic leisure curve, meaning changes to wages or taxes would have a large impact on leisure. The inverse is also true, a low gamma causes changes in the right-hand side of the equation to barely change leisure.

e) Without resolving the entire problem again, it is plain to see that a lumpsum tax would disappear from the equation as partial derivatives come into play. Lump sum taxes, no matter the size, will not change the relative demand for leisure in anyway.