Cantor Space

$(\mathbb{B}^\omega, d)$ with $d(\alpha, \beta) = \begin{cases} 0 & \text{if } \alpha = \beta \\ 2^{-(\min_n \alpha(n) \neq \beta(n))} & \text{else} \end{cases}$

$\frac{1}{2n}$-neighborhood of α: $\alpha[0, n] \cdot \mathbb{B}^\omega$

Cantor Topology

Open Sets: $\mathcal{O} = \{W \cdot \mathbb{B}^\omega \mid W \subseteq B^*\}$

Borel Hierarchy

$\Sigma_1 = \mathcal{O}$
$\Pi_1 = \mathbb{B}^\omega \setminus \mathcal{O}$
$\Sigma_{n+1} = \{\bigcup_{i \in \mathbb{N}} L_i \mid L_i \in \Pi_n\}$
$\Pi_{n+1} = \{\bigcap_{i \in \mathbb{N}} L_i \mid L_i \in \Sigma_n\}$

Relation to Automata

- regular Σ_1 = E-recognizable
- regular Π_1 = A-recognizable
- regular Σ_2 = co-Büchi-recognizable
- regular Π_2 = DBA-recognizable