Definition 0.0.1. Let $\mathcal{A} = (Q, \Sigma, q_0, \delta, c)$ be a deterministic parity automaton. For $w \in \Sigma^* \cup \Sigma^\omega$ and $q \in Q$, we define $\lambda_\mathcal{A}(q, w) \in \mathbb{N}^{1+|w|}$ as follows: Let $q_0 q_1 \cdots \in Q^{1+|w|}$ be the unique run of $\mathcal{A}$ on w. Then $\lambda_\mathcal{A}(q, w)(n) = c(q_n)$.

Two DPAs $\mathcal{A}$ and $\mathcal{B}$ are **priority almost-equivalent**, if for all words $\alpha \in \Sigma^\omega$, $\lambda_\mathcal{A}(q_0^\mathcal{A}, \alpha)$ and $\lambda_\mathcal{B}(q_0^\mathcal{B}, \alpha)$ differ in only finitely many positions. We call two states $p, q \in Q$ of $\mathcal{A}$ priority almost-equivalent, $\mathcal{A}_q$ and $\mathcal{A}_p$ are priority almost-equivalent, where $\mathcal{A}_q$ behaves like $\mathcal{A}$ with initial state q.

We define the **reachability order** $\preceq_\text{reach}^\mathcal{A} \subseteq Q \times Q$ as $p \preceq_\text{reach}^\mathcal{A} q$ iff q is reachable from p. ("p is closer to q_0 than q"). Note that $p \preceq_\text{reach}^\mathcal{A} q$ and $q \preceq_\text{reach}^\mathcal{A} p$ together mean that p and q reside in the same SCC.

Lemma 0.0.1. *Priority almost-equivalence is a congruence relation.*

Definition 0.0.2. Let $\mathcal{A} = (Q, \Sigma, q_0, \delta, c)$ be a DPA and let $\sim \subseteq Q \times Q$ be a congruence relation on $\mathcal{A}$. We define the **Schewe** automaton $\mathcal{S}$ as follows:

For each state q, let $[q]_\sim = \{p \in Q \mid q \sim p\}$ be its equivalence class of $\sim$ and let $Q/_\sim = \{[q]_\sim \mid q \in Q\}$ be the set of equivalence classes. For each such class $\mathfrak{c}$ we fix a representative $r_\mathfrak{c} \in \mathfrak{c}$ which is $\preceq_\text{reach}^\mathcal{A}$-maximal in its class, meaning that all states in $\mathfrak{c}$ that are reachable from $r_\mathfrak{c}$ are also in its SCC.

The automaton is then almost the same as the original DPA, with only a few modifications. Namely, $\mathcal{S} = (Q, \Sigma, r_{[q_0]_\sim}, \delta_\mathcal{S}, c)$.

For each transition $\delta_\mathcal{S}(q, a)$, let $\delta(q, a) = p$. If $q \prec_\text{reach}^\mathcal{A} r_{[p]_\sim}$ (i.e. q is not reachable from the representative of $[p]_\sim$), then $\delta_\mathcal{S}(q, a) = r_{[p]_\sim}$. Otherwise, we keep $\delta_\mathcal{S}(q, a) = p$. In other words, every time a transition moves to a different quotient class, it skips to the representative which lies as "deep" inside the automaton as possible.

Lemma 0.0.2. *For a given $\mathcal{A}$ and $\sim$, the Schewe automaton $\mathcal{S}$ can be computed in $\mathcal{O}(|\mathcal{A}|)$.*

Proof. Using e.g. Kosaraju's algorithm **??**, the SCCs of $\mathcal{A}$ can be computed in $\mathcal{O}(|\mathcal{A}|)$ if we assume an adjacency list or similar as the underlying data structure. A topological sorting of the states and therefore the SCCs can then be computed in $\mathcal{O}(|\mathcal{A}|)$ again, e.g. by DFS. □

We focus on a specialized version of the Schewe automaton. Let $\sim$ be the priority equivalence and let $\mathcal{S}$ be the according automaton. We define $\mathcal{S}_m$ as the Moore-minimization of $\mathcal{S}$.

Lemma 0.0.3. *Priority almost-equivalence implies language equivalence.*

Proof. Let $\mathcal{A} = (Q_\mathcal{A}, \Sigma, q_0^\mathcal{A}, \delta_\mathcal{A}, c_\mathcal{A})$ and $\mathcal{B} = (Q_\mathcal{B}, \Sigma, q_0^\mathcal{B}, \delta_\mathcal{B}, c_\mathcal{B})$ be two DPA that are priority almost-equivalent and assume towards a contradiction that they are not language equivalent. Due to symmetry we can assume that there is a $w \in L(\mathcal{A}) \setminus L(\mathcal{B})$.

Consider $\alpha = \lambda_\mathcal{A}(q_0^\mathcal{A}, w)$ and $\beta = \lambda_\mathcal{B}(q_0^\mathcal{B}, w)$, the priority outputs of the automata on w. By choice of w, we know that $a := \max \mathrm{Inf}(\alpha)$ is even and $b := \max \mathrm{Inf}(\beta)$ is odd. Without loss of generality, assume $a > b$. That means a is seen only finitely often in β but infinitely often in a. Hence, α and β differ at infinitely many positions where a occurs in α. That would mean w is a witness that the two automata are not priority almost-equivalent, contradicting our assumption. □

Definition 0.0.3. Let $\mathcal{A} = (Q, \Sigma, q_0, \delta, c)$ be a DPA and $\sim \subseteq Q \times Q$ a congruence relation. We call $\sim$ a **finite difference relation** if for all $p, q \in Q$ and all $\alpha \in \Sigma^\omega$, there is a finite prefix $w \sqsubset \alpha$ such that $\delta^*(p, w)$ and $\delta^*(q, w)$ are language equivalent.

Examples of finite difference relations include the priority almost-equivalence relation and language equivalence relation.

0.0.1 Structure of the Schewe automaton

For this section, we consider the Schewe automaton $\mathcal{S} = (Q, \Sigma, r_{[q_0]_\sim}, \delta_\mathcal{S}, c)$ of a DPA $\mathcal{A} = (Q, \Sigma, q_0, \delta_\mathcal{A}, c)$ with a congruence relation $\sim$.

Lemma 0.0.4. *Let $p, q \in Q$ such that $p \preceq^\mathcal{A}_{\text{reach}} q$. Then $r_{[q]_\sim} \not\prec^\mathcal{A}_{\text{reach}} r_{[p]_\sim}$.*

Proof. We have to show that $r_{[q]_\sim}$ and $r_{[p]_\sim}$ lie in the same SCC or are not connected at all. For that, let $r_{[q]_\sim} \preceq^\mathcal{A}_{\text{reach}} r_{[p]_\sim}$. We will prove that $r_{[p]_\sim} \preceq^\mathcal{A}_{\text{reach}} r_{[q]_\sim}$ holds as well.

Let $w \in \Sigma^*$ with $p \xrightarrow[\mathcal{A}]{w} q$. Since $p \sim r_{[p]_\sim}$ and $\sim$ is a congruence relation, we also have $r_{[p]_\sim} \xrightarrow[\mathcal{A}]{w} q'$ for some $q' \sim q$. Now we find $r_{[q]_\sim} \preceq^\mathcal{A}_{\text{reach}} r_{[p]_\sim} \preceq^\mathcal{A}_{\text{reach}} q'$. Since $q' \sim r_{[q]_\sim}$ and the representatives are chosen as $\preceq^\mathcal{A}_{\text{reach}}$-maximal elements, we have $q' \preceq^\mathcal{A}_{\text{reach}} r_{[q]_\sim}$ via some word v. Hence, $r_{[q]_\sim}$ is reachable from $r_{[p]_\sim}$ via wv. $\square$

Lemma 0.0.5. *For all $q \in Q$ and $a \in \Sigma$, $\delta_\mathcal{A}(q, a) \sim \delta_\mathcal{S}(q, a)$.*

Proof. This statement is rather trivial. If $\delta_\mathcal{A}(q, a) = \delta_\mathcal{S}(q, a)$, then it is implied by the reflexivity of $\sim$. Otherwise we have $\delta_\mathcal{S}(q, a) = r_{[\delta_\mathcal{A}(q,a)]_\sim}$ which is equivalent to $\delta_\mathcal{A}(q, a)$ by definition of the representative. $\square$

Lemma 0.0.6. *Let $\rho \in Q^\omega$ be an infinite run in $\mathcal{S}$ starting at a reachable state. Then ρ has a suffix that is a run in $\mathcal{A}$.*

Proof. Let $\rho = p_0 \cdots$ be the run and let $r_i = r_{[p_i]_\sim}$ for all i. Let $I \subseteq \mathbb{N}$ be the set of positions at which a "redirected" transition is taken, i.e. $\delta_\mathcal{S}(p_i, \alpha(i)) \neq \delta_\mathcal{A}(p_i, \alpha(i))$.

Claim 1 I is finite.

In particular, it has as most as many elements as there are equivalence classes in $\sim$. Assume the opposite towards a contradiction. There are positions $i_0, \ldots, i_k \in I$ which describe a cycle in the visited equivalence classes. More formally we want the sequence $(i_j)_{0 \le j \le k}$ to satisfy:

- The indices are strictly growing, i.e. $i_0 < i_1 < \cdots < i_k$.
- Between i_0 and i_k, all transitions not at the given indices are transitions from $\mathcal{A}$ (i.e. not redirected transitions).
- $r_{i_j} \prec^\mathcal{A}_{\text{reach}} r_{i_j+1} = p_{i_j+1}$.
- $p_{i_0+1} \sim p_{i_k+1}$.

Let $\mathfrak{C}$ be the set of equivalence classes which occur in the run between i_0 and i_k. For each representative r_{i_j+1}, its SCC in $\mathcal{A}$ contains a state in $\mathfrak{c}$ for every $\mathfrak{c} \in \mathfrak{C}$: there are words to reach r_{i_k+1} from r_{i_j+1} and to reach r_{i_j+1} from r_{i_0+1}. By Lemma **??**, we can use these words to find a run in $\mathcal{A}$ starting at r_{i_j+1}, ending in a state in $[p_{i_j+1}]_\sim$ and visiting all classes $\mathfrak{C}$ inbetween.

Now consider an arbitrary redirected transition, e.g. the first one. The definition of the Schewe automaton tells us that $r_{i_0} \prec^\mathcal{A}_{\text{reach}} r_{i_0+1}$. From r_{i_0+1} we can find a state $t \sim r_{i_0}$ as we have just shown, so $r_{i_0+1} \preceq^\mathcal{A}_{\text{reach}} t$. However, r_{i_0} is a $\preceq^\mathcal{A}_{\text{reach}}$-maximal element in its class, so $t \preceq^\mathcal{A}_{\text{reach}} r_{i_0} \prec^\mathcal{A}_{\text{reach}} r_{i_0+1} \preceq^\mathcal{A}_{\text{reach}} t$, which is a contradiction.

Claim 2 ρ has a suffix that is a run in $\mathcal{A}$.

I is finite, so let $k = 1 + \max I$. That means, from position k onwards, we have $\delta_{\mathcal{S}}(p_i, \alpha(i)) = \delta_{\mathcal{A}}(p_i, \alpha(i))$ for all i. Hence, the suffix $\pi = \rho(k)\rho(k+1)\cdots$ is a run in $\mathcal{A}$ on the word $\alpha(k)\alpha(k+1)\cdots$. $\square$

Lemma 0.0.7. *Let $R \subseteq Q$ be the states that are reachable in $\mathcal{S}$. If $\sim$ implies language equivalence on R in $\mathcal{A}$, then it is also implies language equivalence in $\mathcal{S}$.*

Proof. Let $p, q \in R$ be reachable states with $p \sim q$ and let $\alpha \in \Sigma^\omega$ be arbitrary. We show that $\mathcal{S}_p$ accepts α if and only if $\mathcal{S}_q$ accepts α.

Let ρ_p and ρ_q be the runs of $\mathcal{S}$ on α which start at p and q respectively. By Lemma **??**, there are suffixes π_p and π_q of these runs in $\mathcal{A}$. The "offset" of a suffix, i.e. the starting point in the original string, can be increased without violating this property. By choosing the greater offset of both runs, we can assume that π_p and π_q start at the same offset k.

Because $\sim$ is a congruence relation, $p \sim q$, and Lemma **??**, we know that $p' = \pi_p(0) = \rho_p(k) \sim q' = \pi_q(0) = \rho_q(k)$. Let $\beta = \alpha(k)\alpha(k+1)\cdots$. Since $\sim$ implies language equivalence in $\mathcal{A}$, $\mathcal{A}_{p'}$ accepts β if and only if $\mathcal{A}_{q'}$ accepts β. The parity condition is prefix independent, so this is extended to $\mathcal{S}_p$ and $\mathcal{S}_q$. $\square$

Lemma 0.0.8. *Let p and q be reachable states in $\mathcal{S}$ with $p \sim q$. If there is a path in $\mathcal{A}$ from p to q, then that same path exists in $\mathcal{S}$.*

Proof. Let $\rho = p_0 \cdots p_m$ be the path with $p_0 = p$ and $p_m = q$. Assume towards a contradiction that there is a position k at which this path is not valid in $\mathcal{S}$ anymore, i.e. $\delta_{\mathcal{S}}(p_k, a) \neq p_{k+1}$. Let this k be minimal. We split the word that is used for ρ into parts $u \in \Sigma^*$, $a \in \Sigma$, and $v \in \Sigma^*$ such that $p_0 \overset{u}{\underset{\mathcal{A}}{\rightarrow}} p_k \overset{a}{\underset{\mathcal{A}}{\rightarrow}} p_{k+1} \overset{v}{\underset{\mathcal{A}}{\rightarrow}} p_m$.

We consider the states reached by $\mathcal{A}$ if it would use the redirected transition at k and analyze their relation w.r.t. $\sim$. Let $p'_{k+1} = \delta_{\mathcal{S}}(p_k, a)$ and let $p'_{k+2}, \ldots, p'_m$ be the following run of $\mathcal{A}$ on v. Then let $s = \delta^*_{\mathcal{A}}(p_m, u)$ and $s' = \delta^*_{\mathcal{A}}(p'_m, u)$. As $\sim$ is a congruence relation, $p \sim q$, and Lemma **??**, we have $s \sim s'$. With the same argument we also find $s \sim \delta^*_{\mathcal{A}}(p_0, u) = p_k$, so $s' \sim p_k$.

On the other hand, we can analyze reachability between the states. k is a position where the transition is redirected which implies $r_{[p_k]_\sim} \prec^{\mathcal{A}}_{\text{reach}} r_{[p_{k+1}]_\sim}$ by definition of the construction. Redirected transitions in $\mathcal{S}$ always lead to representatives of the original target, so $p'_{k+1} = r_{[p_{k+1}]_\sim}$. The definition of s' shows $p'_{k+1} \preceq^{\mathcal{A}}_{\text{reach}} s'$. Putting these two statements together we obtain $r_{[p_k]_\sim} \prec^{\mathcal{A}}_{\text{reach}} s'$.

However, $r_{[p_k]_\sim}$ is a $\preceq^{\mathcal{A}}_{\text{reach}}$-maximal element in its class, which implies that $r_{[p_k]_\sim} \not\prec s'$. This stands in contradiction to our previous observation that $p_k \sim s'$. $\square$

Lemma 0.0.9. *Let $\mathcal{A}$ a DPA, $\sim$ the relation of priority almost-equivalence, and $\mathcal{S}$ be the Schewe automaton. Then $\mathcal{A}$ and $\mathcal{S}$ are priority almost-equivalent.*

Proof. Let $\mathcal{A} = (Q, \Sigma, q_0^{\mathcal{A}}, \delta_{\mathcal{A}}, c)$ and $\mathcal{S} = (Q, \Sigma, q_0^{\mathcal{S}}, \delta_{\mathcal{S}}, c)$. Let $\alpha \in \Sigma^\omega$. We have to show that $\lambda_{\mathcal{A}}(q_0^{\mathcal{A}}, \alpha)$ and $\lambda_{\mathcal{S}}(q_0^{\mathcal{S}}, \alpha)$ differ in only finitely many positions. For that, let $a_0 a_1 \cdots \in Q^\omega$ and $s_0 s_1 \cdots \in Q^\omega$ be the respective runs of the automata on α.

Claim 1 For all i, $\mathcal{A}_{a_i}$ and $\mathcal{A}_{s_i}$ are priority almost-equivalent. ($a_i \sim s_i$)

For $i = 0$, we have $a_i = q_0^{\mathcal{A}}$ and $s_i = q_0^{\mathcal{S}} = r_{[q_0^{\mathcal{A}}]_\sim}$. By definition, $s_i = r_{[q_0^{\mathcal{A}}]_\sim} \in [q_0^{\mathcal{A}}]_\sim$, so $s_i \sim a_i$.

Using induction, assume $a_i \sim s_i$ and consider $i+1$. We seperate two cases: If $\delta_{\mathcal{S}}(s_i, \alpha(i)) = \delta_{\mathcal{A}}(s_i, \alpha(i))$, then $a_{i+1} \sim s_{i+1}$ follows from $\sim$ being a congruence relation.

For the second case, consider $\delta_{\mathcal{S}}(s_i, \alpha(i)) = r_{[p]_\sim}$, where $\delta_{\mathcal{A}}(s_i, \alpha(i)) = p$. Again we have $s_{i+1} = r_{[p]_\sim} \in [p]_\sim$, so $s_{i+1} \sim p$. Since $\sim$ is a congruence relation, we have $\delta_{\mathcal{A}}(a_i, \alpha(i)) \sim \delta_{\mathcal{A}}(s_i, \alpha(i))$, which is $a_{i+1} \sim p$ and therefore $a_{i+1} \sim s_{i+1}$ by transitivity.

Claim 2 $\lambda_{\mathcal{A}}(q_0^{\mathcal{A}}, \alpha)$ and $\lambda_{\mathcal{S}}(q_0^{\mathcal{S}}, \alpha)$ differ in only finitely many positions.

We can see from the definition of $\mathcal{S}$ that the "new type of transition" is taken only when the target state of $\delta_{\mathcal{A}}$ is strictly bigger w.r.t. $\preceq^{\mathcal{A}}_{\text{reach}}$. Since this is a partial order on Q, this means in particular that from some point k onwards, only the transition type $\delta_{\mathcal{S}}(q, a) = \delta_{\mathcal{A}}(q, a)$ is taken. Hence, $s_k s_{k+1} \cdots$ is the valid run of $\mathcal{A}_{s_k}$ on some suffix β of α. If now $\lambda_{\mathcal{A}}(q_0^{\mathcal{A}}, \alpha)$ and $\lambda_{\mathcal{S}}(q_0^{\mathcal{S}}, \alpha)$ would differ at infinitely many positions, then also $\lambda_{\mathcal{A}}(a_k, \beta)$ and $\lambda_{\mathcal{A}}(s_k, \beta)$ would (as these are suffixes of the former). However, we have shown in claim 1 that $\mathcal{A}_{a_k}$ and $\mathcal{A}_{s_k}$ are priority almost-equivalent. □

Lemma 0.0.10. *Let $\mathcal{A}$ a DPA, $\sim$ the relation of priority almost-equivalence, and $\mathcal{S}$ be the Schewe automaton. If p and q are priority almost-equivalent and reachable states in $\mathcal{S}$, then they lie in the same SCC.*

Proof. Let $\sim$ the relation of priority almost-equivalence.

Claim 1 If p and q are reachable in $\mathcal{S}$ and $\mathcal{A}_p$ and $\mathcal{A}_q$ are priority almost-equivalent, then $\mathcal{S}_p$ and $\mathcal{S}_q$ are priority almost-equivalent.

Assume towards a contradiction that p, q form a counterexample with $\lambda_{\mathcal{S}}(p, \alpha)$ and $\lambda_{\mathcal{S}}(q, \alpha)$ differing at infinitely many positions. Let ρ_p, ρ_q, π_p, and π_q be the runs of $\mathcal{A}_p$, $\mathcal{A}_q$, $\mathcal{S}_p$, and $\mathcal{S}_q$ on α respectively. Similar to the proof of **??**, we can

Claim 2 Let p and q be reachable states in $\mathcal{S}$ that are priority almost-equivalent and in $\mathcal{A}$ there is a path ρ from p to q. Then ρ is also a valid path in $\mathcal{S}$.

Let $\rho = p_0 \cdots p_m$ with $p_0 = p$ and $p_m = q$. Assume towards a contradiction that k is the first position at which this path is not valid in $\mathcal{S}$, i.e. $\delta_{\mathcal{S}}(p_k, a) \neq p_{k+1}$. That means that the transition was redirected to $\delta_{\mathcal{S}}(p_k, a) = r_{[p_{k+1}]_\sim}$. Therefore, $r_{[p_k]_\sim} \prec^{\mathcal{A}}_{\text{reach}} r_{[p_{k+1}]_\sim}$ by definition of the Schewe automaton. Let $u, v \in \Sigma^*$ with $p_0 \underset{\mathcal{A}}{\overset{u}{\to}} p_k \underset{\mathcal{A}}{\overset{a}{\to}} p_{k+1} \underset{\mathcal{A}}{\overset{v}{\to}} p_m$.

We define $\delta^*_{\mathcal{A}}(p_{k+1}, vu) = s$ and $\delta^*_{\mathcal{A}}(r_{[p_{k+1}]_\sim}, vu) = t$. Since $\mathcal{A}_{p_{k+1}}$ and $\mathcal{A}_{r_{[p_{k+1}]_\sim}}$ are priority almost-equivalent and $\sim$ is a congruence relation, also $\mathcal{A}_s$ and $\mathcal{A}_t$ are priority almost-equivalent. The former is priority-almost equivalent to $\mathcal{A}_{p_k}$. That means, in $\mathcal{A}$, we have $t \sim s \sim p_k \sim r_{[p_k]_\sim}$.

On the other hand we have shown that via vu we can reach t from $r_{[p_{k+1}]_\sim}$, so $r_{[p_{k+1}]_\sim} \preceq^{\mathcal{A}}_{\text{reach}} t$ and therefore $r_{[p_k]_\sim} \preceq^{\mathcal{A}}_{\text{reach}} t$. This together with the fact that $t \sim r_{[p_k]_\sim}$ contradicts the choice of $r_{[p_k]_\sim}$ as a $\preceq^{\mathcal{A}}_{\text{reach}}$-maximal element in its equivalence class. Hence, k cannot exist and ρ is a valid path in $\mathcal{S}$.

Claim 3 In the (not-minimized) Schewe automaton $\mathcal{S}$, for all reachable states q we have $q \preceq^{\mathcal{S}}_{\text{reach}} r_{[q]_\sim}$.

Let q be a reachable state, so there is a run $q_0 \cdots q_m$ in $\mathcal{S}$ on w with $q_m = q$. By definition of the automaton, there is a position k on this run at which a $\sim$-equivalence class $\mathfrak{c}$ is entered with $r_{[q]_\sim} \preceq^{\mathcal{A}}_{\text{reach}} r_{\mathfrak{c}}$. At latest this happens when $[q]_\sim$ is reached for the first time, as then the transition is directed to $r_{[q]_\sim}$ directly. Let $k' \geq k$ be the position in this run at which $[q]_\sim$ is entered for the first time.

Let $p_k \cdots p_{k'}$ be the run of $\mathcal{A}$ on $w(k) \cdots w(k'-1)$ with $p_k = q_k$. Using induction we can show that $\mathcal{A}_{p_i}$ and $\mathcal{A}_{q_i}$ are priority almost-equivalent for all $k \leq i \leq k'$. For $i = k$ this is obvious since $p_k = q_k$. Otherwise consider index $i+1$. Note that because $\sim$ is a congruence relation, $\mathcal{A}_{\delta_{\mathcal{A}}(p_i, w(i))}$ and $\mathcal{A}_{\delta_{\mathcal{A}}(q_i, w(i))}$ are priority almost-equivalent. The definition of the Schewe automaton sets $\delta_{\mathcal{S}}(q_i, w(i))$ either to the state $\delta_{\mathcal{A}}(q_i, w(i))$ or to its representative; in both cases, $\mathcal{A}_{\delta_{\mathcal{S}}(q_i, w(i))} = \mathcal{A}_{q_{i+1}}$ is priority almost-equivalent to $\mathcal{A}_{\delta_{\mathcal{A}}(q_i, w(i))}$ and therefore to $\mathcal{A}_{\delta_{\mathcal{A}}(p_i, w(i))} = \mathcal{A}_{q_i}$.

This observation together with claim 1 implies that all $\mathcal{S}_{p_i}$ and $\mathcal{S}_{q_i}$ are priority almost-equivalent as well. In particular, $p_{k'} \sim q$ (in $\mathcal{S}$).

Claim 4 If p and q are priority almost-equivalent in $\mathcal{S}$ and reachable, then they lie in the same SCC.

$\square$

Lemma 0.0.11. *There is no DPA priority almost-equivalent to $\mathcal{A}$ that is smaller than $\mathcal{S}_m$.*

Lemma 0.0.12. *The priority almost-equivalence of a DPA $\mathcal{A}$ can be computed in $\mathcal{O}(|\mathcal{A}|^2)$.*

Theorem 0.0.13. *For a given DPA $\mathcal{A}$, a minimal almost priority-equivalent automaton can be computed in $\mathcal{O}(|\mathcal{A}|^2)$.*

Definition 0.0.4. Let $\mathcal{A} = (Q, \Sigma, q_0, \delta, c)$ be a DPA. We define the **Moore-minimization** $\mathcal{B}$ as the parity automaton corresponding to the minimal Moore automaton of $\mathcal{A}$. That means it is the minimal automaton such that $\lambda_{\mathcal{A}}(\alpha) = \lambda_{\mathcal{B}}(\alpha)$ for all $\alpha \in \Sigma^\omega$.

More specifically, we define the congruence relation $\sim \subseteq Q \times Q$ by $p \sim q$ iff $\forall w \in \Sigma^* : \lambda_{\mathcal{A}}(p, w) = \lambda_{\mathcal{A}}(q, w)$. Then $\mathcal{A}'$ is constructed from $\mathcal{A}$ by removing unreachable states (from q_0). $\mathcal{B} = (Q/_\sim, \Sigma, [q_0]_\sim, \delta_{\mathcal{B}}, c_{\mathcal{B}})$ is the quotient automaton of $\mathcal{A}'/_\sim$ with $\delta_{\mathcal{B}}([q]_\sim, a) = [\delta(q, a)]_\sim$ and $c_{\mathcal{B}}([q]_\sim) = c(q)$.

Lemma 0.0.14. *For a given DPA $\mathcal{A}$, the Moore-minimization can be computed in $\mathcal{O}$.*