1 Structures

1.1 Words

- alphabet Σ
- Σ^*, Σ^ω
- ω-word $(\alpha : \mathbb{N} \to \Sigma) \in \Sigma^\omega$
- $\mathrm{Occ}(\alpha) = \{\alpha(n) \mid n \in \mathbb{N}\}$
- $\mathrm{Inf}(\alpha) = \bigcap_{i \in \mathbb{N}} \{\alpha(n) \mid n > i\}$

1.2 Finite Ranked Trees

- ranked alphabet $\Sigma_0, \ldots, \Sigma_m$
- trees T_Σ
- trees (inductive definition) $a \in \Sigma_i$, $t_1, \ldots, t_i \in T_\Sigma \Rightarrow a(t_1, \ldots, t_i) \in T_\Sigma$
- trees (labeled definition)
  - tree $t = (\mathrm{dom}_t, \mathrm{val}_t)$
  - $\mathrm{dom}_t \subseteq (\mathbb{N}^*_{>0}$
  - $\mathrm{val}_t : \mathrm{dom}_t \to \Sigma$
  - if $\mathrm{val}_t(w) \in \Sigma_i$, then $wi \in \mathrm{dom}_t$ and $w(i+1) \notin \mathrm{dom}_t$
  - if $w = uv \in \mathrm{dom}_t$, then $u \in \mathrm{dom}_t$
  - if $w(i+1) \in \mathrm{dom}_t$, then $wi \in \mathrm{dom}_t$

1.3 Finite Unranked Trees

- alphabet Σ
- trees T_Σ
- trees (inductive definition)
  - $h \in (T_\Sigma)^*$ is a **hedge**
  - $a \in \Sigma$, h hedge $\Rightarrow a(h) \in T_\Sigma$
- trees (labeled definition)
  - tree $t = (\mathrm{dom}_t, \mathrm{val}_t)$
  - $\mathrm{dom}_t \subseteq (\mathbb{N}^*_{>0}$
  - $\mathrm{val}_t : \mathrm{dom}_t \to \Sigma$
  - if $w = uv \in \mathrm{dom}_t$, then $u \in \mathrm{dom}_t$
  - if $w(i+1) \in \mathrm{dom}_t$, then $wi \in \mathrm{dom}_t$

1.4 Infinite Trees

- alphabet Σ
- trees T_Σ
- tree $t : \{0,1\}^* \to \Sigma$
- path $\pi \in \{0,1\}^\omega$
- $t|_\pi \in \Sigma^\omega : n \mapsto t(\pi(n))$

1.5 Games

- arena / game graph $G = (V_0, V_1, E, c)$
  - V_0, V_1 player vertices, $V = V_0, V_1$
  - $E \subseteq V \times V$
  - colors C finite
  - colors $c : V \to C$
- winning condition $\mathrm{Win} \subseteq C^\omega$
- game $\mathcal{G} = (G, \mathrm{Win})$
- play $\alpha \in V^\omega$
- winner of $\alpha = \begin{cases} 0 & \text{if } c(\alpha) \in \mathrm{Win} \\ 1 & \text{else} \end{cases}$

2 Acceptance Conditions

Q set of states/colors
$\rho \in Q^\omega$

E $F \subseteq Q$. ρ accepted iff $F \cap \mathrm{Occ}(\rho) \neq \emptyset$

A $F \subseteq Q$. ρ accepted iff $\mathrm{Occ}(\rho) \subseteq F$

Staiger-Wagner $\mathcal{F} \subseteq 2^Q$. ρ accepted iff $\mathrm{Occ}(\rho) \in \mathcal{F}$

Büchi $F \subseteq Q$. ρ accepted iff $F \cap \mathrm{Inf}(\rho) \neq \emptyset$

Generalized Büchi $F_1, \dots, F_k \subseteq Q$. ρ accepted iff for all $1 \leq i \leq k$: $F_i \cap \mathrm{Inf}(\rho) \neq \emptyset$

coBüchi $F \subseteq Q$. ρ accepted iff $\mathrm{Inf}(\rho) \subseteq F$

Rabin $\Omega \subseteq 2^Q \times 2^Q$. ρ accepted iff there is $(E, F) \in \Omega$ s.t. $\mathrm{Inf}(\rho) \cap E = \emptyset$ and $\mathrm{Inf} \cap F \neq \emptyset$

- Rabin Chain: $\Omega = \{(E_i, F_i) \mid 1 \leq i \leq k\}$ with $E_k \subseteq F_k \subseteq E_{k-1} \subseteq \cdots \subseteq E_1 \subseteq F_1$

Streett $\Omega \subseteq 2^Q \times 2^Q$. ρ accepted iff for all $(E, F) \in \Omega$: $\mathrm{Inf}(\rho) \cap E \neq \emptyset$ or $\mathrm{Inf} \cap F = \emptyset$

Parity $c : Q \to C$. ρ accepted iff $\max \mathrm{Inf}(c(\rho))$ is even

Muller $\mathcal{F} \subseteq 2^Q$. ρ accepted iff $\mathrm{Occ}(\rho) \in \mathcal{F}$

3 Word Automata

$(Q, \Sigma, q_0, \Delta, \mathrm{Acc})$

Deterministic $\delta : Q \times \Sigma \to Q$

Non-Deterministic $\Delta \subseteq Q \times \Sigma \times Q$

Alternating $\delta : Q \times \Sigma \to B_+(Q)$ where $B_+(Q)$ are boolean formulas over $Q \cup \{0, 1\}$ without negation

3.1 Deterministic Word Automata

- DBA
- weak DBA
- coDBA
- E-automaton
- A-automaton
- Staiger-Wagner automaton
- DMA
- DRA
- DPA

3.2 Nondeterministic & Alternating Word Automata

- NBA
- ABA

4 Finite Tree Automata

4.1 Ranked Top-Down Automata

(Q, Σ, Δ, F) where $\Delta \subseteq \bigcup_{i=0}^{m} Q^i \times \Sigma_i \times Q$ or deterministic δ

- DTA
- NTA

4.2 Ranked Bottom-Up Automata

(Q, Σ, Q_0, Δ) where $Q_0 \subseteq Q$ initial states, $\Delta \subseteq \bigcup_{i=0}^{m} Q \times \Sigma_i \times Q^i$ or deterministic δ

- $\downarrow$DTA
- $\downarrow$NTA

4.3 Unranked Bottom-Up Automata

(Q, Σ, Δ, F) where $\Delta \subseteq \mathrm{Reg}(Q) \times \Sigma \times Q$

Normalized For all $(L_1, a_1, q_1), (L_2, a_2, q_2)$: If $a_1 = a_2$ and $q_1 = q_2$, then $L_1 = L_2$

Deterministic For all $(L_1, a_1, q_1), (L_2, a_2, q_2)$: $L_1 \cap L_2 = \emptyset$ or $a_1 \neq a_2$ or $q_1 = q_2$

- DUTA
- NUTA

4.4 Ranked Tree Walking Automata

Types $= \{\mathrm{root}, 1, \ldots, m\}$
Dir $= \{\uparrow, 0, 1, \ldots m\}$

$$\mathrm{type}_t(u) = \begin{cases} \mathrm{root} & \text{if } u = \epsilon \\ i & \text{if } u = vi \end{cases}$$

$(Q, \Sigma, q_0, \Delta, F)$ where $\Delta \subseteq Q \times \mathrm{Types} \times \Sigma \times Q \times \mathrm{Dir}$
$(\text{current state}, \text{current type}, \text{current symbol}, \text{new state}, \text{movement}) \in \Delta$

- TWA

5 Infinite Tree Automata

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