Vision and Image Processing: Camera Models

François Lauze
Department of Computer Science
University of Copenhagen

Plan for today

- Before we start: vectors and matrices.
- Motivation and a bit of history.
- A general Introduction to Camera Models, specifically the pinhole model.
- Notions of projection and projective geometry
- 2 Views and Epipolar Geometry.

Outline

Vectors

- A *n*-vector is a *n*-uple of real values:

$$v = \begin{pmatrix} x_1 \\ \dots \\ x_n \end{pmatrix}$$

- Addition

$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} + \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} x_1 + y_1 \\ \vdots \\ x_n + y_n \end{pmatrix}$$

- Multiplication by a scalar

$$\lambda \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} \lambda x_1 \\ \vdots \\ \lambda x_n \end{pmatrix}$$

Linear Mapping

- Mapping between vectors with only addition of coordinates, multiplications by scalar and no constant terms.
- Example
$$f\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}x+3y\\z-2x\end{pmatrix}$$
- Non linear example
$$g\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}x^2+3yz\\z-2x^2+1\end{pmatrix}$$

  There are powers and constant terms.

Linearity

- This means $f(v+\lambda v') = f(v) + \lambda f(v')$

$$
\begin{aligned}
f\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} + \lambda \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}\right) &= f\begin{pmatrix} x+\lambda x' \\ y+\lambda y' \\ z+\lambda z' \end{pmatrix} \\
&= \begin{pmatrix} x+\lambda x' + 3(y+\lambda y') \\ z+\lambda z' - 2(x+\lambda x') \end{pmatrix} \\
&= \begin{pmatrix} x+3y \\ z-2x \end{pmatrix} + \lambda \begin{pmatrix} x'+3y' \\ z'-2x' \end{pmatrix} \\
&= f\begin{pmatrix} x \\ y \\ z \end{pmatrix} + \lambda f \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}
\end{aligned}
$$

- f is linear.

Matrices

- A $n \times m$ matrix is an array of numbers with n rows and m columns
- A 2×3 matrix F

$$F = \begin{pmatrix} 1 & 3 & 0 \\ -2 & 0 & 1 \end{pmatrix}$$

- 2 matrices of the same size can be added together: just add the entries:

$$\begin{pmatrix} 1 & 3 & 0 \\ -2 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 4 & -2 & 1 \\ 7 & -3 & 0 \end{pmatrix} = \ ?$$

Product of a Matrix and a Vector

- A matrix of size $m \times n$ and a vector of length m can be multiplied to form a vector of length n.
- Formal rule:

$$A = \begin{pmatrix} a_{11} & \dots & a_{1m} \\ \vdots & \vdots & \vdots \\ a_{n1} & \dots & a_{nm} \end{pmatrix}, v = \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}$$

$$Av = \begin{pmatrix} a_{11}v_1 + a_{12}v_2 + \dots a_{1m}v_n \\ a_{21}v_1 + a_{22}v_2 + \dots a_{2m}v_n \\ \vdots \\ a_{n1}v_1 + a_{n2}v_2 + \dots a_{mn}v_n \end{pmatrix}$$

- Each line of A is multiplied in “inner product way” with v.

- Example: Compute the product of

$$A = \begin{pmatrix} 1 & 3 & 0 \\ -2 & 0 & 1 \end{pmatrix} \text{ and } v = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

- Example: Compute the product of

$$A = \begin{pmatrix} 1 & 3 & 0 \\ -2 & 0 & 1 \end{pmatrix} \text{ and } v = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

- We find precisely the value of

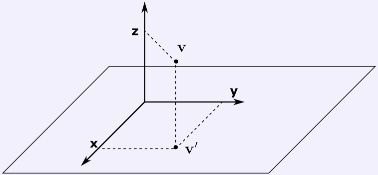
$$f\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x + 3y \\ -2x + z \end{pmatrix} = \begin{pmatrix} x + 3y \\ z - 2x \end{pmatrix}$$

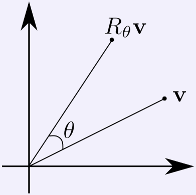
- Each linear mapping can be written that way. Often use the same notation for the matrix and the linear mapping.

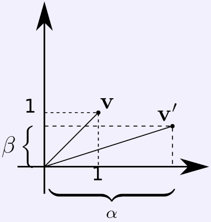
Matrices /linear mappings as geometric transformations

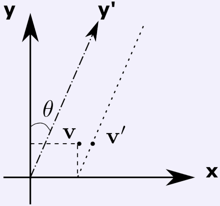
.

Projection on $x - y$ plane









Rotation of angle θ anisotropic scaling shear

- projection $\mathbb{R}^3 \to \mathbb{R}^2$

$$F\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}x\\y\end{pmatrix}, \quad F = \begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 0\end{pmatrix}$$

- Rotation of angle θ from $\mathbb{R}^2 \to \mathbb{R}^2$:

$$R_\theta\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}x\cos\theta - y\sin\theta\\x\sin\theta + y\cos\theta\end{pmatrix}, \quad R_\theta = \begin{pmatrix}\cos\theta & -\sin\theta\\\sin\theta & \cos\theta\end{pmatrix}$$

- Scaling by a factor α in x and β in y:

$$S\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}\alpha x\\\beta y\end{pmatrix}, \quad S = \begin{pmatrix}\alpha & 0\\0 & \beta\end{pmatrix}$$

- Shear of the y-axis with angle θ:

$$\mathcal{S}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}x + \sin\theta y\\y\end{pmatrix}, \quad \mathcal{S} = \begin{pmatrix}1 & \sin\theta\\0 & 1\end{pmatrix}$$

Product of Matrices

- A matrix of size $m \times n$ and a matrix of size $n \times p$ can be multiplied to produce a matrix of size $m \times p$.
- Algebraic rule: a_{ij} entry (i,j) of A, b_{jk} entry (j,k) of B

$$A = (a_{ij})_{\substack{i=1\dots m,\\ j=1\dots n}} \quad B = (b_{jk})_{\substack{j=1\dots n\\ k=1\dots p}}$$

  Denote entry (i,k) of product $C = AB$ by c_{ik}:

$$c_{ik} = \sum_{j=1}^{n} a_{ij} b_{jk}$$

- Matrix vector multiplication is in fact a special case of it!
- Example

$$\begin{pmatrix} 2 & 2 \\ 1 & 3 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 3 & 0 \\ -2 & 0 & 1 \end{pmatrix} =$$

Product of Matrices

- A matrix of size $m \times n$ and a matrix of size $n \times p$ can be multiplied to produce a matrix of size $m \times p$.
- Algebraic rule: a_{ij} entry (i,j) of A, b_{jk} entry (j,k) of B

$$A = (a_{ij})_{\substack{i=1\ldots m,\\ j=1\ldots n}} \quad B = (b_{jk})_{\substack{j=1\ldots n\\ k=1\ldots p}}$$

Denote entry (i,k) of product $C = AB$ by c_{ik}:

$$c_{ik} = \sum_{j=1}^{n} a_{ij} b_{jk}$$

- Matrix vector multiplication is in fact a special case of it!
- Example

$$\begin{pmatrix} 2 & 2 \\ 1 & 3 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 3 & 0 \\ -2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 6 & 2 \\ -5 & 3 & 3 \\ 3 & 3 & -1 \end{pmatrix}$$

- What does matrix multiplication means?

Meaning of the Product

- M and N the linear mappings $\begin{pmatrix} 2 & 2 \\ 1 & 3 \\ 1 & -1 \end{pmatrix}$ and $\begin{pmatrix} 1 & 3 & 0 \\ -2 & 0 & 1 \end{pmatrix}$
- Apply N to $\mathbf{v} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and M to the result:

$$N\mathbf{v} = \begin{pmatrix} 1 & 3 & 0 \\ -2 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x + 3y \\ -2x + z \end{pmatrix}$$

- and

$$\begin{pmatrix} 2 & 2 \\ 1 & 3 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x + 3y \\ -2x + z \end{pmatrix} =$$

Meaning of the Product

- M and N the linear mappings $\begin{pmatrix} 2 & 2 \\ 1 & 3 \\ 1 & -1 \end{pmatrix}$ and $\begin{pmatrix} 1 & 3 & 0 \\ -2 & 0 & 1 \end{pmatrix}$
- Apply N to $\mathbf{v} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and M to the result:

$$N\mathbf{v} = \begin{pmatrix} 1 & 3 & 0 \\ -2 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x + 3y \\ -2x + z \end{pmatrix}$$

- and

$$\begin{pmatrix} 2 & 2 \\ 1 & 3 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x + 3y \\ -2x + z \end{pmatrix} = \begin{pmatrix} -2x + 6y + 2z \\ -5x + 3y + 3z \\ 3x + 3y - z \end{pmatrix} =$$

Meaning of the Product

- M and N the linear mappings $\begin{pmatrix} 2 & 2 \\ 1 & 3 \\ 1 & -1 \end{pmatrix}$ and $\begin{pmatrix} 1 & 3 & 0 \\ -2 & 0 & 1 \end{pmatrix}$
- Apply N to $\mathbf{v} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and M to the result:

$$N\mathbf{v} = \begin{pmatrix} 1 & 3 & 0 \\ -2 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x + 3y \\ -2x + z \end{pmatrix}$$

- and

$$\begin{pmatrix} 2 & 2 \\ 1 & 3 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x + 3y \\ -2x + z \end{pmatrix} = \begin{pmatrix} -2x + 6y + 2z \\ -5x + 3y + 3z \\ 3x + 3y - z \end{pmatrix} = \begin{pmatrix} -2 & 6 & 2 \\ -5 & 3 & 3 \\ 3 & 3 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Matrix Product as Chain Application of Linear Mappings

- We found that

$$M\,(N\mathbf{v}) = \underbrace{MN}_{\text{Matrix product}}\;\mathbf{v}$$

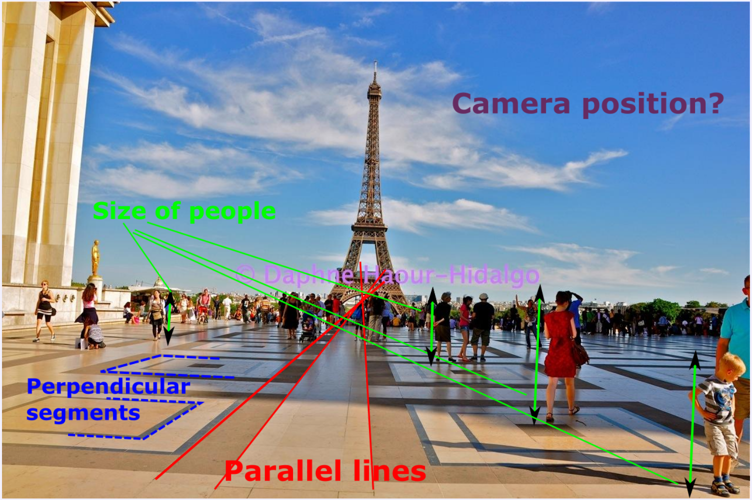
- Very Important Property: Matrix product corresponds to chain application (composition) of linear mappings!

Read the Linear Algebra Tutorial and Reference on Absalon!

This will also be useful for other courses!

Outline

Motivation


Camera position?
Size of people
© Daphne Haour-Hidalgo
Perpendicular
segments
Parallel lines

Questions

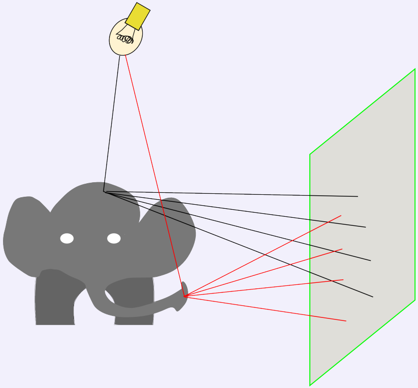
Previous picture raises some questions about:

- Lines?
- Parallelism?
- Angles / orthogonality?
- Sizes?
- Camera position / Horizon?

What happens when you take a picture (or Daphné Haour-Hidalgo in the previous case :-))

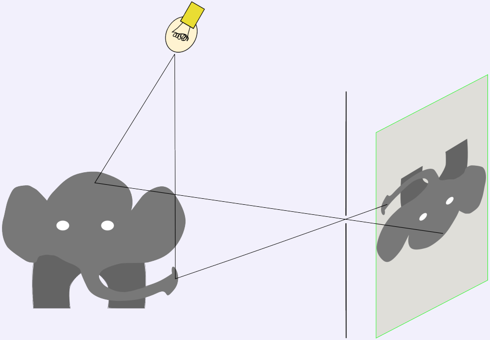
Outline

Getting an Image – I



Many rays emanating from the same position touch the image sensitive array at many location: big blur!

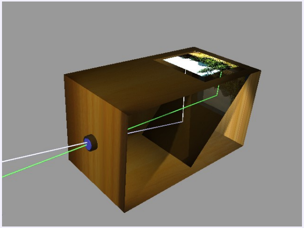
Getting an Image – II



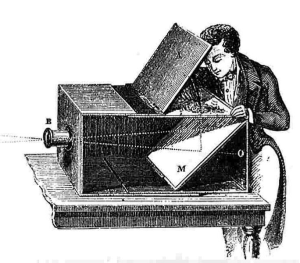
Filtering the rays via a pinhole: get an (inverted) image.
Principle of the Camera Obscura (dark room).

Outline

Camera Obscura



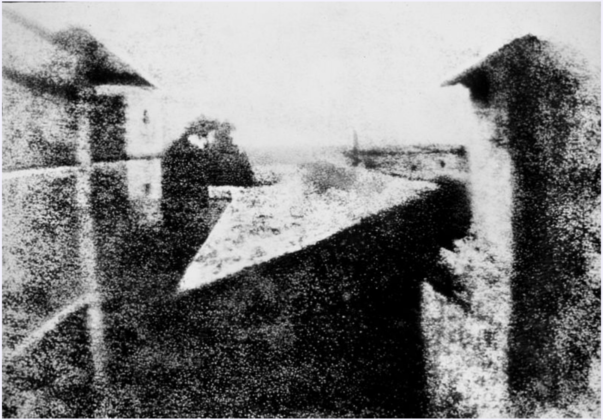
Principle of Camera Obscura



18th Century Camera Obscura

- Known from old chines writings
- Mentioned by Aristotle
- Plaque with photosensitive material: Photographic camera!

The Very First Photography, 1826



J.N. Nièpce, View from the window at Le Gras, Saint Loup de Varennes, France – Now at University of Texas at Austin.

The Pioneers

J. Nicéphore Nièpce

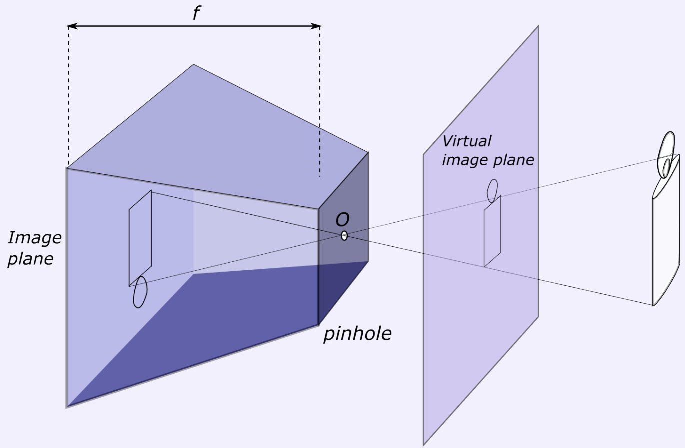
Louis Daguerre

Henri F. Talbot

Now...

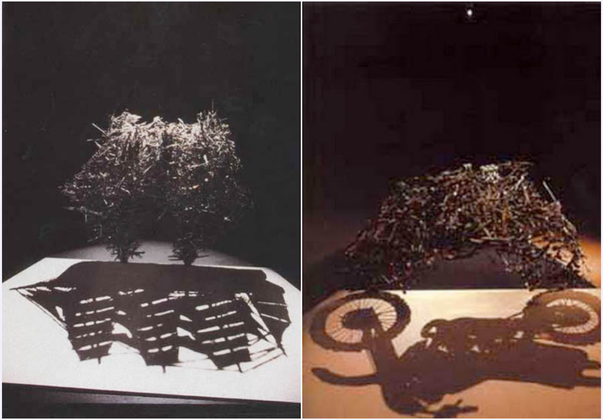
Outline

The Pinhole Camera Model



- f is the focal length,
- O is the camera center.

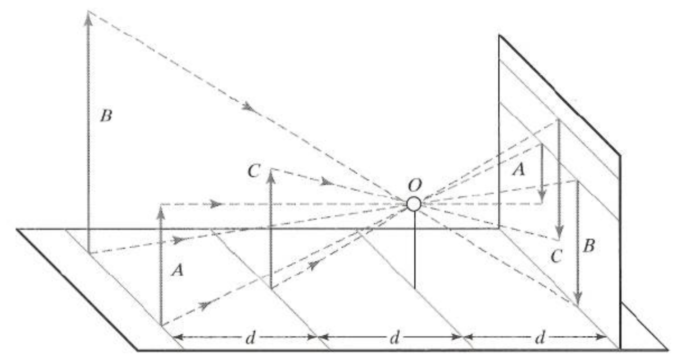
Projection is Tricky!



Some illusions from Shigeo Fukuda.

Perspective Effects

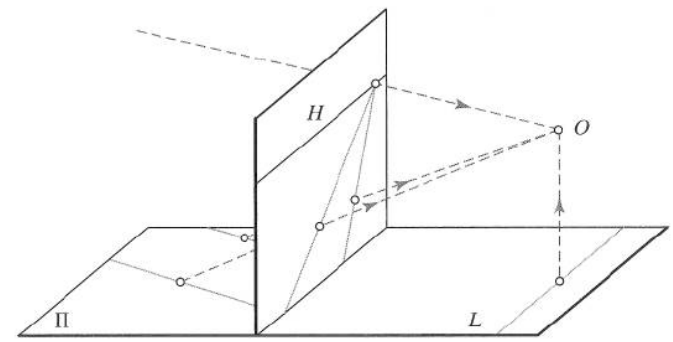
Remember from Kim's first lecture



Far objects appear smaller that close ones.

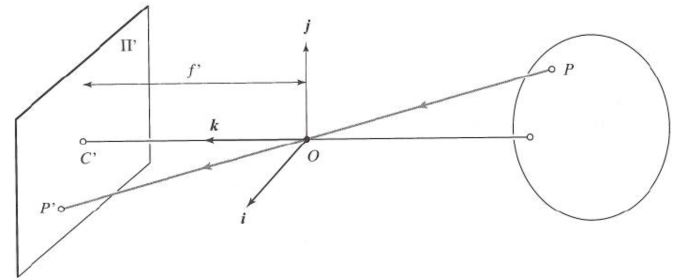
Perspective Effects

Remember from Kim's first lecture again



Images of parallel lines intersect at the horizon (virtual image plane).

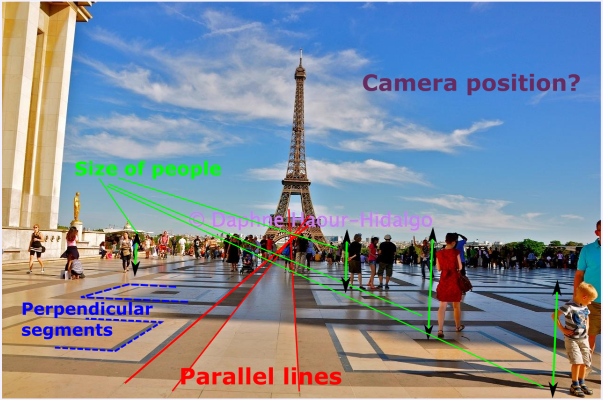
Projection Equations



- $P(x, y, z)$, $P'(x', y', z')$. P' in the image plane $\implies z' = f$
- Remember optical flow lecture: similar triangles:

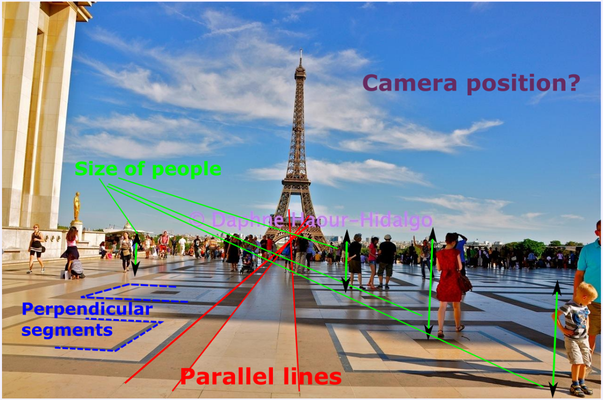
$$x' = f\frac{x}{z}, \quad y' = f\frac{y}{z}$$

Lost in projection



Angles, size, parallelism.

Preserved by projection

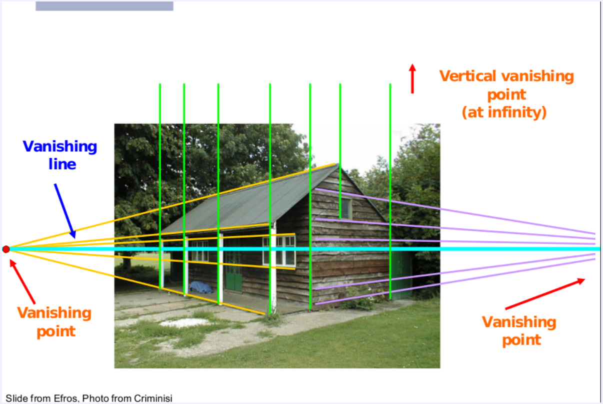


Straight lines.

Vanishing Points

Projections of parallel lines intersect at common points.

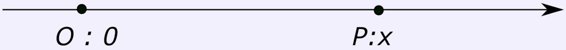
Vanishing line



Vanishing points intersect at a common line. Intersection can be at infinity.

Homogeneous coordinates 1D

- In 1D coordinate is just 1 number.



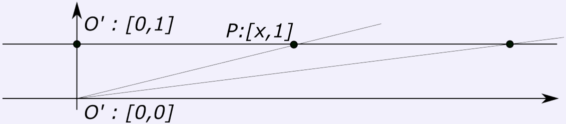
- 1D coordinate to 1D Homogeneous coordinates:

$$x \sim \begin{bmatrix} x \\ 1 \end{bmatrix}$$

- 1D homogeneous coordinate to 1 D coordinate

$$\begin{bmatrix} x \\ w \end{bmatrix} \sim x/w$$

- What can we do with that? we can “tame” infinity!
- A point with homogeneous coordinate $[x, 0]^T$ has “normal” coordinate $x/0 = \infty$ as if we took homogeneous coordinate $[\infty, 1]^T$



Homogeneous coordinates, 2D

"Natural Coordinates" for projective geometry

- From 2D point coordinate to 2D Homogeneous coordinate

$$(x, y) \Longrightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- From 2D homogeneous coordinates to 2D coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Longrightarrow (x/w, y/w)$$

- Exercise: P with coordinates $(x, y, 1)$. $Q(x', y', w)$ point on the line through $O(0, 0, 0)$ and P. Compute x' and y'. Considering $[x', y', w]$ as homogeneous 2D coordinates, what are the corresponding 2D coordinates?

Homogeneous coordinates, 3D

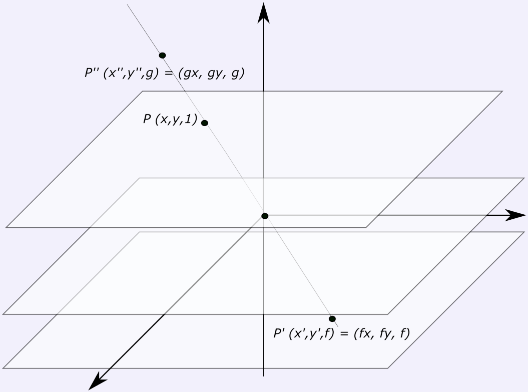
- From 3D point coordinate to 3D Homogeneous coordinate

$$(x, y, z) \Longrightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- From 3D homogeneous coordinates to 3D coordinates

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Longrightarrow (x/w, y/w, z/w)$$

Homogeneous coordinates



Homogeneous coordinates in 2D correspond to points in plane $z = 1$ but also to lines through the origin and this point.

Why Are They Useful

- Projection to image plane in standard coordinates:

$$P : (x, y, z) \mapsto P' : (f\frac{x}{z}, f\frac{y}{z})$$

- In homogeneous coordinates:

Why Are They Useful

- Projection to image plane in standard coordinates:

$$P : (x, y, z) \mapsto P' : (f\frac{x}{z}, f\frac{y}{z})$$

- In homogeneous coordinates:

$$P : \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \mapsto P' : \begin{bmatrix} fx \\ fy \\ z \end{bmatrix}$$

- Matrix notation

$$\begin{bmatrix} fx \\ fy \\ z \end{bmatrix} = \underbrace{\begin{pmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}}_{M} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- M is the Camera Matrix

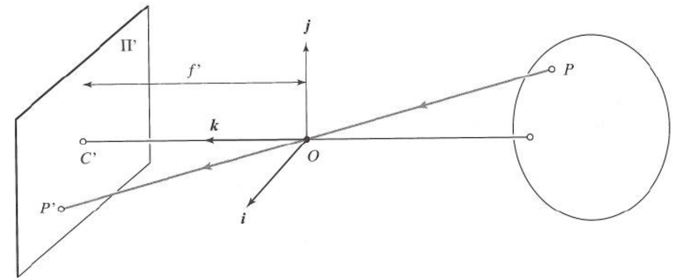
World,Camera and Image Coordinates

In the previous slides, Many coordinate systems are implicitly known:

- 3D World Coordinates: Coordinate system of the 3D world.
- Camera Coordinates: 3D coordinate system attached to the camera.
- Image Coordinates: 2D Coordinate system attached to the image plane.

Not that simple in practice!

Assumptions Behind the Simple Model



- The camera center 0 is the same as the world coordinates origin,
- The axes **i**, **j** and **k** are common for the camera and the world
- Pixels are squared and perfectly aligned with the camera coordinates.
- Image coordinates have their origin at C', projection of the camera center O.

Intrinsic vs Extrinsic Camera Parameters

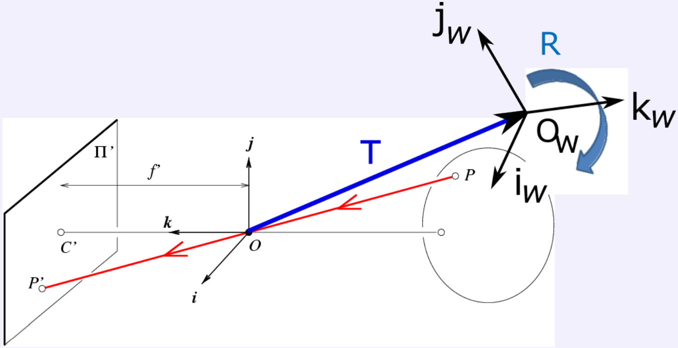
Intrinsic parameters refer to internal parameters:

- Position of the image center wrt projection of the camera center: 2 parameters
- Scale factors for the pixels sizes in both x and y directions: 2 parameters,
- Skewness of pixels: 1 parameter.

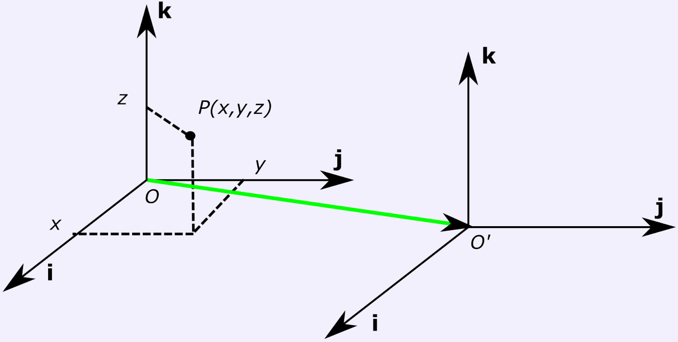
Extrinsic Camera parameters refer

- position of the camera coordinate system vs the world coordinates system: translation,
- orientation of the camera coordinate system vs the world coordinates system: rotation.

Oriented and Translated Camera



Translating Coordinate System



Assume O' at coordinates $(x_{O'}, y_{O'}, z_{O'})$ in coordinate system $(O, \mathbf{i}, \mathbf{j}, \mathbf{k})$ and P has coordinates (x, y, z) also in $(O, \mathbf{i}, \mathbf{j}, \mathbf{k})$. What are its coordinates in $(O', \mathbf{i}, \mathbf{j}, \mathbf{k})$?

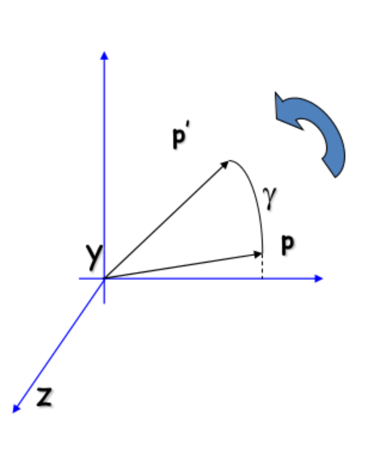
- Let (x', y', z') its coordinates in $(O, \mathbf{i}, \mathbf{j}, \mathbf{k})$.
- Write $\overrightarrow{O'P} = \overrightarrow{OP} - \overrightarrow{OO'}$
- Develop:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} x_{O'} \\ y_{O'} \\ z_{O'} \end{pmatrix} = \begin{pmatrix} x - x_{O'} \\ y - y_{O'} \\ z - z_{O'} \end{pmatrix}$$

- Transformation in homogeneous coordinates:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{pmatrix} 1 & 0 & 0 & -x_{O'} \\ 0 & 1 & 0 & -y_{O'} \\ 0 & 0 & 1 & -z_{O'} \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Rotating Coordinate System



Rotations along coordinate axes

$$R_{x\alpha} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{pmatrix}$$

$$R_{y\beta} = \begin{pmatrix} \cos\beta & 0 & -\sin\beta \\ 0 & 1 & 0 \\ \sin\beta & 0 & \cos\beta \end{pmatrix}$$

$$R_{y\gamma} = \begin{pmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Can also be written in homogeneous coordinates

Camera Matrix

- Combine world vs camera coordinates with
- Simple Camera matrix and
- Image plane transformation (axis scalings, shear, translation)

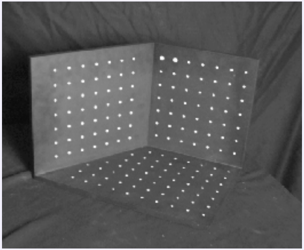
$$\mathbf{C} = \mathbf{K}\,[\mathbf{R}\ \mathbf{t}]$$

- $\mathbf{K}$ 3×3 matrix encoding the homogeneous transformations inside the camera: calibration matrix.
- $[\mathbf{R}\ \mathbf{t}]$ Concatenation of world coordinates rotation and origin translation to align camera and world coordinates.

$$w \begin{bmatrix} u \\ b \\ 1 \end{bmatrix} = \underbrace{\begin{pmatrix} \alpha & s & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{pmatrix}}_{\mathbf{K}} \underbrace{\begin{pmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{pmatrix}}_{[\mathbf{R}\ \mathbf{t}]} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Geometric Calibration

- Computing the camera matrix is called geometric calibration.
- Extrinsic parameters: usually easy.
- Calibration focuses more on intrinsic parameters (calibration matrix **K**).
- Often difficult.



- Use an object with known geometry
- Use vanishing points / lines
- Use other cues...

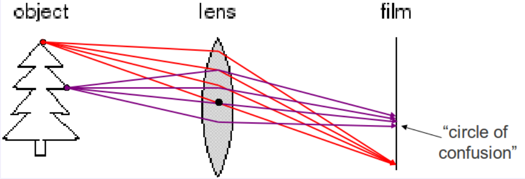
Without knowledge of object geometry, calibration can be very problematic (Ames Room illusion).

Outline

Shrinking the aperture

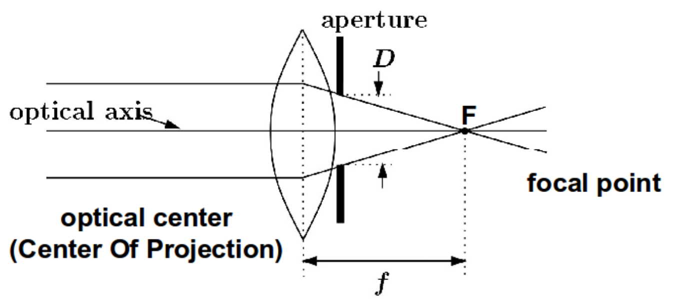
Less light in, diffraction.

Adding Lens



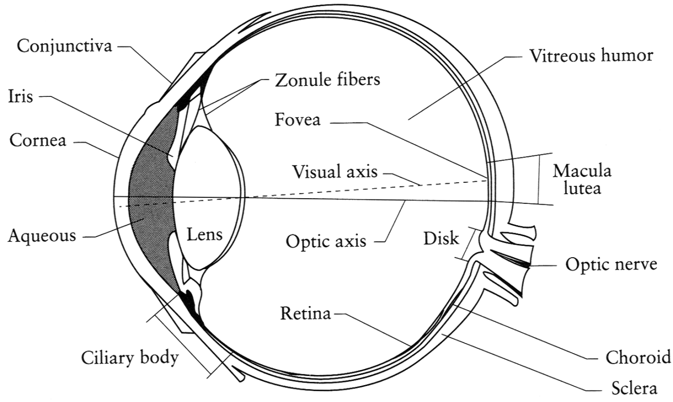
- Specific distance for which objects are in focus
- Changing shape of lens changes the focus distance.

Focal Length, Aperture, Depth of Field

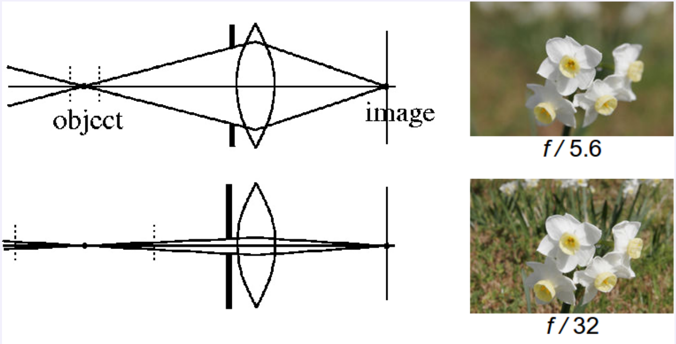


- Lens focuses parallel rays into a single point.
- Aperture restricts range of rays.

The Eye is a Camera with Lens



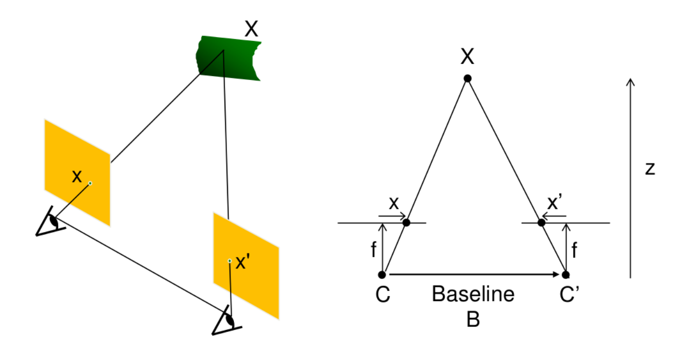
Depth of Field



Controlled by aperture size and focal length.

Outline

Multiple View Correspondences

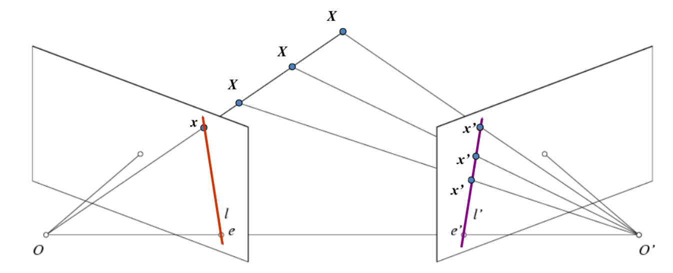


If we can recover x' from x we can recover depth.

Image Correspondence

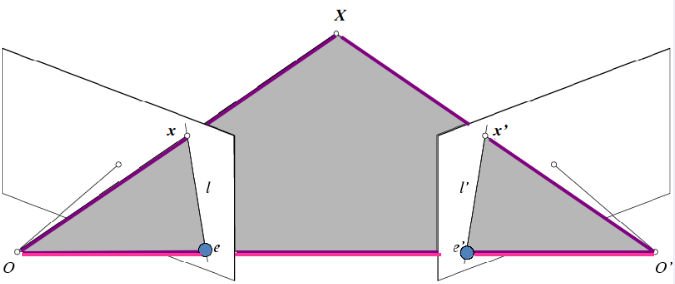
How do we match points from image 1 to image 2: you should have seen some of it with Kim, but this is not the end of the story!

Epipolar Constraints



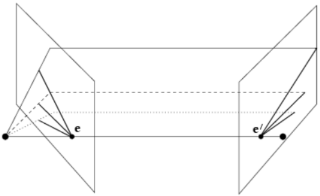
- Potential match for x must lie in corresponding line l'
- Potential match for x' must lie in corresponding line l

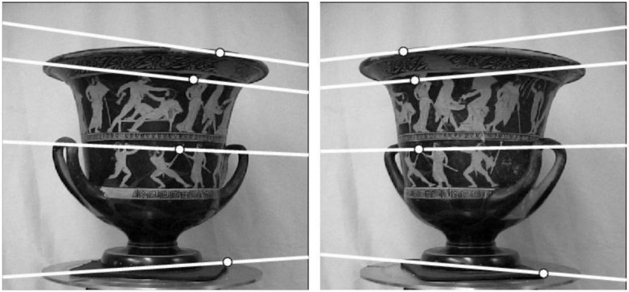
Epipolar Constraints



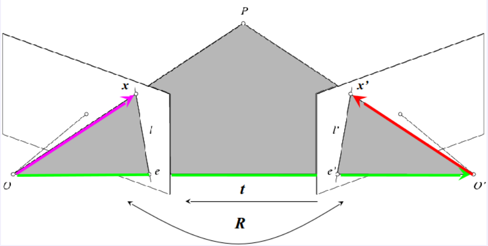
- Line connecting O and O': baseline
- Plane through baseline x and x': Epipolar Plane
- Epipoles: intersection of baseline and image planes: projection of the other camera center.
- Epipolar Lines - intersections of epipolar plane with image planes (always come in corresponding pairs)

Example: Converging cameras





Calibrated Case



Camera parameters known for the two cameras: calibration matrices K and K'

- x and x' (in 3D coords but not the same) are related by rotation and translation.

$$x' = R(x - \mathbf{t})$$

- Their homogeneous coordinates y and y' are related by a simple matrix E built from R and t

$$y^T E y = 0.$$

- E is called the essential matrix (Longuet-Higgins 1981).
- E can be estimated from images.
- The position and orientation of camera 1 vs camera 2 (i.e., R and $\mathbf{t}$) can be recovered from E.

Note on the geometric transformation between image 1 and image2

- The transformation between y and y' is not linear: it is an homography between the 2 image planes:
- An homography conserves straight lines, but not parallelism.
- Two parallel lines intersect at infinity: after an homography they may intersect at finite distance.

Uncalibrated Case

- No way to directly compare x and x': they relate through the unknown transformations K and K'.
- A relation however still exists (Faugeras, Luong, 1992)

$$y^T \underbrace{K^{-\top} E K'^{-1}}_{F} y = 0$$

- F is called the fundamental matrix.
- Though K and K' are unknown F can be esitimated (complicated) thus used for the correspondence problem!
- $K^\top$ is the transposed of K: inverse row and line indices. For a vector in column, write it in line.
- K^{-1} is the inverse of K: provides the inverse change of coordinates from unnormalized to normalized image coordinate.
- $K^{-\top}$ means inverse of the transposed matrix: The same as transposed of the inverse.