Vision and Image Processing: Disparity, Stereo Matching

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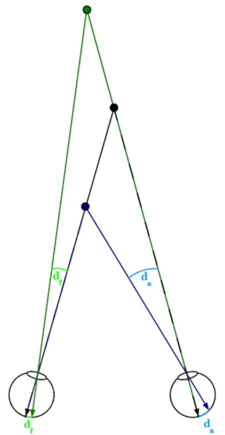
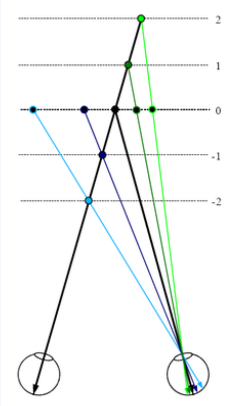
Topics for today's lecture

- Binocular Disparity
- Depth and Disparity
- Stereo Matching
- Dense Matching vs. Optical Flow

Binocular Disparity[1]



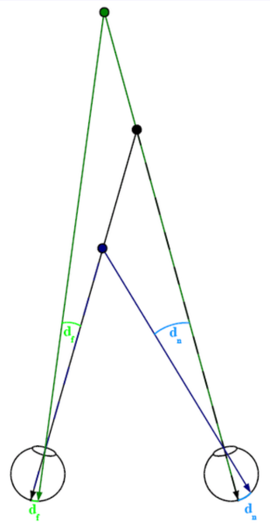
Binocular Disparity



Disparity simulation.

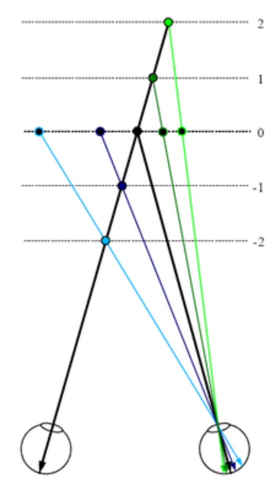
[1] Pictures from Wikipedia.

Binocular Disparity I



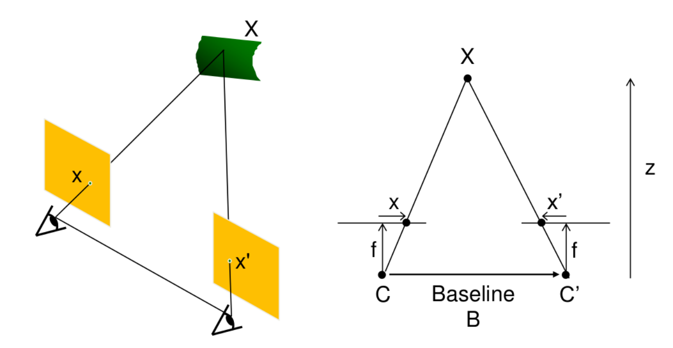
- Black dot: fixation point.
- Green dot: point with far disparity. Angle with direction of fixation point is small.
- Blue dot: point with near disparity. Angle with direction of fixation point is large.

Binocular Disparity II



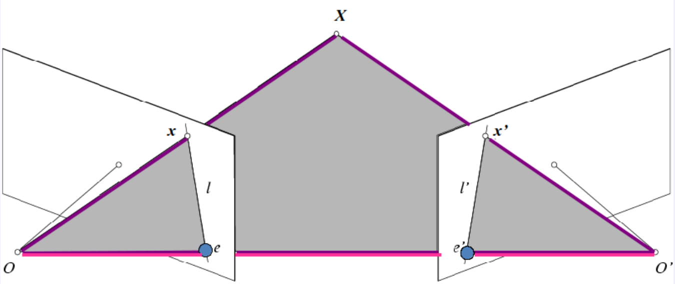
- Object at depth $\neq$ fixation depth.
- Can be simulated by presenting laterally shifted image to the other eye.
- Principle behind 3D movies.
- Some explanations to follow!

A Reminder from Last Week I



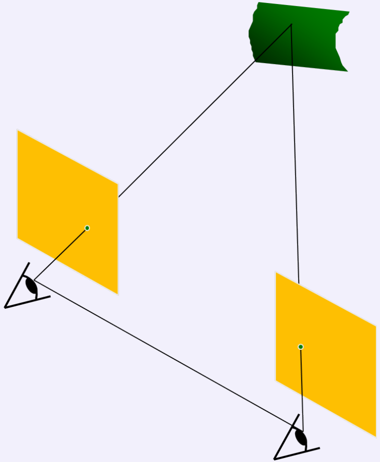
If we can recover x' from x we can recover depth.

Reminder II



- Line connecting O and O': baseline
- Plane through baseline x and x': Epipolar Plane
- Epipoles: intersection of baseline and image planes: projection of the other camera center.
- Epipolar Lines - intersections of epipolar plane with image planes (always come in corresponding pairs)

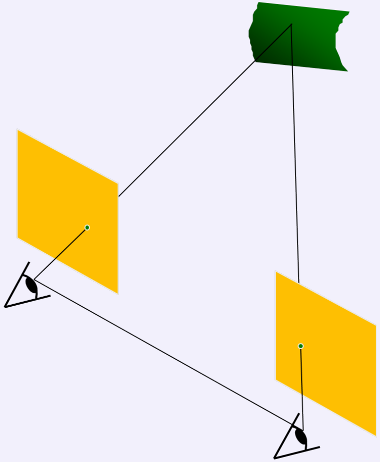
How: Parallel Case



Simple situation:

- Parallel image planes and parallel to baseline.
- Camera centers at same height
- same focal length.

How: Parallel Case



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For instance:

- Two pictures obtained from the same camera, translating along its x-axis.

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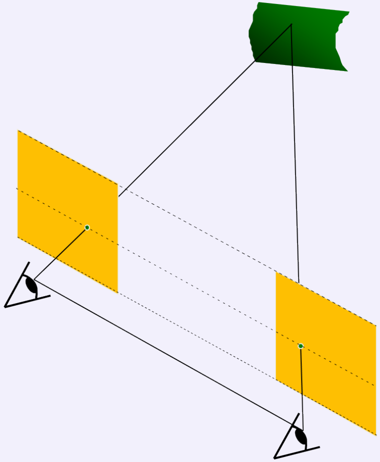

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For instance:

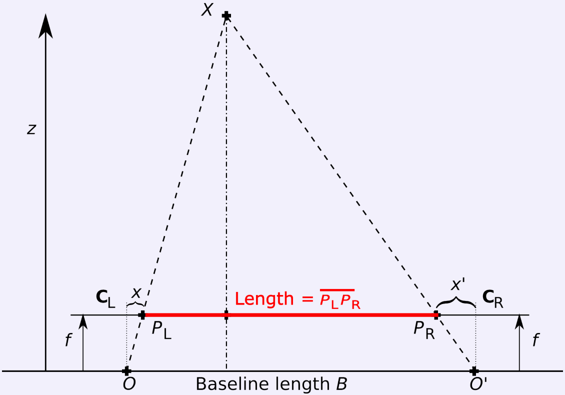
- Two pictures obtained from the same camera, translating along its *x*-axis.

Implication:

- Epipolar lines are horizontal scan lines!
- Epipoles are at ∞.

Depth from Disparity

Intercept Theorem again!



- disparity := $x' - x = B - \overline{P_L P_R}$

Apply Intercept Theorem to previous figure:

$$\frac{\overline{P_L P_R}}{B} = \frac{z - f}{z} \iff z \times \overline{P_L P_R} = B \times (z - f)$$

$$\Updownarrow$$

$$z \times \underbrace{\left(B - \overline{P_L P_R}\right)}_{\text{disparity}} = B \times f \iff \text{disparity} = \frac{B \times f}{z}$$

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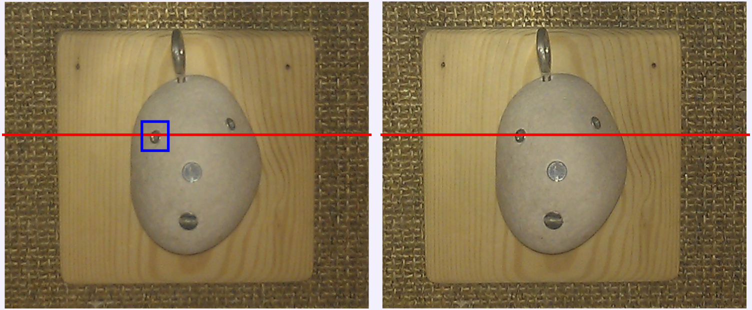
Depth and Disparity

Disparity is inversely proportional to depth.

Computing Disparity: Simple Settings

- Calibrated parallel cameras, with identical parameters.
- Search matches along scan lines, they are the epipolar lines of the 2-views problem.
- Extract disparity from Match positions.

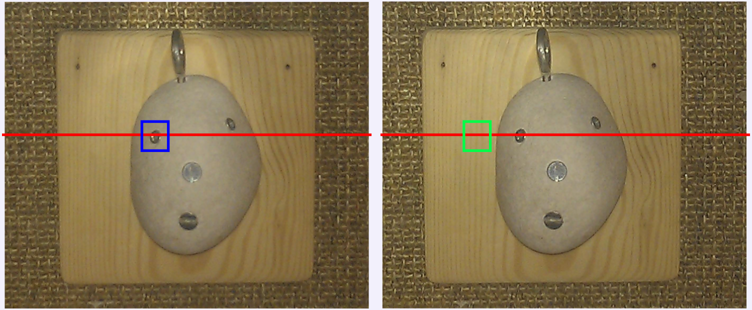
Example



Searching for match along scan lines.

- Feature based Disparity maps:
- Search interest point matches along scan lines.
- Disparities computed only at these interest points:
- If more needed, interpolation or dense disparity maps?

Example



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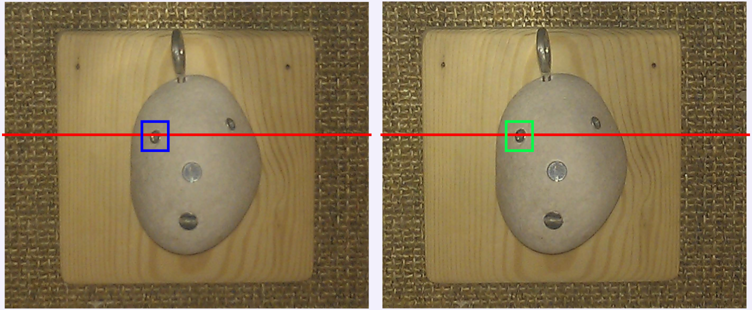

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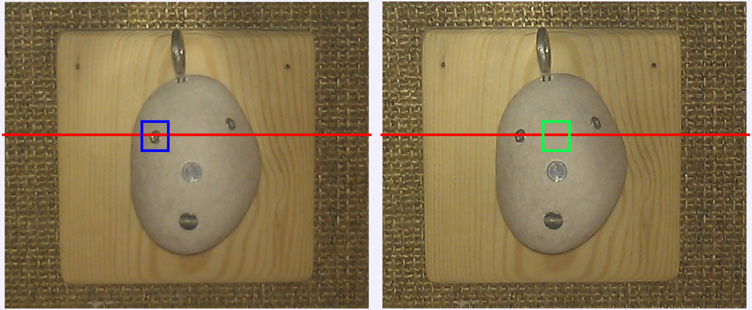

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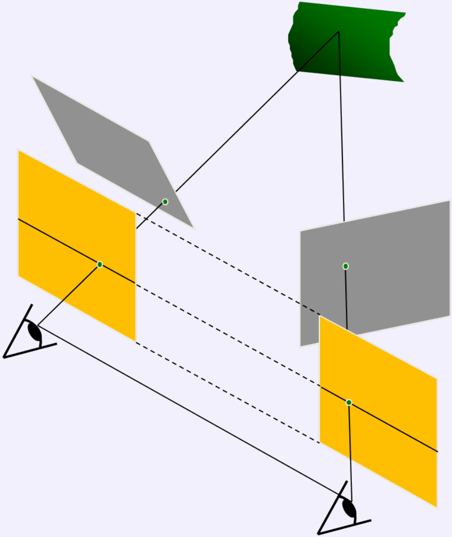
Non horizontal Scan lines

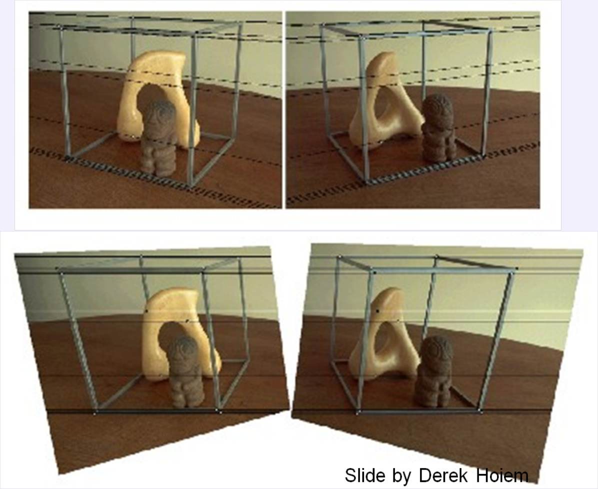
- What when cameras are in more general position?, calibration is unknown?
- If calibration known, the essential matrix provides epipolar constraints.
- Non calibrated views: Estimate the fundamental matrix.
- Knowing Essential or Fundamental matrix allows for image rectification.
- To know more on fundamental matrices: follow the link!

Projective Rectification



- Reproject onto a common plane parallel to line between camera centers
- Projections are homographies!
- Pixel motion is horizontal after reprojection.
- Cf Loop-Zhang, CVPR 1999.

Projective Rectification example



Disparity Map

- Still simple model: Horizontal scan lines.
- $d(x, y)$ = disparity of pixel at position (x, y). Assume intensity conserved

$$I_2(x + d(x, y), y) = I_1(x, y)$$

- Similar to Displaced Frame Difference in Optical Flow

$$I_2(x + v_1(x, y), y + v_2(x, y)) = I_1(x, y)$$

- Displacement only along one direction: compared to Optical Flow, no aperture problem!
- This does not mean that computing disparity is easy...

Differential Form

- Mimic Optical Flow idea: linearize the disparity constraint

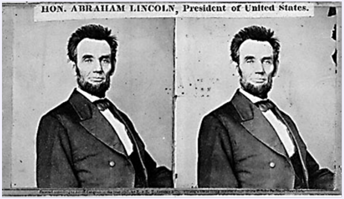
$$I_2(x + d(x,y), y) - I_1(x,y) \approx \underbrace{I_2(x,y) - I_1(x,y)}_{\tilde{I}_t} + \frac{\partial I_2}{\partial x} d(x,y)$$

$$\approx 0$$

- At each pixel: 1 equation and 1 unknown: can be solved.
- Would give the solution

$$d(x,y) = -\frac{\tilde{I}_t}{I_{2x}}$$

- Similar to 1D optical Flow.
- Above equation does not often hold because of: Noise in measurement, flat regions, shadows, repetitions...



Textureless surfaces

Occlusions, repetition

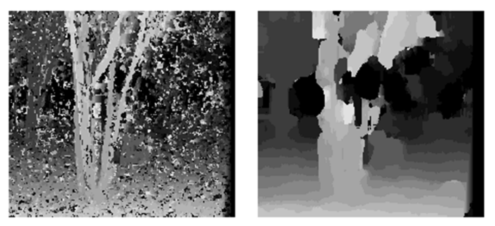
Non-Lambertian surfaces, specularities

Slide by Derek Hoiem

Regularization of the problem

- Assume generally slowly varying disparity map, solve in least-squares sense.
- Many possibilities:
  - Block Matching approach.
  - Adaptation of the Lucas-Kanade algorithm in that setting.
  - More global algorithms à la Horn and Schunck. PDE based solutions, Graph-Cuts...

Disparity Map By Dense Block Matching[2]



W = 3 W = 20

- Window size 3: Noise but details.
- Window size 20: smoother, but missing details.

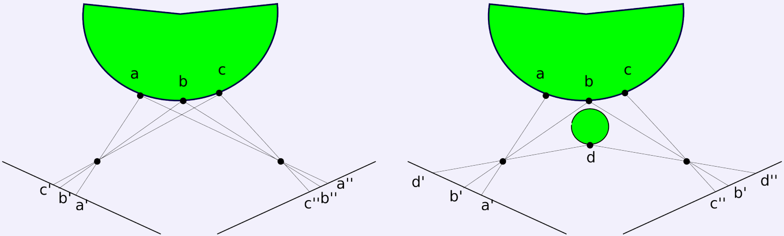
[2] Slide adapted from Derek Hoiem

Disparity vs Optical Flow

- Disparity map problem better posed.
- Restricted only to camera motion: homography between geometric configurations.
- Optical Flow can recover non rigid and multiple motion in a scene.
- Pure camera motion: Optical flow should be equivalent to disparity map.

Constraints

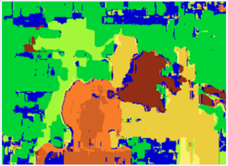
- Already discussed 1: disparity should change slowly almost everywhere.
- More coming from geometry:



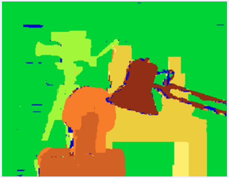
Order preserved — Order not preserved.

- Point ordering not preserved $\Longrightarrow$ different depths $\Longrightarrow$ disparity map discontinuity.

Graph Cuts Example

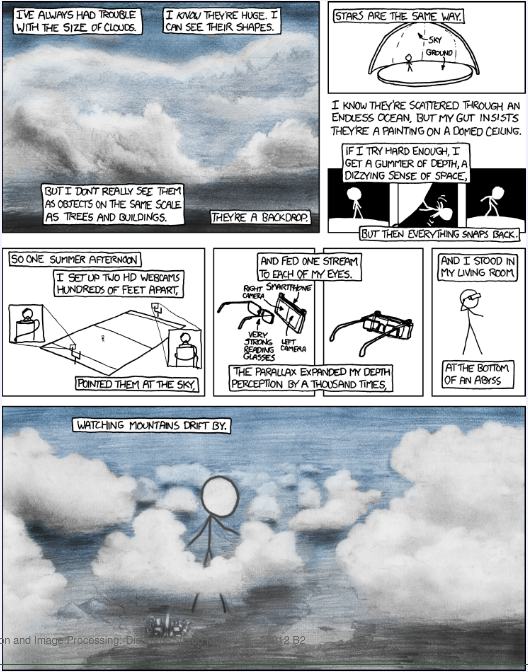


No extras constraints



Boykov et al. 2001

Ground truth


I'VE ALWAYS HAD TROUBLE WITH THE SIZE OF CLOUDS.
I KNOW THEY'RE HUGE. I CAN SEE THEIR SHAPES.
BUT I DON'T REALLY SEE THEM AS OBJECTS ON THE SAME SCALE AS TREES AND BUILDINGS.
THEY'RE A BACKDROP.
STARS ARE THE SAME WAY.
SKY
GROUND
I KNOW THEY'RE SCATTERED THROUGH AN ENDLESS OCEAN, BUT MY GUT INSISTS THEY'RE A PAINTING ON A DOMED CEILING.
IF I TRY HARD ENOUGH, I GET A GLIMMER OF DEPTH, A DIZZYING SENSE OF SPACE,
BUT THEN EVERYTHING SNAPS BACK.
SO ONE SUMMER AFTERNOON
I SET UP TWO HD WEBCAMS HUNDREDS OF FEET APART,
POINTED THEM AT THE SKY,
AND FED ONE STREAM TO EACH OF MY EYES.
RIGHT CAMERA
SMARTPHONE
VERY STRONG READING GLASSES
LEFT CAMERA
THE PARALLAX EXPANDED MY DEPTH PERCEPTION BY A THOUSAND TIMES,
AND I STOOD IN MY LIVING ROOM
AT THE BOTTOM OF AN ABYSS
WATCHING MOUNTAINS DRIFT BY.