State Space Reduction For Parity Automata

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Overview

We establish the notion of **merger functions**. Using that definition, we present three of our newly developed heuristic techniques to reduce the number of states in deterministic parity automata.

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ω-automata

ω-words are words of one-sided infinite length: $\alpha \in \Sigma^\omega \Leftrightarrow \alpha : \mathbb{N} \to \Sigma$
a^ω, $aa(ba)^\omega$, $(abc)^\omega$

ω-automata are finite transition structures that describe a language
$L(\mathcal{A}) \subseteq \Sigma^\omega$
$\{a^n b^\omega \mid n \in \mathbb{N}\}$

Deterministic parity automata (DPA):

- State set Q
- Alphabet Σ
- Transition function $\delta : Q \times \Sigma \to Q$
- Priority function $c : Q \to \mathbb{N}$

An ω-word α starting in a state $q_0 \in Q$ induces a run $q_0 q_1 q_2 \dots$. The DPA accepts α iff the **smallest** priority that occurs infinitely often in the sequence $c(q_0)c(q_1)c(q_2)\dots$ is **even**.

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Why do we need heuristic reduction?

Goal: Reduce number of states in the automaton to ease run time of follow up algorithms.

Minimization Problem: Given an automaton $\mathcal{A}$, what is the smallest number of states required to recognize the same language as $\mathcal{A}$?

For deterministic finite automata (on finite words): Minimization is solvable in $\mathcal{O}(n \log n)$.

For DPAs: Minimization is NP-hard. []

Moore Minimization

A DPA can be interpreted as a Moore automaton with c being the output function.

Theorem.

Deterministic Moore automata can be minimized in log-linear time.

Idea: Compute equivalence $\equiv_M$ with $p \equiv_M q$ iff $\forall w \in \Sigma^* : c(\delta^*(p, w)) = c(\delta^*(q, w))$. Build the quotient automaton w.r.t. $\equiv_M$.

The same algorithm can be used to reduce DPAs but will not give minimal DPAs in general.

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Merger functions

Definition.

Let $\mathcal{A} = (Q, \Sigma, \delta, c)$ be a DPA. A **merger function** is a function $\mu : D \to 2^Q \setminus \{\emptyset\}$ such that

- all sets in D are pairwise disjoint
- for all $X \in D$, $\mu(X) \cap (U \setminus X) = \emptyset$, where $U = \bigcup D$

$\mu(M) = C$
Merge all states in $M \subseteq Q$ into any one representative of $C \subseteq Q$.

For a congruence relation $\sim$, the quotient automaton is defined by state set $Q_\sim = \{[q]_\sim \mid q \in Q\}$.
This is captured by the merger function $\mu_\div : Q_\sim \to 2^Q, \kappa \mapsto \kappa$.

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Delayed Simulation

Definition.

$p \equiv_{\mathrm{de}} q$ iff for all $w \in \Sigma^*$, every run that starts in $\delta^*(p, w)$ or $\delta^*(q, w)$ eventually sees a priority of at most $\min\{c(\delta^*(p, w)), c(\delta^*(q, w))\}$.

Definition.

Let $\mathfrak{C}_{\mathrm{de}} = \{[q]_{\equiv_{\mathrm{de}}} \mid q \in Q\}$ be the set of $\equiv_{\mathrm{de}}$-equivalence classes. Define the **delayed simulation merger** as $\mu_{\mathrm{de}} : \mathfrak{C}_{\mathrm{de}} \to 2^Q, \kappa \mapsto c^{-1}(\min c(\kappa))$.

Theorem.

Merging states according to μ_{de} preserves language.

Computing Delayed Simulation

We define a deterministic Büchi automaton $\mathcal{G}_{\mathrm{de}}$ such that $p \equiv_{\mathrm{de}} q$ iff both $L(\mathcal{G}_{\mathrm{de}}, q^0_{\mathrm{de}}(p,q))$ and $L(\mathcal{G}_{\mathrm{de}}, q^0_{\mathrm{de}}(q,p))$ are universal, i.e. Σ^ω.
This automaton uses the state set $Q_{\mathrm{de}} = Q \times Q \times (c(Q) \cup \{\checkmark\})$.
Computing states of universal language in a DBA requires linear time.

Theorem.

$\equiv_{de}$ can be computed in $\mathcal{O}(n^2 k)$.

Delayed Simulation Automaton

$\mathcal{G}_{\mathrm{de}} = (Q_{\mathrm{de}}, \Sigma, \delta_{\mathrm{de}}, F_{\mathrm{de}})$
States are $Q_{\mathrm{de}} = Q \times Q \times (c(Q) \cup \{\checkmark\})$.
The first two components are a "simulation" of the original DPA. The third component are the so called "obligations".
Transitions δ_{de}.
The first two components mimic the transitions of $\mathcal{A}$. The third component is defined by $\gamma : Q_{\mathrm{de}} \times \Sigma \to c(Q) \cup \{\checkmark\}$. (next slide)
Accepting states are $F_{\mathrm{de}} = Q \times Q \times \{\checkmark\}$.

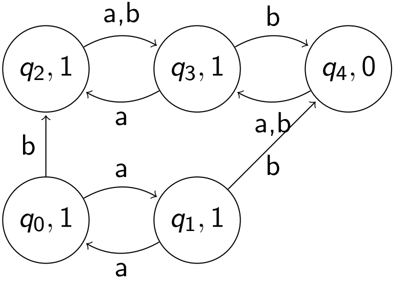
Delayed Simulation Automaton: γ

Let $0 \leq_{\checkmark} 1 \leq_{\checkmark} 2 \leq_{\checkmark} \cdots \leq_{\checkmark} \checkmark$.
For $p, q \in Q$, $k \in c(Q) \cup \{\checkmark\}$, $a \in \Sigma$, set
$\gamma((p, q, k), a) = \gamma'(\delta^*(p, a), \delta^*(q, a), k)$, where γ' is defined as follows:
If any of the following is true, then $\gamma'(i, j, k) = \checkmark$.

- i is odd, j is even, and $i \leq_{\checkmark} k$
- i is odd, j is even, and $j \leq_{\checkmark} k$
- i is odd, j is odd, $j \geq i$, and $i \leq_{\checkmark} k$
- i is even, j is even, $j \leq i$, and $j \leq_{\checkmark} k$

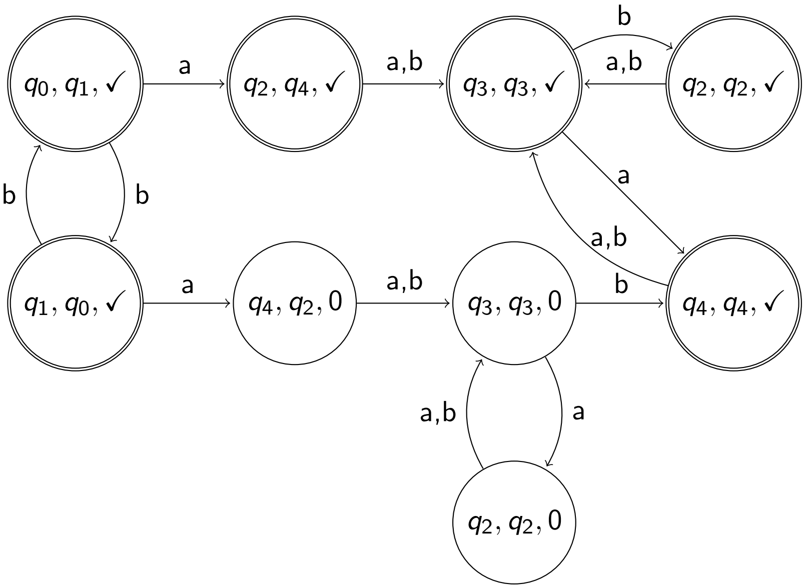
Otherwise, $\gamma'(i, j, k) = min_{\leq_{\checkmark}}\{i, j, k\}$.
$q^0_{\mathrm{de}}(p, q) = (p, q, \gamma'(c(p), c(q), \checkmark))$.

Delayed Simulation Automaton



A DPA with 5 states. We want to check whether $q_0 \equiv_{\mathrm{de}} q_1$ is true.

Delayed Simulation Automaton



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Congruence Path Refinement

Definition.

Let $\sim$ be a congruence relation and let $\lambda \subseteq Q$ be an equivalence class of $\sim$. The **path refinement** equivalence $\equiv_{\mathsf{PR}}^{\lambda}$ is the smallest relation