$$U^*(x, y, z) \geq z + \sqrt{z} - \frac{1}{z}$$

Let a, b, and c be the side lengths of right-angled triangle. Then, we know that:

$$a^2 + b^2 = c^2$$

Prove by induction:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{i=0}^{n} a_i = 2^{1+i} + x_1$$

$$\begin{Bmatrix} 1 & 2 & \dots & 10 \\ 2 & 2 & \dots & 10 \\ \vdots & \vdots & \ddots & \vdots \\ 10 & 10 & \dots & 10 \end{Bmatrix} + z + \sqrt{z} - \frac{1}{z} \Rightarrow \sum_{i=0}^{n} a_i = 2^{1+i} + x_1$$