1 List Of Models

1.1 Infinite Words

- DBA
- NBA
- GBA
- Rabin automaton
- Muller automaton
- Parity automaton
- E automaton
- A automaton
- coBA
- weak BA
- Staiger-Wagner automaton
- ABA
- LTL
- S1S
- ∃S1S
- $S1S_0$

1.2 Finite Trees

- DTA
- NTA
- ↓DTA
- ↓NTA
- DUTA
- NUTA
- deterministic DTD
- DTD
- deterministic EDTD

- single-type EDTD
- EDTD
- Relax NG
- FO
- MSO
- Regular expressions
- DTWA
- TWA

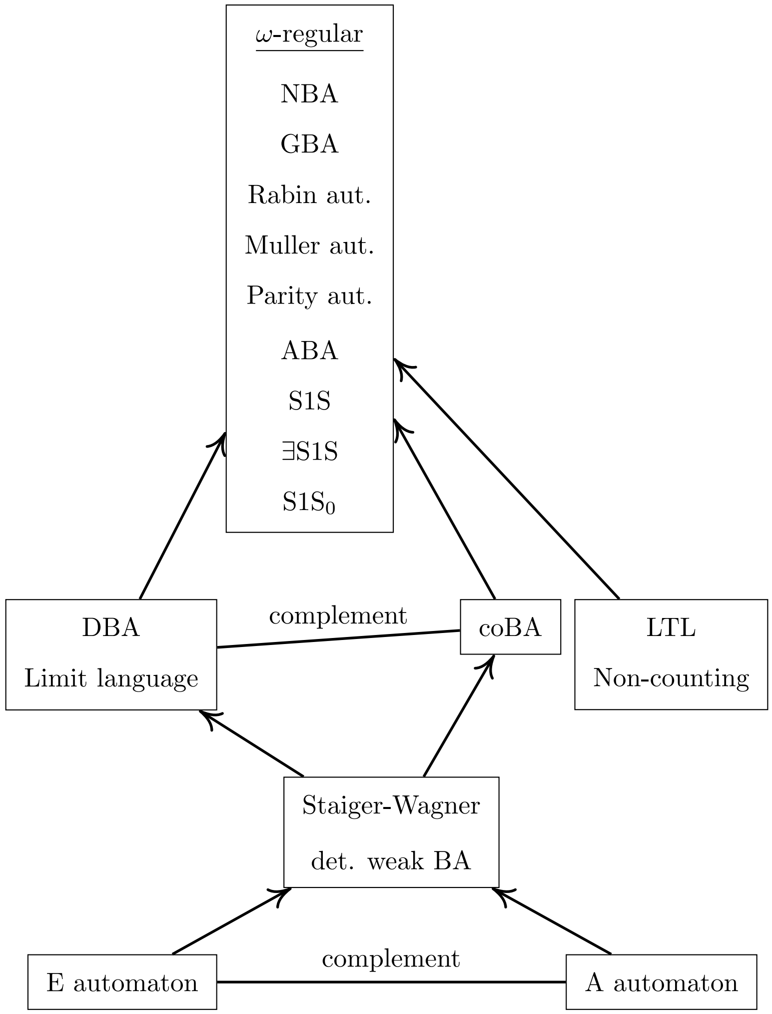
1.3 Infinite Trees

- BTA
- Muller TA
- Parity TA
- DMTA
- S2S (MSO / WMSO)
- $S2S_0$ (MSO / WMSO)

2 List Of Games

- Büchi
- Staiger-Wagner
- weak Parity
- Reachability
- Safety
- Muller
- Parity
- Rabin
- Streett
- Gale-Stewart
- Wadge

3 Infinite Word Models



3.1 Class Inclusions

- E aut. $\subseteq$ Staiger-Wagner
  Proof: SWA with $\mathcal{F} = \{Q' \subseteq Q \mid F \cap Q' \neq \emptyset\}$.

- A. aut. $\subseteq$ Staiger-Wagner
  Proof: SW closed under complement,

- Staiger-Wagner $\subseteq$ DBA / coBA
  Proof: $\mathcal{A}$ SWA $\Rightarrow \mathcal{A}' = (Q \times 2^Q, \Sigma, (q_0, \emptyset), \delta', F')$
  Collect all visited states and accept if that set stays in $\mathcal{F}$.

- DBA $\subseteq$ NBA
  trivial

- coBA $\subseteq$ NBA
  Proof: NBA closed under complement.

- LTL $\subseteq$ NBA
  Proof: **??**

- LTL $\subseteq$ ABA
  Proof: **??**

3.2 Class Exclusions

- E aut. $\not\subseteq$ A aut.
  Example: $(a+b)^*a(a+b)^\omega$
  Proof: **??**

- A aut. $\not\subseteq$ E aut.
  Example: $\{a^\omega\}$
  Proof: **??**

- DBA $\not\subseteq$ coBA
  Example: $(a^*b)^\omega$
  Proof: **??**

- coBA $\not\subseteq$ DBA
  Example: $(a+b)^*a^\omega$
  Proof: **??**

- LTL $\not\subseteq$ NBA
  Example: $((a+b)a)^\omega$
  Proof: **??**

3.3 Class Equalities

3.3.1 NBA

- NBA $\Rightarrow$ ω-regular
  Proof: **??**

- ω-regular $\Rightarrow$ NBA
  Proof: **??**

- NBA $\Rightarrow$ $\exists$S1S
  Proof: **??**

- S1S $\Rightarrow$ $\mathrm{S1S}_0$
  Proof: **??**

- $\mathrm{S1S}_0 \Rightarrow$ NBA
  Proof: **??**

- Det. Muller $\Rightarrow$ NBA
  Proof: NBA with $L(\mathcal{A}) = \bigcup_{F \in \mathcal{F}} \left(\bigcap_{q \in F} L(\mathcal{A}_q) \cap \bigcap_{q \notin F} \overline{L(\mathcal{A}_q)} \right)$ where $\mathcal{A}_q$ is $\mathcal{A}$ starting in q.

- NBA $\Rightarrow$ det. Muller
  Proof: **??**

- (det.) Muller $\Rightarrow$ (det.) Parity
  Proof: **??**

- ABA $\Rightarrow$ NBA
  Proof: **??**

3.3.2 LTL

LTL $\Leftrightarrow$ Non-counting
No proof. Remarks in F8.

3.3.3 SW

Staiger-Wagner $\Leftrightarrow$ det. weak BA
Proof: **??**

3.4 Closures

3.4.1 NBA

- Closed under union
  Proof: **??**

- Closed under intersection
  Proof: **??**

- Closed under complement
  Proof: **??**

3.4.2 DBA

- Not closed under complement (inf. many $a \leftrightarrow$ fin. many a)

3.4.3 SW

- Closed under union
  Proof: **??**

- Closed under intersection
  Proof: **??**

- Closed under complement
  Proof: **??**

3.5 Characterizations

- Parity conditions are directly convertible to Rabin chain conditions and vice-versa.
  Proof: Assign priorities in ascending order; $E_k \to 0$, $F_k \setminus E_k \to 1$, $E_{k-1} \setminus F_k \to 2 \ldots$
- U is ω-regular iff U is a Boolean combination of DBA-languages
  Proof: NBAs are closed under Boolean operations.
- U is DBA-recog. iff $U = \lim(L)$ for some regular $L \subseteq \Sigma^*$.
  Proof: **??**
- U is E-recog. iff $U = L \cdot \Sigma^*$ for some regular $L \subseteq \Sigma^*$.
  Proof: **??**
- Landweber's theorem
  Proof: **??**
- DBA $\cap$ coBA $\Rightarrow$ SW
  Proof: **??**

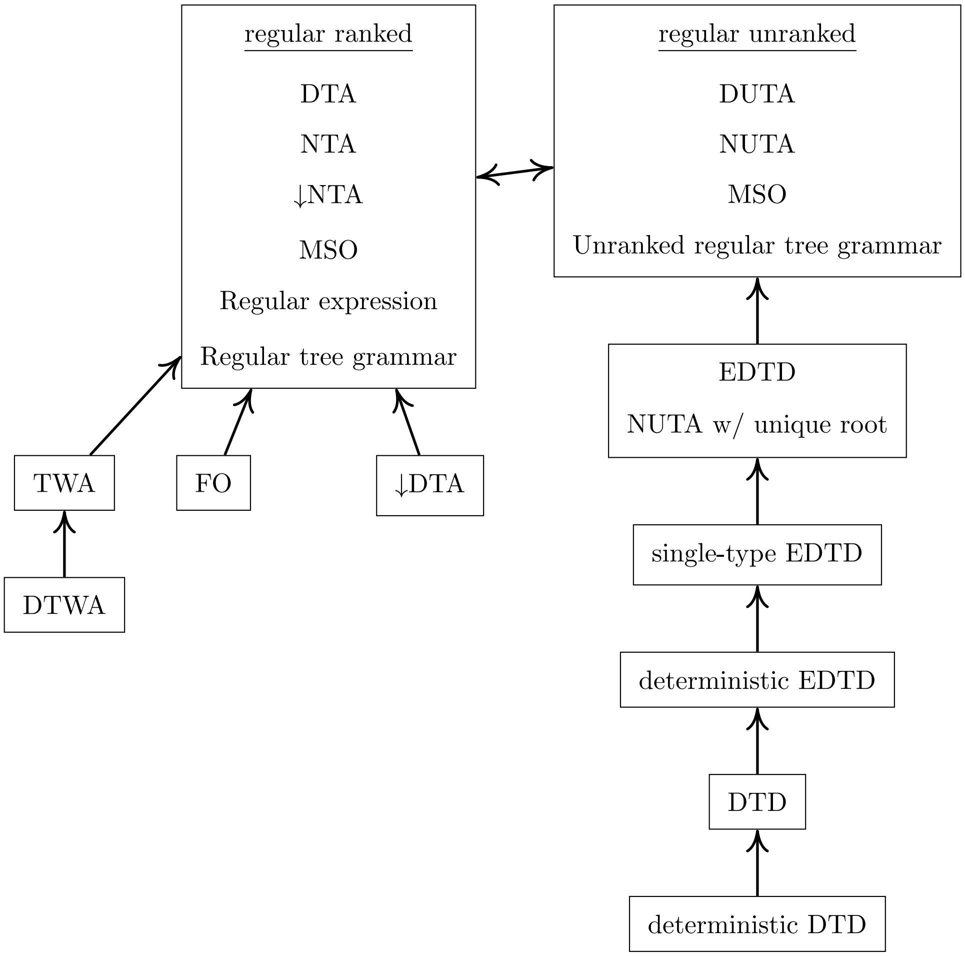
3.6 Duality

- U is A-recog. iff $\Sigma^\omega \setminus U$ is E-recog.
  Proof: **??**
- U is coBA-recog. iff $\Sigma^\omega \setminus U$ is DBA-recog.
  Proof: **??**

3.7 Problems / Complexity

- Emptiness problem for NBAs is decidable in poly. time.
  Proof: **??**
- Emptiness problem for ABAs is PSPACE-complete.
  No proof. Remarks in F23.
- Membership problem for ABAs is decidable in poly. time.

4 Finite Tree Models



4.1 Class Inclusions

- Regular Ranked $\subseteq$ Regular Unranked
  Proof:
- det. DTD $\subseteq$ DTD $\subseteq$ det. EDTD $\subseteq$ single-type EDTD $\subseteq$ EDTD
  trivial
- EDTD $\subseteq$ Regular tree grammar
  Proof: $N = \Sigma'$, $P_{\text{gram}} = P \cup \{a^{(n)} \to a \mid a, n\}$
- TWA $\subseteq$ NTA
  Proof: **??**

4.2 Class Exclusions

- $\downarrow$DTA $\not\subseteq$ NTA
  Example: $T = \{f(a,b), f(b,a)\}$
  Proof: **??**

- DTD $\not\subseteq$ single-type EDTD
  Example: $T = \{t \in T_{\{a,b\}} \mid$ there is a path in t on which a occurs exactly twice

- NUTA w/ unique root $\not\subseteq$ NUTA
  Example: $T = \{a, b\}$

- FO $\not\subseteq$ MSO
  Example: T = positive boolean terms that evaluate to true

- DTWA $\not\subseteq$ TWA
  Example: $T_{N \setminus D}$
  $\Sigma_0 = \{a, b\}$, $\Sigma_2 = \{f\}$
  $t \in T_{N \setminus D}$ iff $|t|_a = 3 \wedge \mathrm{lca}(u,v) \sqsubseteq \mathrm{lca}(v,w)$ **Proof**: **??**

- TWA $\not\subseteq$ NTA
  Example: all paths in the skeleton have even length
  Proof: **??**

4.3 Class Equalities

4.3.1 Regular Ranked

- NTA $\Rightarrow$ DTA
  Proof: Subset construction.

- NTA $\Leftrightarrow$ $\downarrow$NTA
  Proof: **??**

- $\downarrow$NTA $\Leftrightarrow$ Regular Tree Grammar
  Proof: **??**

- MSO $\Leftrightarrow$ NTA
  Proof: **??**

- Reg. exp. $\Leftrightarrow$ NTA
  Proof: **??**

4.3.2 Regular Unranked

- NUTA $\Rightarrow$ DUTA
  Proof:

- MSO $\Leftrightarrow$ NUTA
  Proof:

4.3.3 EDTD

- NUTA with unique root $\Leftrightarrow$ EDTD (in poly. time)
  Proof:

4.4 Closures

4.4.1 Regular Ranked

- Regular (ranked) trees are closed under complement.
  Proof: **??**

- Regular (ranked) trees are closed under union.
  Proof: **??**

- Regular (ranked) trees are closed under intersection.
  Proof: **??**

4.4.2 Regular Unranked

- Regular unranked trees are closed under complement, union, and intersection.
  Proof: via FCNS

4.4.3 TWA

- TWAs are closed under union.
  Proof:

- TWAs and DTWAs are closed under intersection.
  Proof:

- Open Problem: Are TWAs closed under complement?

- DTWAs are closed under complement.
  Proof:

4.5 Problems / Complexity

4.5.1 Regular Ranked

- Membership problem for NTAs is decidable.

- Reachable states of NTAs can be computed in linear time in $|\mathcal{A}|$.

- Emptiness of an NTA can be decided in linear time in $|\mathcal{A}|$.
  Algorithm: $T(\mathcal{A}) = \emptyset$ iff Reachable$(\mathcal{A}) \cap F = \emptyset$

- Given a DTA $\mathcal{A}$, $\sim_{T(\mathcal{A})}$ can be computed in time poly$(|Q^m \times \Sigma \times Q|)$ where m is the maximal arity in Σ.
  Algorithm:

4.5.2 Regular Unranked

- Emptiness / membership / inclusion for NUTAs is decidable in polynomial / polynomial / exponential time.
  Algorithm:

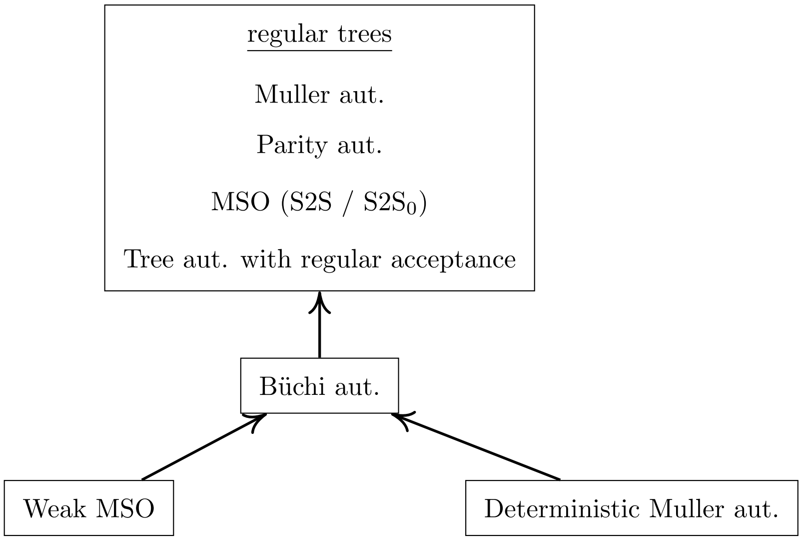
- Inclusion for complete DUTAs is decidable in polynomial time.
  Algorithm:

4.5.3 Grammars

- Emptiness / membership for EDTDs is decidable in polynomial time.
  Proof: EDTD can be converted to NUTA in polynomial time.

- Inclusion for deterministic EDTDs is decidable in polynomial time.
  Proof:

5 Infinite Tree Models



5.1 Class Exclusions

- BTA $\not\subseteq$ Regular tree
  Example: $T_{\text{fin}} = \{t \in T_{\{a,b\}} \mid \text{every infinite path has only fin. many } b\}$
  Proof:

- DMTA $\not\subseteq$ BTA
  Example: $T_{\text{fin}}^{\complement} = \{t \in T_{\{a,b\}} \mid \text{there is a path with inf. many } b\}$
  Proof: BTA-recognizable (see below) but not a path tree language.

- WMSO $\not\subseteq$ BTA
  Example: $\{t \in T_{\{a,b\}} \mid \text{every infinite path has infinitely many } b\}$
  Proof:

5.2 Class Equalities

5.2.1 Regular Trees

- PTA, MTA $\Rightarrow$ TA with regular acceptance
  Proof: By definition and PA/MA regularity.

- TA with reg. acc. $\Rightarrow$ PTA
  Proof: TA $\mathcal{A}$, DPA $\mathcal{A}'$ over alphabet Q that defines Acc. Define PTA with state space $Q \times P$.
  $(q, a, q_1, q_2) \in \Delta \Rightarrow ((q, p), a, (q_1, \delta(p, q)), (q_2, \delta(p, q))) \in \Delta'$

- PTA $\Leftrightarrow$ MSO (S2S) **Proof**: **??**

5.3 Closures

5.3.1 Regular Trees

- The class of regular tree languages is closed under union and intersection.
  Proof: **??**
- The class of regular tree languages is closed under projection.
  Proof: **??**
- PTAs are closed under complement ($2^{\mathcal{O}(kn\cdot\log(kn))}$ states for $|Q| = n$, $|\mathrm{img}(c)| = k$) **Proof**: **??**

5.3.2 BTA

- BTAs are not closed under complement.
  Proof: $T_{\text{fin}}^{\complement} = \{t \in T_{\{a,b\}} \mid \text{there is a path with inf. many } b\}$
  $T_{\text{fin}}^{\complement}$ is BTA-recognizable but its complement is not.

5.3.3 DMTA

- DMTAs are not closed under union or complement.
- DMTAs are closed under intersection.
  Proof: product automaton

5.4 Characterizations

- $T \subseteq T_\Sigma$ is DMTA-recognizable iff T is a path tree language.
  $T : 2^{(\{0,1\}\times\Sigma)^\omega} \to 2^{T_\Sigma}, L \mapsto \{t \in T_\Sigma \mid \forall \pi \in \{0,1\}^\omega : \pi^\frown(t|_\pi) \in L\}$
  Proof:
- $T \subseteq T_\Sigma$ is WMSO-definable iff T and $T^{\complement}$ are BTA-recognizable.

5.5 Problems

5.5.1 Regular Trees

- The membership problem for regular tree languages is decidable.
  Proof: via membership game as in complementation
- The emptiness problem for regular tree languages is decidable.
  Proof: **??**