# 1 List Of Models

## 1.1 Infinite Words

- DBA
- NBA
- GBA
- Rabin automaton
- Muller automaton
- Parity automaton
- det. E automaton
- det. A automaton
- det. coBA
- weak BA
- Staiger-Wagner automaton
- ABA
- LTL
- S1S
- ∃S1S
- $S1S_0$

## 1.2 Finite Trees

- DTA
- NTA
- ↓DTA
- ↓NTA
- DUTA
- NUTA
- deterministic DTD
- DTD
- deterministic EDTD

- single-type EDTD
- EDTD
- Relax NG
- FO
- MSO
- Regular expressions
- DTWA
- TWA

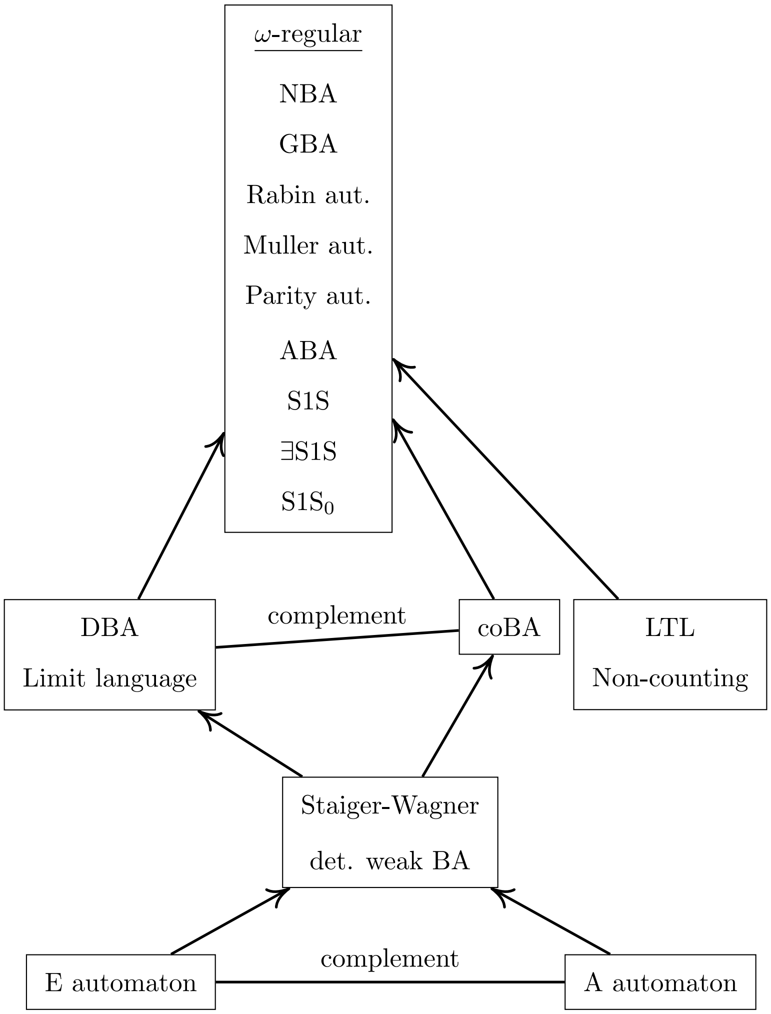
### 1.3 Infinite Trees

- BTA
- Muller TA
- Parity TA
- DMTA
- S2S (MSO / WMSO)
- $S2S_0$ (MSO / WMSO)

## 2 List Of Games

- Büchi
- Staiger-Wagner
- weak Parity
- Reachability
- Safety
- Muller
- Parity
- Rabin
- Streett
- Gale-Stewart
- Wadge

# 3 Infinite Word Models




## 3.1 Class Inclusions

- E aut. $\subseteq$ Staiger-Wagner
  **Proof**: SWA with $\mathcal{F} = \{Q' \subseteq Q \mid F \cap Q' \neq \emptyset\}$.

- A. aut. $\subseteq$ Staiger-Wagner
  **Proof**: SW closed under complement,

- Staiger-Wagner $\subseteq$ DBA / coBA
  **Proof**: $\mathcal{A}$ SWA $\Rightarrow \mathcal{A}' = (Q \times 2^Q, \Sigma, (q_0, \emptyset), \delta', F')$
  Collect all visited states and accept if that set stays in $\mathcal{F}$.

- DBA $\subseteq$ NBA
  trivial

- coBA $\subseteq$ NBA
  **Proof**: NBA closed under complement.

- LTL $\subseteq$ NBA (exponential size transformation)
  **Proof**: Theorem 2

- LTL $\subseteq$ ABA (linear size transformation)
  **Proof**: Similar to theorem 2. $\mathcal{A}_\varphi = (\mathrm{cl}(\varphi), \mathbb{B}^n, \varphi, \delta, F)$ with $F = \{\psi \in \mathrm{cl}(\varphi) \mid \psi = G\vartheta\}$ and $\delta$ is defined as follows:

  - $\delta(p_i, a)$ and $\delta(\neg p_i, a)$ are tt or ff depending on the $i$-th component of $a$.
  - $\delta(\psi_1 \oplus \psi_2, a) = \delta(\psi_1, a) \oplus \delta(\psi_2, a)$ for $\oplus \in \{\wedge, \vee\}$.
  - $\delta(X\psi, a) = \psi$.
  - $\delta(G\psi, a) = \delta(\psi, a) \wedge G\psi$.
  - $\delta(\psi_1 U \psi_2, a) = (\delta(\psi_1, a) \wedge \psi_1 U \psi_2) \vee \delta(\psi_2, a)$.

### 3.2 Class Exclusions

- E aut. $\not\subseteq$ A aut.
  **Example**: $b^*a(a+b)^\omega$
  **Proof**: Assume the A automaton $\mathcal{A} = (Q, \Sigma, q_0, \delta, F)$ recognizes $L$, so for every $b^n ab^\omega$, the run $\rho_n$ is accepting. Let $\rho^*(n) = \rho_{n+1}(n)$. This is an accepting run on $b^\omega$, which is a contradiction.

- A aut. $\not\subseteq$ E aut.
  **Example**: $\{a^\omega\}$
  **Proof**: Assume the E automaton $\mathcal{A} = (Q, \Sigma, q_0, \delta, F)$ recognizes $L$, so the run $\rho$ of $\mathcal{A}$ on $a^\omega$ is accepting. That means there is an $n$ s.t. $\rho(n) \in F$. Therefore, $\mathcal{A}$ accepts the word $a^n b^\omega$, which is a contradiction.

- DBA $\not\subseteq$ coBA
  **Example**: $(a^*b)^\omega$
  **Proof**: Assume $\mathcal{A} = (Q, \Sigma, q_0, \delta, F)$ is a coBA recognizing $L$. Let $n = |Q|$, $w = (a^n b)^\omega$, and $\rho$ the run of $\mathcal{A}$ on $w$. From some $m$ on, $\rho$ only visits states in $F$. Consider $(a^n b)^m a^n$. By choice of $n$, there must be $i < j$ such that after $(a^n b)^m a^i$ and $(a^n b)^m a^j$, the automaton is in the same state $q \in F$. Therefore, the run on $(a^n b)^m a^\omega$ is accepting, which is a contradiction.

- coBA $\not\subseteq$ DBA
  **Example**: $(a+b)^* a^\omega$
  **Proof**: Assume $\mathcal{A} = (Q, \Sigma, q_0, \delta, F)$ is a DBA recognizing $L$. We inductively define words $w_n$ with runs $\rho_n$ s.t. $w_n \sqsubseteq w_{n+1}$ and $\rho_n$ visits $F$ at least $n$ times and $|w_n|_b = n$. Then the "limit" of those words contains infinitely many $b$ but is accepted by $\mathcal{A}$, which is a contradiction.
  Let $w_0 = \varepsilon$ and $\rho_0 = q_0$. For $n+1$, consider $w_n a^\omega$ with the run $\rho_n \pi$. Since $w_n a^\omega \in L$, there is a $k$ s.t. $\pi(k) \in F$. Let $w_{n+1} = w_n a^k b$ and $\rho_{n+1}$ accordingly.

- LTL $\not\subseteq$ NBA
  **Example**: $((a+b)a)^\omega$
  **Proof**: Show that $L$ is counting. Then it follows that it is not LTL-definable. Assume that $L$ is non-counting, so there is an $n_0$ according to the definition. Let $n = n_0 + 1$, $u = \varepsilon$, $v = a$, and $\beta = ba^\omega$. Due to symmetry, we can assume that $n_0$ is even, so $uv^n\beta \notin L$ but $uv^{n+1}\beta \in L$.

## 3.3 Class Equalities

### 3.3.1 NBA

- NBA $\Leftrightarrow$ $\omega$-regular
  **Proof**: $\Leftarrow$ All three operations used in regular expressions $(\cup, \cdot, ^\omega)$ correspond to easy NBA constructions.

  $\Rightarrow$ Let $\mathcal{A} = (Q, \Sigma, q_0, \Delta, F)$ be an NBA. For every final state $q \in F$, we consider the finite words $U_q$ which lead from $q_0$ to $q$, and the finite words $V_q$ which lead from $q$ back to $q$ itself. Then $L(\mathcal{A})$ is the union of $U_q \cdot V_q^\omega$ for all final states $q$.

- GBA $\Rightarrow$ NBA
  **Proof**: Let $\mathcal{A} = (Q, \Sigma, q_0, \Delta, (F_1, \ldots, F_k))$ be a GBA. An equivalent NBA is $\mathcal{A}' = (Q \times \{1, \ldots, k\}, \Sigma, (q_0, 1), \Delta', F_k \times \{k\})$. The transitions $\Delta'$ contain all those from $\Delta$ and allow to switch from $(q, i)$ to $(q, i+1)$ if $q \in F_i$.

- NBA $\Leftrightarrow$ S1S
  **Proof**: Theorem 4.

- Det. Muller $\Leftrightarrow$ NBA
  **Proof**: Theorem 5.

- (det.) Muller $\Rightarrow$ (det.) Parity
  **Proof**: Theorem 6.

- ABA $\Rightarrow$ NBA
  **Proof**: Theorem 7.

### 3.3.2 LTL

LTL $\Leftrightarrow$ Non-counting
No proof. Remarks in F8.

A language $L$ is called **non-counting** if there is an $n_0$ such that for all $n > n_0$, all $u, v \in \Sigma^*$, and all $\beta \in \Sigma^\omega$: $uv^n\beta \in L \leftrightarrow uv^{n+1}\beta \in L$.

### 3.3.3 SW

Staiger-Wagner $\Leftrightarrow$ det. weak BA
**Proof**: **??**

## 3.4 Closures

### 3.4.1 NBA

- Closed under union
  **Proof**: Product automaton with $F = (F_1 \times Q_2) \cup (Q_1 \times F_2)$.

- Closed under intersection
  **Proof**: GBA $\mathcal{A} = (Q_1 \times Q_2, \Sigma, (q_0^1, q_0^2), \Delta, (F_1 \times Q_2, Q_1 \times F_2))$ where
  $\Delta = \{((p_1, p_2), a, (q_1, q_2)) \mid (p_1, a, q_1) \in \Delta_1, (p_2, a, q_2) \in \Delta_2\}$.

- Closed under complement
  **Proof**: Theorem 1.

### 3.4.2 DBA

- Closed under union and intersection
  **Proof**: Same construction as for NBA.

- Not closed under complement (inf. many $a \leftrightarrow$ fin. many $a$)

### 3.4.3 SW

- Closed under union and intersection
  **Proof**: Product automaton with $\mathcal{F}_\cap = \{F \subseteq Q_1 \times Q_2 \mid \pi_1(F) \in \mathcal{F}_1, \pi_2(F) \in \mathcal{F}_2\}$ where $\pi_i((x_1, x_2)) = x_1$.

- Closed under complement
  **Proof**: $\overline{\mathcal{F}} = 2^Q \setminus \mathcal{F}$.

## 3.5 Characterizations

- Parity conditions are directly convertible to Rabin chain conditions and vice-versa.
  **Proof**: Assign priorities in ascending order; $E_k \to 0$, $F_k \setminus E_k \to 1$, $E_{k-1} \setminus F_k \to 2 \ldots$

- $U$ is $\omega$-regular iff $U$ is a Boolean combination of DBA-languages
  **Proof**: $\Leftarrow$ NBAs are closed under Boolean operations.

  $\Rightarrow$ Let $\mathcal{A}$ be a DRA for $U$. It suffices to consider $\Omega = \{(E, F)\}$ as any other language is a finite union of these conditions. Let $\mathcal{A}_E$ and $\mathcal{A}_F$ be the modifications of $\mathcal{A}$ with conditions $\{(E, Q)\}$ and $\{\emptyset, F\}$ respectively. Then $L(\mathcal{A}_E)^\complement$ and $L(\mathcal{A}_F)$ are DBA-recognizable.

- $U$ is DBA-recog. iff $U = \lim(L)$ for some regular $L \subseteq \Sigma^*$.
  **Proof**: Use the DBA as a DFA or vice-versa.

- $U$ is E-recog. iff $U = L \cdot \Sigma^*$ for some regular $L \subseteq \Sigma^*$.
  **Proof**: $\Rightarrow$ Let $\mathcal{A}$ be an E-automaton for $U$. For every $q \in F, a \in \Sigma$, add a transition $(q, a, q)$. The resulting automaton as an NFA accepts $L$ with $U = L \cdot \Sigma^*$.

  $\Leftarrow$ Let $\mathcal{A}$ be a DFA for $L$. The same automaton as an E-automaton recognizes $L \cdot \Sigma^*$.

- Landweber's theorem
  **Proof**: Theorem 3

- DBA $\cap$ coBA $\Rightarrow$ SW
  **Proof**: **??**

- SW $\Leftrightarrow$ Boolean combination of E-recognizable languages.
  **Proof**: $\Leftarrow$ SWAs are closed under Boolean operations.

  $\Rightarrow$ Let $\mathcal{A} = (Q, \Sigma, q_0, \delta, \mathcal{F})$ be the SWA and let $\mathcal{A}_q = (Q, \Sigma, q_0, \delta, \{q\})$. For every $q \in Q$, the language $L(\mathcal{A}_q)$ is E-recognizable. For every $F \subseteq Q$, let $L_F = \bigcap\limits_{q \in F} L(A_q) \cap \bigcap\limits_{q \notin F} L(A_q)^{\complement}$, which is a Boolean combination of E-recognizable languages. Then $L(\mathcal{A}) = \bigcup\{L_F \mid F \in \mathcal{F}\}$.

### 3.6 Duality

- $U$ is A-recog. iff $\Sigma^\omega \setminus U$ is E-recog.
  **Proof**: same automaton, $\overline{F} = Q \setminus F$

- $U$ is coBA-recog. iff $\Sigma^\omega \setminus U$ is DBA-recog.
  **Proof**: same automaton, $\overline{F} = Q \setminus F$

### 3.7 Problems / Complexity

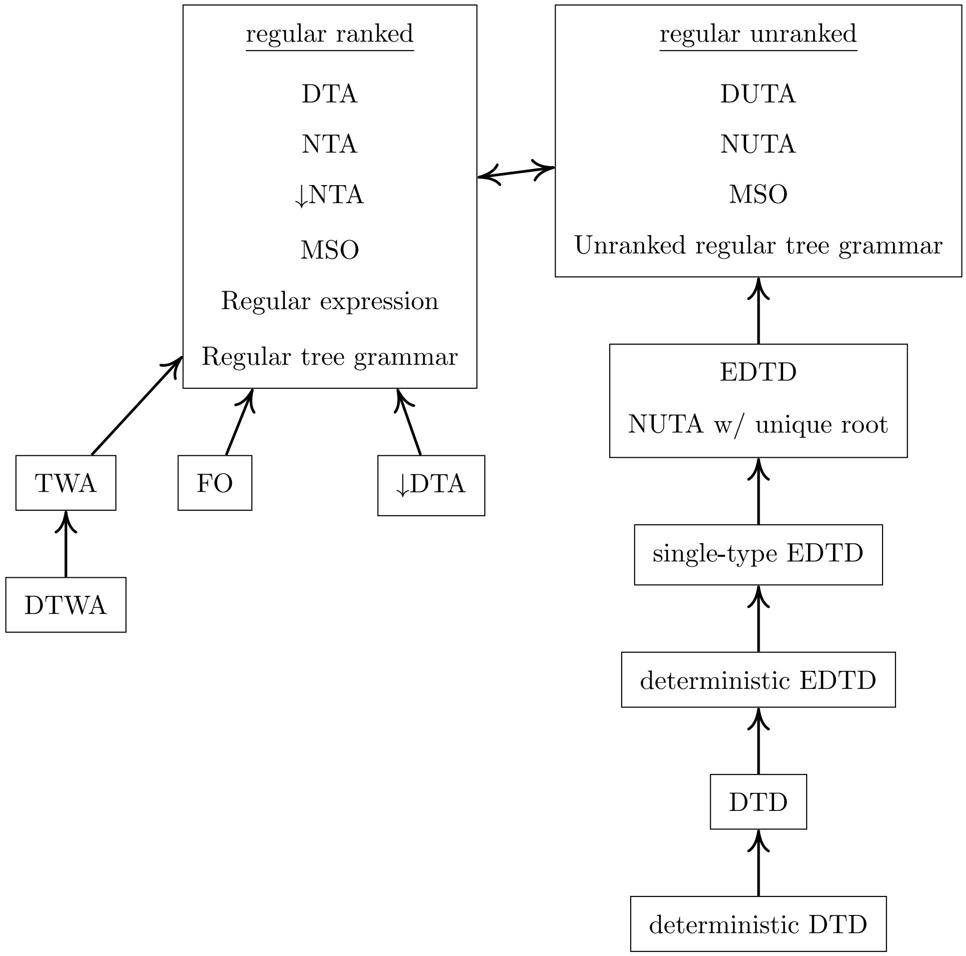
- Emptiness problem for NBAs is decidable in poly. time.
  **Algorithm**:

  1. Compute all states that are reachable from $q_0$. ($\mathcal{O}(|Q| + |\Delta|)$).
  2. For each of these reachable states that is also accepting, check if there is a path to itself. ($\mathcal{O}(|F| \cdot (|Q| + |\Delta|))$).
  3. If any state like that was found, the language is not empty.

- Emptiness problem for ABAs is PSPACE-complete.

- Membership problem for ABAs is decidable in poly. time.
  **Proof**: Model the problem as a Büchi game. In each turn, starting at a node $v \in Q$, player 1 chooses an $a \in \Sigma$. Then, according to the Boolean operators of $\delta(v, a)$, the players choose a next state. If player 0 wins, the choices of player 1 model an element of the ABA language.

# 4 Finite Tree Models




## 4.1 Class Inclusions

- Regular Ranked $\subseteq$ Regular Unranked
  **FCNS**: Let $\Sigma$ be an unranked alphabet. We define $\Gamma_0 = \{\#\}$ and $\Gamma_2 = \Sigma$. Let $\bar{t} = t_1 \dots t_n \in T_\Sigma^*$ with $t_1 = a(t'_1 \dots t'_m)$. We define

$$\mathrm{fcns}(\bar{t}) = \begin{cases} \# & \text{if } n = 0 \\ a(\mathrm{fcns}(t'_1, \dots, t'_m), \mathrm{fcns}(t_2 \dots t_n)) & \text{else} \end{cases}$$

  See Theorem 12.

- det. DTD $\subseteq$ DTD $\subseteq$ det. EDTD $\subseteq$ single-type EDTD $\subseteq$ EDTD
  trivial

- EDTD $\subseteq$ Regular tree grammar
  **Proof**: $N = \Sigma'$, $P_{\text{gram}} = P \cup \{a^{(n)} \to a \mid a, n\}$

- TWA $\subseteq$ NTA
  **Proof**: Theorem 8

## 4.2 Class Exclusions

- $\downarrow$DTA $\not\subseteq$ NTA
  **Example**: $T = \{f(a,b), f(b,a)\}$
  **Proof**: Assume the $\downarrow$DTA $\mathcal{A} = (Q, \Sigma, q_0, \Delta)$ accepts $T$. Let $(q_0, f, (q_1, q_2)) \in \Delta$, so also $(q_1, a), (q_1, b), (q_2, a), (q_2, b) \in \Delta$. However, that means the tree $f(a,a)$ is accepted by the run $q_0(q_1, q_2)$ which is a contradiction. On the other hand, NTAs can clearly recognize this property.

- DTD $\not\subseteq$ single-type EDTD
  **Example**: $T = \{t \in T_{\{a,b\}}$ | there is a path in $t$ on which $a$ occurs exactly twice

- NUTA w/ unique root $\not\subseteq$ NUTA
  **Example**: $T = \{a, b\}$

- FO $\not\subseteq$ MSO
  **Example**: $T$ = positive boolean terms that evaluate to true

- DTWA $\not\subseteq$ TWA
  **Example**: $T_{N \setminus D}$
  $\Sigma_0 = \{a, b\}$, $\Sigma_2 = \{f\}$
  $t \in T_{N \setminus D}$ iff $|t|_a = 3 \wedge \text{lca}(u, v) \sqsubseteq \text{lca}(v, w)$ **Proof**: DTWAs cannot recognize this tree language (no proof). TWAs can:

  - Check whether there are exactly three $a$ and move to the right-most one (DFS).
  - While going up, guess a node and go the the left-most ancestor.
  - The tree is in $T_{N \setminus D}$ iff there are exactly two leafs labeled $a$ right of that node. (DFS)

- TWA $\not\subseteq$ NTA
  **Example**: all paths in the skeleton have even length
  **Proof**: $\Sigma_0 = \{a, c\}$, $\Sigma_2 = \{b\}$
  Skeleton of a tree $t$: replace all subtrees that contain exactly one $a$.
  TWAs cannot recognize this tree language (no proof). NTAs can (find "merge“ points and count the parity)

## 4.3 Class Equalities

### 4.3.1 Regular Ranked

- NTA $\Rightarrow$ DTA
  **Proof**: Subset construction.

- NTA $\Leftrightarrow$ $\downarrow$NTA
  **Proof**: Reverse the transitions and initial states $\leftrightarrow$ final states.

- $\downarrow$NTA $\Leftrightarrow$ Regular Tree Grammar
  **Proof**: Non-terminals correspond to states and transitions $(q, a, q_1, \dots, q_n)$ correspond to rules $A_q \to a(A_{q_1}, \dots, A_{q_n})$

- MSO $\Leftrightarrow$ NTA
  **Proof**: same as S2S

- Reg. exp. $\Leftrightarrow$ NTA
  **Proof**: Theorem 9

#### 4.3.2 Regular Unranked

- NUTA $\Rightarrow$ DUTA
  **Proof**: Specialized subset construction.
  $\mathcal{A} = (Q, \Sigma, \Delta, F) \Rightarrow \mathcal{A}' = (2^Q, \Sigma, \Delta', \{P \subseteq Q \mid P \cap F \neq \emptyset\})$
  $\Delta' = \{(L_{a,P}, a, P) \mid a \in \Sigma, P \subseteq Q\}$ with $L_{a,P} = \bigcap_{p \in P} K_{a,p} \cap \bigcap_{p \notin P} K_{a,p}^{\complement}$
  $K_{a,p} = \{P_1 \dots P_n \in (2^Q)^* \mid \exists p_1 \in P_1, \dots, p_n \in P_n : p_1 \dots p_n \in L_{a,p}\}$.

- MSO $\Leftrightarrow$ NUTA

#### 4.3.3 EDTD

- NUTA with unique root $\Leftrightarrow$ EDTD (in poly. time)
  **Proof**: $\Rightarrow$ Let $q_0, \dots, q_n$ be an enumeration of $Q$. We then set $D = (\{a^{(i)} \mid a \in \Sigma, 0 \leq i \leq n\}, P, a^{q_0})$ where $a$ is the unique root symbol of the automaton. Rules in $P$ guess arbitrary symbols for fitting states.

  $\Leftarrow$ Analogously, we use $Q = \Sigma \times \{1, \dots, n\}$ where $n$ is the maximal type in $\Sigma'$ and rules mimic the transitions.

### 4.4 Closures

#### 4.4.1 Regular Ranked

- Regular (ranked) trees are closed under complement.
  **Proof**: Make $\Delta$ total (add sink state) and set $F' := Q \setminus F$.

- Regular (ranked) trees are closed under union and intersection.
  **Proof**: Product construction ($F_\cap = F_1 \times F_2$, $F_\cup = (Q_1 \times F_2) \cup (F_1 \times Q_2)$)

#### 4.4.2 Regular Unranked

- Regular unranked trees are closed under complement, union, and intersection.
  **Proof**: via FCNS, because ranked trees are closed under these operations

### 4.4.3 TWA

- TWAs are closed under union.
  **Proof**: Non-deterministically choose at the start whether to execute $\mathcal{A}_1$ or $\mathcal{A}_2$.
- TWAs and DTWAs are closed under intersection.
  **Proof**: Execute $\mathcal{A}_1$. If it accepts, move to the root and execute $\mathcal{A}_2$.
- Open Problem: Are TWAs closed under complement?
- DTWAs are closed under complement.
  **Proof**: Theorem 11

## 4.5 Problems / Complexity

### 4.5.1 Regular Ranked

- Membership problem for NTAs is decidable.
- Reachable states of NTAs can be computed in linear time in $|\mathcal{A}|$. **Algorithm**:
  1. Initial reachable states $R = \{q \in Q \mid \exists a \in \Sigma_0 : (a, q) \in \Delta\}$
  2. For each transition $\tau = \{\overline{q}, a, p\} \in \Delta$, count $\text{in}(\tau) = |\overline{q}|$ and remember $\forall q \in \overline{q} : \tau \in \text{tr}(q)$.
  3. Grow $R$ by processing each reachable state once: Decrement $\text{in}(\tau)$ by 1; if that value reaches 0, every ingoing state of $\tau$ is reachable and therefore, the outgoing state is.
- Emptiness of an NTA can be decided in linear time in $|\mathcal{A}|$.
  **Algorithm**: $T(\mathcal{A}) = \emptyset$ iff $\text{Reachable}(\mathcal{A}) \cap F = \emptyset$
- Given a DTA $\mathcal{A}$, $\sim_{T(\mathcal{A})}$ can be computed in time $\text{poly}(|Q^m \times \Sigma \times Q|)$ where $m$ is the maximal arity in $\Sigma$.
  **Algorithm**:
  1. Mark all $(q, q')$ with $(q, q') \in (F \times Q \setminus F) \cup (Q \setminus F \times F)$.
  2. As long as there is still change in the marks, execute step 3:
  3. If $(p, p')$ is already marked and there are $q_1, \ldots, q_{i-1}, q, q', q_{i+1}, \ldots, q_n \in Q$ and $a \in \Sigma_n$ such that $p = \delta(q_1, \ldots, q, \ldots, q_n, a)$ and $p' = \delta(q_1, \ldots, q', \ldots, q_n, a)$, then mark the pair $(q, q')$.
  4. $p \sim_{T(\mathcal{A})} q$ iff $(p, q)$ is not marked.

### 4.5.2 Regular Unranked

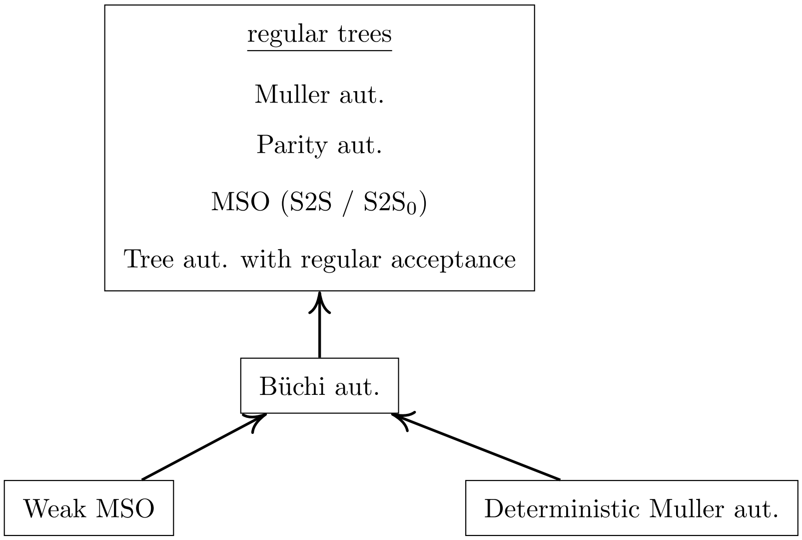
- Emptiness / membership / inclusion for NUTAs is decidable in polynomial / polynomial / exponential time.
  **Algorithm** (Emptiness): Use FCNS transformation.
  **Algorithm** (Membership): Construct $\mathcal{A}_t$ with $T(\mathcal{A}_t) = \{t\}$ and check $\mathcal{A} \cap \mathcal{A}_t \stackrel{?}{=} \emptyset$.
  **Algorithm** (Inclusion): Check $T(\mathcal{A}_1) \cap T(\mathcal{A}_2)^{\complement} \stackrel{?}{=} \emptyset$. Complementation is exponential.
- Inclusion for complete DUTAs is decidable in polynomial time.
  **Algorithm**: Same algorithm as for general NUTAs but complementation can be done in polynomial time.

### 4.5.3 Grammars

- Emptiness / membership for EDTDs is decidable in polynomial time.
  **Proof**: EDTD can be converted to NUTA in polynomial time.

- Inclusion for deterministic EDTDs is decidable in polynomial time.
  **Proof**: Let $D_1, D_2$ be deterministic EDTDs. Using the previous results and theorem 10, we can construct NUTAs $\mathcal{A}, \mathcal{B}$ with $T(\mathcal{A}) = T(D_1)$ and $T(\mathcal{B}) = T(D_2)^{\complement}$ in polynomial time. Then $T(D_1) \subseteq T(D_2)$ iff $T(\mathcal{A}) \cap T(\mathcal{B}) = \emptyset$.

# 5 Infinite Tree Models




## 5.1 Class Exclusions

- BTA $\not\subseteq$ Regular tree
  **Example**: $T_{\text{fin}} = \{t \in T_{\{a,b\}} \mid \text{every infinite path has only fin. many } b\}$
  **Proof**:

- DMTA $\not\subseteq$ BTA
  **Example**: $T_{\text{fin}}^{\complement} = \{t \in T_{\{a,b\}} \mid \text{there is a path with inf. many } b\}$
  **Proof**: BTA-recognizable (see below) but not a path tree language.

- WMSO $\not\subseteq BTA$
  **Example**: $\{t \in T_{\{a,b\}} \mid \text{every infinite path has infinitely many } b\}$
  **Proof**:

## 5.2 Class Equalities

### 5.2.1 Regular Trees

- PTA, MTA $\Rightarrow$ TA with regular acceptance
  **Proof**: By definition and PA/MA regularity.

- TA with reg. acc. $\Rightarrow$ PTA
  **Proof**: TA $\mathcal{A}$, DPA $\mathcal{A}'$ over alphabet $Q$ that defines Acc. Define PTA with state space $Q \times P$.
  $(q, a, q_1, q_2) \in \Delta \Rightarrow ((q, p), a, (q_1, \delta(p, q)), (q_2, \delta(p, q))) \in \Delta'$

- PTA $\Leftrightarrow$ MSO (S2S) **Proof**: **??**

## 5.3 Closures

### 5.3.1 Regular Trees

- The class of regular tree languages is closed under union and intersection.
  **Proof**: **??**

- The class of regular tree languages is closed under projection.
  **Proof**: **??**

- PTAs are closed under complement ($2^{\mathcal{O}(kn\cdot\log(kn))}$ states for $|Q| = n$, $|\mathrm{img}(c)| = k$) **Proof**: **??**

### 5.3.2 BTA

- BTAs are not closed under complement.
  **Proof**: $T_{\text{fin}}^{\complement} = \{t \in T_{\{a,b\}} \mid \text{there is a path with inf. many } b\}$
  $T_{\text{fin}}^{\complement}$ is BTA-recognizable but its complement is not.

### 5.3.3 DMTA

- DMTAs are not closed under union or complement.

- DMTAs are closed under intersection.
  **Proof**: product automaton

## 5.4 Characterizations

- $T \subseteq T_\Sigma$ is DMTA-recognizable iff $T$ is a path tree language.
  $T : 2^{(\{0,1\}\times\Sigma)^\omega} \to 2^{T_\Sigma}, L \mapsto \{t \in T_\Sigma \mid \forall \pi \in \{0,1\}^\omega : \pi^\frown(t|_\pi) \in L\}$
  **Proof**:

- $T \subseteq T_\Sigma$ is WMSO-definable iff $T$ and $T^{\complement}$ are BTA-recognizable.

## 5.5 Problems

### 5.5.1 Regular Trees

- The membership problem for regular tree languages is decidable.
  **Proof**: via membership game as in complementation

- The emptiness problem for regular tree languages is decidable.
  **Proof**: **??**