Theorem 1. *Every non-empty ω-regular language contains an ultimately periodic word.*

Theorem 2. *For a Kripke structure $\mathcal{K}$ with initial state s and $\varphi \in LTL$, the model checking problem $L(\mathcal{K}, s) \subseteq L(\varphi)$? is PSPACE-complete.*

Proof. **PSPACE** Compute the intersection automaton for $L(\mathcal{K}, s) \cap L(\neg\varphi)$ and test it for emptiness.

PSPACE-hard Encode a poly.-length Turing tape as a Kripke structure and its correct behavior in LTL. $\square$

Theorem 3 (Büchi)**.** *The MSO theory of* $(\mathbb{N}, +1, <, 0)$ *is decidable.*

Proof. Corresponds to S1S formula. Can be checked with NBA emptiness test. $\square$

Theorem 4. *The FO theory of* $(\mathbb{R}, +, <, 0)$ *is decidable.*

Proof. Encode real numbers x by triples of sets (X_s, X_i, X_f) with the number's sign ($X_s = \emptyset$ or $\{0\}$), the positive decimal digits in binary encoding, and the positive fractional digits in binary encoding. Then an FO sentence can be transformed to an equi-satisfiable MSO sentence over $(\mathbb{N}, +1, <, 0)$. $\square$