Population Ageing and Labour Markets: Synthesizing Macroeconomic Accounts and Individual Behavior in an Economic-Demographic Growth Model

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Abstract

A macroeconomic-demographic growth model developed at IIASA extends the Blanchet-Kessler model by treating population in single-year age groups and adding modules covering pensions, health, and long-term care. This model has been used for, among other things, an extended analysis of the impacts of current and alternative demographic trends on the Japanese economy in a linked, globally consistent framework. A major weakness of the model in its present form is that the treatment of labor force is rudimentary. New research is adding a micro-simulation component that can be used to model labor market behaviors (entry to and exit from the labor force and human capital formation) endogenously, as well as closely related demographic behaviors (fertility, mortality / morbidity, migration, and household formation). These micro-level behaviors simultaneously determine macroeconomic variables (for example, the size of the labor force depends on the aggregation of individual decisions) and are influenced by them (for example, individual labor supply behavior depends on the average wage rate). In this paper, we present a prototype "MicroSSR" model which combines micro-simulation of fertility and human capital formation and a reduced version of the full macroeconomic model. Illustrative simulation results are shown and directions for future work are discussed. The paper is organized in two parts, the first describing the macro-model, the second the microsimulation extensions as currently developed.

Note: This draft consists of elements cut and pasted from several documents, and only limited attention has been given to continuity.

Part I

The IIASA Social Security Forecasting and Simulation Model

1.1 Macro-model structure

The IIASA model, which extends work originally presented by Blanchet and Kessler (1992), is theoretically heterodox. It is neoclassical in that average wage and profit rates are given by marginal productivity conditions. Thus, the basic features of the neoclassical analysis of population ageing are maintained: a decline in the ratio of active labor force members to inactive persons results in a higher capital-output ratio (i.e., capital is less productive because labor is scarce), a lower rate of profit relative to the wage rate, and a decline *pari passu* in income of the elderly relative to income of the young. However, saving in the model is Kaldorian, with age-specific saving rates out of disposable wage income and profit income representing exogenous assumptions. Sometimes, although not here, this ultimately Marxian approach is taken to the limit of assuming that all wages are consumed and all profits are saved. Consumer utility maximization, based in most models on the life cycle hypothesis and some form of forward-looking behavior, plays no role in the IIASA model save to the extent that current income can be considered a proxy for lifetime wealth.

The full model structure is presented in Annex 1, and we give only a brief textual description here. The main function of the model is to track income, expenditure, and assets of a single-year cohort as it ages, utilizing an accounting system closely mirroring the OECD System of National Accounts (SNA). Members of the cohort accumulate assets while young, then annuitize these assets on retirement, bequeathing to the living population whatever is left over when they die. In this way, consumers' inter-temporal budget constraint is respected. A Pay As You Go (PAYG) defined benefit public pension system collects contributions (at a rate calculated to maintain system balance) out of wages and profits of the self-employed and pays benefits to the retired population. In addition, some workers participate in a private sector PAYG defined benefit pension system where, again the contribution rate is calculated endogenously to maintain system balance. A private fully funded defined contribution pension system also collects contributions and, after retirement, pays out annuities; in addition to which, households engage in saving outside the pension system.

A health system, assumed to be public, is financed on a PAYG basis, with patient co-payments calculated as an exogenous share of age-specific health care expenditure and payroll tax contributions collected at a rate calculated to fund total expenditure minus total co-payments. The "underlying" rate of health care expenditure growth, reflecting new technology, changes in utilization rates, rising salaries of health care workers, etc., is an exogenous assumption. Relative age-specific health care costs (for example, the health care costs of persons aged 70-74 relative to persons aged 0-4) are assumed to be constant at their base-year value. Thus, aggregate health care spending changes to reflect the underlying rate of cost increase and changes in the age structure of the population. There is a long-term care system structurally identical to the health care system; only the parameters differ.

Since public pension and public health and long-term care system contribution rates are calculated to balance the respective systems, the government fiscal balance is "de-linked," in this model, from demography; it is private, not public, savings that decline in response to population aging. When the population ages, social contribution rates of the working population increase, unless the model user changes exogenous assumptions such as the retirement age, the rate of defined benefit pensions indexation to real wages, the exogenous accrual rate parameter which links PAYG pension benefits to wages, and health or long-term care system co-payment rates. Subsidies from general government to the pension and health system can also be taken into account, thus "re-linking" demographic change and the fiscal balance. However, none of these adjustments occur automatically when the model is solved; the judgment and intuition of the model user are required. One way of characterizing this approach is that, absent adjustments imposed by the user, demographic risk is borne by the working-age population, whose social contribution rates change to accommodate population.

Savings consist of personal household savings (which can, in turn, be split into savings captured by the private pension system and other savings), government savings and corporate savings. For accounting purposes (asset formation must be age-indexed), the latter two are distributed over households using exogenous coefficients reflecting the typical age-structure of asset ownership. Having been thus augmented, household savings are allocated over three asset classes -- residential capital, capital operated by private unincorporated enterprises, and capital held on households' behalf by non-pension financial institutions – using exogenous share coefficients. In the first two cases, profits are directly credited to households. In the latter case, as well as capital held on households' behalf by the private pension system, profits are credited to firms, who distribute dividends to the financial intermediary. Non-pension financial institutions are assumed to pay out these dividends immediately; pension institutions, by contrast, retain dividends on participants' behalf.

Since government savings have no impact on household savings (although such a link would be simple enough to implement) there is no Ricardian equivalence in the model; a dollar of government debt translates into a dollar of foregone capital formation. In order to impose inter-temporal consistency on the government budget, an automatic adjustment mechanism is built in which limits government debt as a proportion of GDP by adjusting consumption expenditure. Interest on government debt is credited to households as income.

Underscoring the partial equilibrium nature of the model, there is no investment demand function in the model; total capital formation equals domestic savings minus acquisition of net foreign assets. The model is multi-regional and tracks bilateral capital flows, but as with the domestic portfolio allocation process, the treatment of the international allocation process is largely *ad hoc*. Pension funds and non-pension financial institutions are assumed to allocate capital to investment projects located in the domestic region and foreign regions in accordance with exogenous coefficients. This approach adds little from a behavioral point of view, but from a scenario-building point of view, it forces the model-user to make explicit assumptions regarding the foreign-sector scenario and enforces global consistency on capital stocks and flows.

1.2 An illustrative baseline scenario

In order to give a feel for the model, we briefly present some baseline projection results for Japan.

Model initialization assumptions

The initial year of the solution here presented is 1995, and the model has been solved for 55 time periods, i.e. out to 2050. Since this is a "test run" of the model, only limited attention was given to fine-tuning assumptions. Therefore, the following scenario should not be interpreted as a forecast (or even as a particularly accurate depiction of the situation in 1995, the initial year).

The model has been solved for four countries/regions: Japan, U.S., Other Industrial Countries (UN definition; this is effectively OECD minus U.S. and Japan plus Eastern Europe and the Former Soviet Union), and Less Developed Countries (LDCs). The age-specific (single-year) population data for Japan and the U.S. are from the UN Population Division *1998 Revision*. Population data for world as a whole and for the industrial and developing country regions are from the International Institute for Applied Systems Analysis (IIASA) population projections.. Hence, we solve for the Other Industrial Country region as a residual (IIASA world total minus IIASA LDC minus U.N. Japan and U.S.)[1]

Details of initalisation assumptions are given in Annex 2. Initial-year total GDP was calculated from initial-year total population and the estimated initial level of per capita GDP. In combination with an assumed initial-year capital-output ratio, this allowed estimation of the total capital stock. This was split into initial-year residential capital, capital operated by private unincorporated enterprises, and capital operated by firms using notional shares. Capital operated by firms was further split into capital held on households' behalf by the private pension system (disaggregated into defined contribution and defined benefit components) and other (non-pension system) financial institutions. Admittedly sketchy information on the age structure of capital ownership was then used to share down assets in each class over the population by single-year age groups. Data sources described in MacKellar *et al*. (1999) were used to estimate cross-border holdings of pension systems and other financial institutions.

In all regions, the capital coefficient in the Cobb-Douglas production function was set equal to 0.33 and the rate of total factor productivity growth was set equal to 1% per year. The depreciation rate was assumed to be 5% in Japan, 4% in the U.S. and the other industrial countries, and 6% in the LDCs. Assumptions on government consumption as a proportion of GDP, the direct tax rate (as a proportion of pre-tax income, constant across all age groups and sources of income), and the indirect tax rate (as a proportion of GDP) are also given in Annex 3.2, Table 1. Tax rates were not varied over the simulation horizon; the government consumption share was adjusted to maintain target levels of government debt as a share of GDP. Finally, several assumptions relevant to corporate saving are also presented.

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[1] The reason for this rather complicated approach is that, whereas IIASA makes available alternative fertility, mortality, and international migration scenarios, the U.N. publishes only a low fertility scenario. On the other hand, whereas the U.N. performs projections for individual countries, IIASA does only regional projections. Combiining the two sets of projections makes it possible to do alternative fertility, mortality, and migration scenarios for Japan, the U.S., Other Industrial Countries, and LDCs.

Private consumption rates out of disposable income, assumed for simplicity to be time- and age-invariant, are shown in Annex 2. Given the assumed consumption rates and the fact that the elderly consume out of annuity income in addition to current income, persons aged 15-59 are net savers whereas persons aged 60+ are net dissavers. Age-specific labor force participation rate assumptions (for both sexes combined) are based on data from the International Labour Organisation. These were subdivided into assumptions for 1995-99, 2000-09, and 2010-50. No significant changes over time were assumed in Japanese labor force participation rates, i.e. the baseline scenario represents "business as usual" in terms of labor market institutions and behavior.

Annex 2 presents assumptions regarding the three components of the pension system – the public pension PAYG DB system, the private PAYG DB pension system, and the private fully funded DC pension system. In Japan, 65% of workers were assumed to participate in the government DB pension scheme. In order to make the simulation more relevant to Japan, 20% of the retirement-age population was assumed to be eligible for a flat (non-earnings related) benefit basic scheme financed on a PAYG basis by a levy on wages and entrepreneurial income. The flat basic pension scheme was assumed to pay a pension equal to 10% of the average wage. In all other regions, where no flat-benefit plan was assumed to exist, the rate of workers' participation in the earnings-related public pension scheme was assumed to be 75%. 80% of all wage income (as well as income from capital operated by private unincorporated enterprises) was assumed to be subject to social insurance contributions (pensions, health, and long-term care). Earnings-related public pension schemes in all regions were assumed to have an accrual rate of 1% per year, so that a participant who contributed for 40 years would have an initial-year pension entitlement equal to 40% of his/her final wage. Earnings-related public pensions in Japan were assumed to be 20% indexed to wages (i.e., after retirement, the pension increases by one-fifth the rate of increase in the wage rate); in regions other than Japan lacking the flat-rate pension, the rate of indexation was set equal to 50%.

Also in Annex 2 are given assumptions regarding the two components of the private pension system. The private DB pension system was assumed to have a 1% annual accrual rate in all regions and retirement benefits were assumed to be 10% indexed to wages. 15% of workers in Japan were assumed to participate in the private DB system, as opposed to 20 % in the U.S. and other industrial countries and 10% in LDCs. 30% of Japanese and U.S. workers were assumed to participate in private DC pension schemes, 50% of other industrial country workers and 20% of workers in LDCs. The contribution rate to the private DC pension system was assumed to be 5% of participants' pre-tax income. It was assumed that every year 3% of DC pension system assets are withdrawn by job-switchers who do not roll over their accumulation into a new plan (this is not shown in Annex 2). These withdrawn assets are shared out between consumption and investment in non-pension assets.

DC pension assets were assumed to be decumulated between age 60 and age 80; other types of assets were assumed to be decumulated between 60 and 105.

The health and long-term care systems were initialized using relative age-specific spending coefficients described by Mayhew (2001); these assumptions are not shown in Annex 2. Japanese weights were from national data sources; for the U.S. and other industrial countries, weights for the U.K. described in Mayhew (2000) were used. In the LDC region, as described in Mayhew (2000), the assumption was made that health care expenditure in each age group is proportional to deaths (on the assumption that most health expenditure occurs in the period running up to death). The calculation of age-specific long-term care expenditure is exactly analogous to health spending. Co-payment rates for the health care and long-term care systems are also given in Annex 2.

Domestic savings outside the pension system must be allocated between residential capital, capital operated by private unincorporated enterprises, and capital operated by firms and held on households' behalf by non-pension financial institutions. The first step in this process, as shown in Annex 2, is to allocate 30% to residential investment, leaving 70% for non-residential investment. The next step is to allocate 33% of non-residential capital formation to investment in capital operated by private unincorporated enterprises and the remaining 67% to capital operated by corporations and held on households' behalf by non-pension financial institutions. These coefficients were assumed to be the same in all regions and to be invariant over time.

Residential capital and capital operated by private unincorporated enterprises is, by definition, installed in the domestic region. Capital operated by firms, which is held for households by the private pension system and other financial institutions, may be invested in projects located at home or abroad. In Annex 2, we give the allocation shares used to assign change in pension system capital and change in capital held by other financial institutions across regions. Thus, for example, it was assumed that Japanese pension fund managers invest 90% of the net change in their assets every year in Japan and 3.3% apiece in each of the other three regions.

Summary of Results

Baseline simulation results for Japan are summarised in Table 1 and illustrated in Figure 1. According to the U.N. Population Division projections, the total population is expected to begin to shrink between 2000 and 2010. The dependency ratio (defined as persons over 60 to persons 15-59) is projected to rise from about one in three at present to about three in four.

Table 1
Baseline scenario: summary presentation

	1995	2000	2010	2020	2030	2040	2050
Demography							
Population (annual % change)	0.2%	0.2%	-0.1%	-0.4%	-0.5%	-0.6%	-0.6%
Labour force (annual % change)	0.5%	-0.1%	-1.8%	-0.7%	-1.0%	-1.1%	-0.6%
Population 60+ : Population 15-59 (%)	30.4%	35.2%	49.0%	57.1%	61.9%	73.1%	74.9%
Macroeconomy							
GDP per capita (US $)	36078	39550	45049	50912	58395	65496	73579
GDP per capita (annual % change)	2.1%	1.7%	2.3%	1.4%	1.3%	1.1%	1.5%
Capital-output ratio	2.59	2.69	2.98	3.20	3.31	3.47	3.48
Rate of return to capital (%)	5.8%	5.4%	4.4%	3.7%	3.2%	2.6%	2.4%
Net national savings (% of GDP)	7.17%	7.62%	6.33%	4.45%	3.81%	2.94%	1.65%
Change in net foreign assets (% of GDP)	0.2%	0.3%	0.2%	0.0%	0.0%	-0.1%	-0.2%
Gross foreign investment in Japan (% of GDP)	4.6%	4.9%	4.1%	3.1%	2.6%	1.9%	1.1%
Gross Japanese investment abroad (% of GDP)	4.4%	4.5%	3.9%	3.0%	2.6%	2.0%	1.3%
Social insurance							
Public pension system							
Contribution rate (%)	18.2%	20.3%	26.7%	30.9%	33.7%	39.5%	40.9%
Contributions (% of GDP)	6.8%	7.6%	9.8%	11.2%	12.2%	14.3%	14.7%
Health system							
Contribution rate (%)	5.8%	6.3%	6.9%	8.2%	8.5%	8.5%	10.3%
Contributions (% of GDP)	4.2%	4.5%	4.9%	5.8%	5.9%	5.9%	7.1%
Long-term care system							
Contribution rate (%)	1.9%	2.2%	2.5%	3.0%	3.1%	3.1%	3.8%
Contributions (% of GDP)	1.4%	1.6%	1.7%	2.1%	2.2%	2.2%	2.6%
Intergenerational distribution							
Adjusted disposable income per capita, pop. 15-59 : pop. 60+ (%)	107.0%	110.5%	115.6%	116.4%	113.8%	113.1%	106.6%
Adjusted disposable income per capita, pop. 15-59	31158	33350	37389	40442	45315	50614	55402
Adjusted disposable income per capita, pop. 60+	29115	30184	32353	34755	39831	44755	51975
Non-health consumption per capita, pop. 15-59 : pop. 60+ (%)	98.6%	101.5%	102.7%	102.2%	98.2%	93.4%	89.5%
Assets per capita, pop. 15-59 : pop. 60+ (%)	48.6%	44.7%	47.4%	49.4%	50.7%	48.1%	47.4%

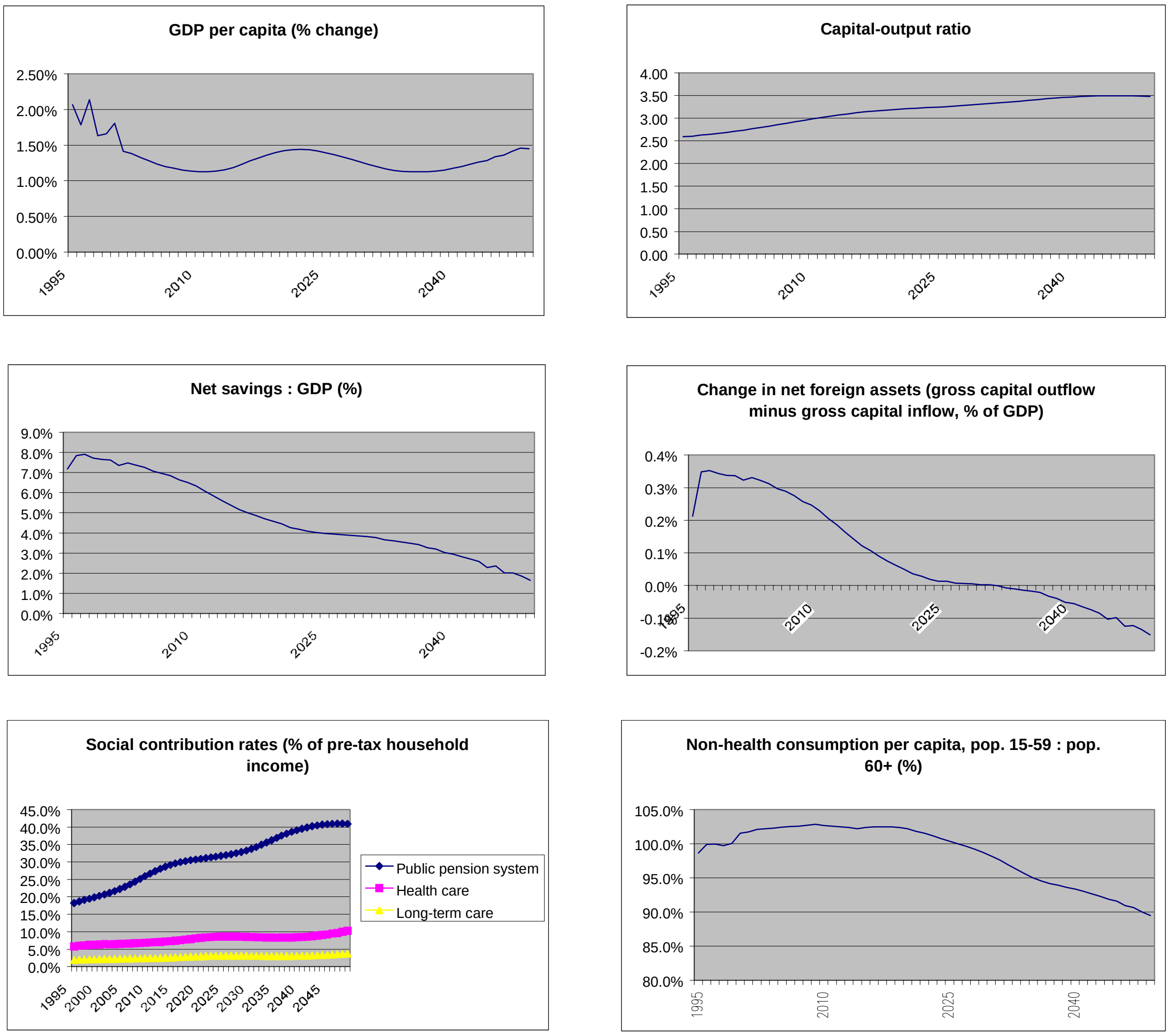


Figure 1 - Baseline scenario: summary results

Assuming constant 1% per year total factor productivity growth, the economic consequences of these demographic trends correspond to what Keynes termed "demographic stagnation." Per capita GDP growth in Japan is projected to decelerate to the range of 1-1.5% per year while net national savings decline to less than 2% of GDP by 2050. This decline results entirely from the drop in the household saving rate as the proportional weight of the dissaving elderly in the population increases. The capital-output ratio is projected to rise from roughly 2.5 at the turn of the century to 3.5 at mid-century, with corresponding reductions in the rate of return to capital. If the model took account of diminished demand for investment and the diminished attraction of Japan as the locus of net foreign investment, the deterioration in the productivity of capital and the rate of return to capital would be attenuated, however, this would slow per capita GDP growth even more. Change in net foreign assets is projected to switch from margnally positive to marginally negative between the first and second halves of the simulation period.

As expected by virtually all analysts, social insurance contribution rates are projected to rise, roughly doubling over the simulation period in the case of public pension contributions; increasing less, proportionally speaking, in the case of health and long-term care.

Despite increases in social contribution rates, adjusted disposable income (i.e. income after adjustment for changes in pension wealth) of the working-age population is projected to increase from approximately $30,000 per year at the beginning of the simulation period to about $50,000 at its end. This favorable trend highlights the importance of the increase in the real wage rate as labor force declines. Adjusted disposable income of the working age population is projected to rise relative to that of the retirement age population in the first half of the simulation period and then decline. However, adjusted disposable income is a poor measure of welfare of the elderly population, since it includes health and long-term care benefits (which finance the consumption of "bads" instead of "goods") and excludes defined-contribution pension benefit receipts (since these represent a decline in wealth). Probably the best welfare measure to be found among variables tracked by the model is non-health related consumption (i.e., consumption minus consumption of health care minus consumption of long-term care). Non-health related consumption of the working-age population is projected to increase (relative to that of the elderly) until about 2015 then decline for the rest of the simulation period, ending up about 10% below its initial level.

Are these results reasonable? One way of answering this question is to compare them with results of other projections made using economic-demographic models. In Annex 3, results of a number of studies are summarized and compared with our own. Broadly speaking, studies agree that the aggregate saving rate is likely to decline and perhaps even turn negative, that Japan is likely to switch from being a net lender to the international financial system to being a net borrower from it, and that social contribution rates will rise. However, there is no unanimity regarding trends in the rate of economic growth, the capital-output ratio and the rate of return to capital Moreover, while some models project extreme shifts in variables such as the saving rate and current account balance, others (like the IIASA model) project only modest trends.

Part II

The use of microsimulation in economic growth and social security modeling in the context of demographic change

This section describes the first steps in trying to develop a modeling and simulation framework that allows to incorporate the strengths of microsimulation in economic growth modeling in the context of demographic change. This is done by restating and programming the existing IIASA "Social Security Forecasting and Simulation Model" in terms of microsimulation. The first "micro-SSR" version presented here was developed in the context of a feasibility study in order to explore and demonstrate some of the features microsimulation techniques can possibly "add" to this kind of modeling. The IIASA macro-model explicitly introduces "realistic demography" by disaggregating the household sector (and all model outputs) by age cohorts. This kind of economic modeling is incorporated in a dynamic microsimulation framework by further disaggregation of the cohorts to the individual micro-level. Allowing for heterogeneous individual agents, economic and demographic behavior can be modeled taking into account a wide set of individual and household characteristics.

2.1 What is microsimulation?

Unfortunately, microsimulation is a rather confusing term, both due to the wide range of models it does address as well as the very different concepts, in that the terms simulation and modeling are used. The term micro thereby only indicates the smallest unit of analysis, usually individuals or households, but not necessarily the unit, that's behavior is modeled.

- Models of **tax-benefit systems** are the most typical application of **static microsimulation**, where individuals or households (represented in a micro-database) are only used as accounting units having the necessary characteristics to calculate taxes and benefits. Simulation is used as means of "testing" the outcomes of different policies.
- In **dynamic microsimulation**, the behavior of the micro-units is modeled – most importantly it's behavior over time. Typical behavioral models are statistical models, that for a given set of personal characteristics determine probabilities for a defined set of possible transitions like marriage, pregnancy or death. **Monte-Carlo simulation** is then used to determine, if a transition takes place in the simulation experiment. This allows to dynamically update personal characteristics over time and to add and remove micro units to or from the population due to birth, death or migration. Another type of dynamic behavior is policy response, that might be modeled using econometrical approaches or based on theory like utility maximization.
- In **agent based simulation**, the micro units or agents are fully defined by their micro behavior based on theoretical assumptions and simulation is used in order to study what happens, when these units are put together, that is, the resulting dynamics and patterns on the population level. In this approach, simulation consti-

tutes a type of modeling (named **theoretical modeling**) and a method of theory development. Agents usually follow goals and have a certain range of abilities. There is a wide range of possibilities to model individual behavior, from the modeling of a rational forward looking utility optimizing "homo economicus" to more realistic human behavior including learning processes etc. as it is done in agent based simulation based on the artificial intelligence approach.

While especially the first two mainly "**data driven**" approaches and the last "**context driven**" approach evolved in almost total ignorance of each other, both increasingly use concepts of each other, and a synthesis might be desirable. Certainly there is a trade-off between the theoretical foundation of behavior and the grade of detail and data fit in microsimulation models, as it can be found in macro-models as-well.

In data driven microsimulation, strong behavioral characteristics are usually sacrificed to get a very detailed model with a good fit to the data. Behavior is mostly modeled implicitly, and so are corresponding assumptions what can make models difficult to understand. In contrast, context driven agent based microsimulation incorporates behavior explicitly. Agents are defined by their behavior and act according the environmental context they are put in. In the field of economic macro-models, the same contrast can be found between OLG models that incorporate behavior as fully as possible at the expense of having to accept a simple highly stylized structure and – for example – the IIASA SSR model with its fine-grained accounting treatment of age-specific stocks and flows. Social security modeling in the context of demographic change – as it is done in this exercise of restating the existing IIASA SSR macro model in terms of microsimulation – should combine all three listed microsimulation approaches.

- The IIASA SSR model is characterized by its detailed **accounting treatment** of age-specific stocks and flows. The population is represented by one representative agent by age cohort. This kind of economic modeling can be incorporated in a microsimulation framework by further disaggregation of the cohorts to the individual micro-level. This in principle can be done without any change in outcome, as representing a cohort by one "weighted" representative agent or the same number of identical individuals would not make any difference. Of course, one of the motivation to do so, is to extend the possible scope of analysis to distributional issues allowing for heterogeneous agents in each cohort. Regarding the accounting treatment in the tax and benefit framework, microsimulation allows to model tax and social security systems in much more detail, as policies usually tie taxes and benefits in a nonlinear fashion to several individual and household attributes. Regarding the modeling of possible **tax-benefit and social security systems** as well as the **accounting treatment**, the micro-population has to be linked to the accounting scheme as it is already defined in the macro model. In addition, various other ways of aggregation might be defined, in order to allow for i.e. **generational accounting** and detailed **distributional analysis** in a cross-sectional way as well as over the life cycle. As micro-units hold individual accounts of different capital types and pension funds etc., assumptions of initial distributions can and have to be made when using stylized scenarios, but the simulation can also be based on micro-data containing this kind of information as it is done in typical static microsimulation tax-benefit models.
- The macro model produces or imports **population scenarios** usually based on cohort-component population projections. Population scenarios can be described in matrices giving the population number for both sexes per age for each calendar

year. In microsimulation models, populations are usually represented by means of a micro-database describing the individual units. In order to project this population into the future, detailed household projections are needed, that are produced by **dynamic microsimulation**. As projections of this kind are usually not available from outside sources, they have to be produced within the model. There is a wide range of possibilities how a starting-population can be synthetically generated or built from empirical micro data, and how the dynamics are modeled. Models differ in the extent, in that behavior is made endogenous or set in exogenous scenarios, to that the model has to be aligned. Fertility scenarios are prominent candidates of this type of exogenous scenarios. The statistical modeling of behavior can be limited to demographic behavior like partnerships, births and death or also include economic characteristics like labor market participation and income variables. Most of the big microsimulation projects use very detailed statistical and econometrical models in order to project detailed household characteristics into the future. In multi-level models, micro behavior is linked to the macro-outcome of individual behavior, what is especially of importance in economic modeling i.e. the determination of wages etc. Demographic and economic behavior might be modeled simultaneous – with i.e. fertility depending on economic characteristics – or independently, as it is done in the SSR macro model. The modeling framework developed here should allow for various alternative modeling approaches. Methods that are used to determine transition probabilities for a defined set of transitions in a defined set of considered parallel careers might range from exogenously given rates and scenarios to event history or longitudinal models that make demographic behavior endogenous.

- In order to make long term economic growth models understandable, **economic behavior** has to be made explicit and, in order to make it endogenous, it has to be **based on theory**. What the microsimulation approach can add to this kind of modeling of the household sector, is that decisions can be modeled at the level at that they are taken (the individual and/or the household) and that they can be based on much more detailed information on individual and family characteristics. This kind of model representation should make it easier, to move from the "fixed coefficient" approach of describing **economic behavior** to explicit economic modeling.

2.2 The general strengths of microsimulation

One of the central strengths of microsimulation lies in the fact that it permits **inclusion of more variables than other methods**, which is especially important in projection and planning applications, as this allows for more detailed research. For example, when trying to estimate future demand for health care facilities, etc. based on population projections, a large set of household characteristics, such as household size, family composition, age and income can be used.

MS allows the construction of behavioral **models at the level on which the relevant decisions are made**, i.e. on the micro-level. There is no need to translate behavioral relations from the micro-level to the macro level. This also implies that no information is lost through aggregation.

From the view of policy-makers the main strength of microsimulation lies in its ability to test new policies in a virtual world before they are introduced into practice. In comparison to more traditional policy evaluation modeling exercises, microsimulation is especially powerful in **addressing distributional issues**, both in a "static" cross-sectional way and over time.

Based on micro-data, microsimulation allows **flexible aggregation** as the information may be cross-tabulated in any form, while in aggregate approaches the aggregation scheme is determined a priori. Simulation results can be displayed and accounted for simultaneously in various ways -- in aggregate time series, cross-sectional joint distributions, and individual- and family life paths. Flexible aggregation helps to **determine "winners and losers"** of policy changes by various characteristics. An example is the possibility to study and compare contribution and benefit histories over a whole individual lifespan, permitting the calculation of return.

Based on multivariate methods such as history event analysis or rule-based behavioral models, microsimulation permits **study of the interaction between variables** and **the life course interactions between various parallel carriers and roles**, such as education, work, partnership and parenthood within a changing socio-economic context.

Microsimulation allows to study the **interaction between individuals**. While modeling takes place on the individual level, simulation is used to study the resulting dynamics and patterns of change on the macro-level. This is the key element of most agent-based simulation, where societies are "grown" by "putting together" micro units defined by their behavior in order to study the resulting dynamics. The use of models to compose complex processes from simple processes has been termed theoretical modeling (Burch, 1999, p. 4) as opposed to empirical modeling. In the empirical "data-based" tradition of microsimulation, the possibility to study the interaction between individuals is mainly used to study changes in **family and kinship networks**. Direct applications can be found in the field of elderly care and other aspects of aging societies, where knowledge of the detailed household and family characteristics is valuable information when designing policies. The knowledge of kinship patterns additionally allows for detailed study of intergenerational transfers and bequests.

The potential to handle large state spaces in projections implies the possibility to handle not only a wider set of individual characteristics and categories, but also spatial and other environmental characteristics that allow for detailed modeling and studying of the **interaction between individuals and the environment**. The study of these interactions is of central importance to most agent-based and multilevel microsimulation models.

Due to the inclusion of stochastic elements - i.e. Monte-Carlo simulation - resulting in different outcomes of each single simulation experiment, microsimulation allows for the **exploration of the distribution of events rather than its point-estimates**, thus leading to more adequate representation of uncertainty and risk.

The advantages described certainly come at a price, fortunately a price that decreases over time, at least with regard to two of the most frequently listed **drawbacks** of microsimulation: **(1)** the usually **large investments** with respect to both manpower and hardware required might be considerably reduced over time as hardware prices fall and more powerful and efficient object-oriented computer languages become available; and **(2) data problems** are reduced over time, as more and better data, especially longitudinal data become available – and this increasingly in standardized and internationally comparable form.

Demand for microsimulation models in social security research does not only result from the special advantages of these kinds of models compared to others, but also from the fact that there is no alternative modeling strategy to address a series of related critical policy and research issues. Caldwell and Morrison (2000) give the following examples:

- analysis of projected winners and losers on period-specific or lifetime basis;
- analysis focused on families and individuals simultaneously;
- exploration at the micro-level of the operation of social security programs in the context of the broader tax/transfer system;
- quantifications of incentives to work, to save, or to retire at particular life course or period junctures;
- cross-subsidies across population segments or cohorts;
- feedback effects of government programs on population demographics; and
- longer-term consequences of social trends in marriage, divorce and fertility.

2.3 Expected improvements to the IIASA SSR model

The exercise of restating the SSR model in terms of microsimulation should lead rather to a flexible modeling platform suitable for a broad range of policy analysis than to one single economic model. Microsimulation supports this planned **modular design**, that provides flexibility as it allows to combine alternative behavioral models. The main expected gains can be summarized in the following points.

- A more adequate and consistent **model representation**: both, the accounting treatment and the behavioral modeling is made more transparent, as it happens at the micro level. This also allows for more efficient object-oriented programming;
- a broader range of **policy questions** can be investigated, including distributional impacts and progressive tax and benefit schemes. In general it is possible to model social security and other tax-benefit systems at any level of detail;
- the possibility to extend the number of individual and **household characteristics**, i.e., by kinship information or individual risk factors necessary to assess health and elderly care costs, etc.
- the possibility to either use stylized data or to base the model on empirical **populations**; as this populations are dynamic, it is possible to model and account for changing family and household structures;

2.4 The IIASA SSR model restated as microsimulation model

This section explores ways to restate the IIASA SSR model in terms of microsimulation. A dynamic approach is chosen that handles both demographic and other life-course events, and economic stocks and flows, mostly in the form of individual accounts representing claims on different types of capital assets. While this gives a consistent representation of the household sector, additional decision-making agents have to be modeled, namely financial institutions (including private pension funds) which are allocating savings across alternative investment projects, the enterprise sector which allocates earnings between corporate retentions and dividends, and the government which is setting net taxes and running a public pension system.

As in the macro-model, behavior is mostly expressed implicitly by transition matrices and exogenously assumed age- and time-specific rates such as saving rates. Following the terminology of object-oriented programming, these matrices are methods of determining transition probabilities and rates. This representation can serve as a starting point for more advanced behavioral modeling, with these exogenously assumed rates being subsequently replaced by expressions based on more advanced statistical models for better empirical foundation (such as event history analyses) or derived from economic theory (like utility maximization). While the endogenization of economic behavior such as consumption and labor supply is beyond the scope of this paper, the framework developed does serve as a logical starting point of this exercise. The microsimulation approach supports this goal, both by providing all necessary information usually used for individual decision-making on the individual and household-level, and by a representation of the model where behavioral modeling can be introduced on a level on which individual and household decisions are made: the micro-level. The object-oriented software-design of the "microSSR" software platform explicitly supports the definition of alternative methods determining behavior.

The micro-population

The most obvious difference between the IIASA macro-model and its representation as a microsimulation model is the representation of the population. In the macro-framework, this is done by an age-period population matrix for each sex (if distinguished) that might be imported from another source or be produced by the cohort-component method within the model. In the micro-representation, a micro-population has to be imported, e.g. from empirical micro-data containing all characteristics used, or it may also be generated within the model according to specific assumptions. As micro-units are, unlike the “representative individual” macro-framework, distinguished per definition in microsimulation models, births can be explicitly linked to parents and the model can keep track of kinship networks as well as account for different family sizes and types – information that might be valuable in modeling various socio-economic behaviors such as labor supply or savings, etc.

Switching from a representative behavior to the linked lives of individual agents allows for and implies the necessity to model a multitude of aspects of human life-course interactions: how do people select their partners, what determines the stability of partnerships, how do education, professional and partnership careers interact, what determines fertility, etc. While considerable effort is made in most microsimulation projects to model these behaviors – and microsimulation can be seen as the best tool to study the resulting dynamics – it has to be noted that any population dynamics produced by the cohort-component method can be produced by microsimulation just by interpreting the underlying transition rates as transition probabilities (ignoring some Monte-Carlo variability depending on population size).

For the purpose of this pilot study, a closed synthetic micro-population with all demographic characteristics is produced first, on which all later simulations will be based. All individuals enter the simulation at birth, which implies that the model has to be “run” for 100 years – the assumed maximum age a person can reach – to build up the population. From the point in time, when the first female age cohort reaches the end of her fertile period, all population dynamics (births, deaths) can be produced within the simulation. For the initial period, the number of births has to be exogenously set in order to account as well for births by women outside the simulated population.

This synthetic populations might be preferred in highly stylized model designs, as typical for most economic modeling. In this way relationships between different processes and their individual contributions to the resulting aggregates can be studied free from "unwanted irregularities" of empirical populations, or the influence of shocks can be isolated or assessed directly by comparing shock scenarios to regular ones.

Creating a population "from the scratch" obviously limits the simulation of accumulation processes to those that can be directly linked to simulated individuals. This implies the necessity of additional assumptions regarding initial endowments or inheritances and other related issues, both with regard to initial stocks and distributional issues.

Demographic behavior and education

While the cohort-component method is restricted to very few additional variables beside age and sex, behavioral modeling on the individual level allows to process data on past life histories, current states, as well as other individual risk factors. Compared to the component-component method underlying the demographic macro-scenario, two initial extensions are made, one by introducing education and the other by applying parity-dependent fertility measures instead of overall measures.

The base model developed here distinguishes three education levels. For simplicity, education level and school leaving age are determined at birth. Once the education level is determined, the school leaving age is also determined by Monte-Carlo simulation for given age-specific probabilities by education level and birth cohort.

Fertility is modeled by applying cohort rather than period measures in a two-step process. First the parity of a woman is determined at birth (assuming survival) for given parity progression rates by age cohort and education level. In a second step, where applicable the age at first birth (again depending on the women’s year of birth and education) and the spacing between births is determined. Resulting dates are checked for survival of the mother and the simulated “babies” are immediately simulated in a recursive process. Sex is determined by random for a given sex ratio.

In this simple model version, mortality risks do not depend on detailed individual and household characteristics, but by applying a set of mortality tables by sex and education. A better assessment of mortality (as well as morbidity and disability) on the individual level will be especially important when studying kinship networks of elderly people and (related) topics such as elderly care (Wolf, 1999).

Digression: Demographic change and retiring age in Japan

Japan currently experiencing the fastest process of population ageing in world history, leading both to a rapid change of the age distribution as a decline of total population, with total population expected to be only at halve of the current number at the end of the century.

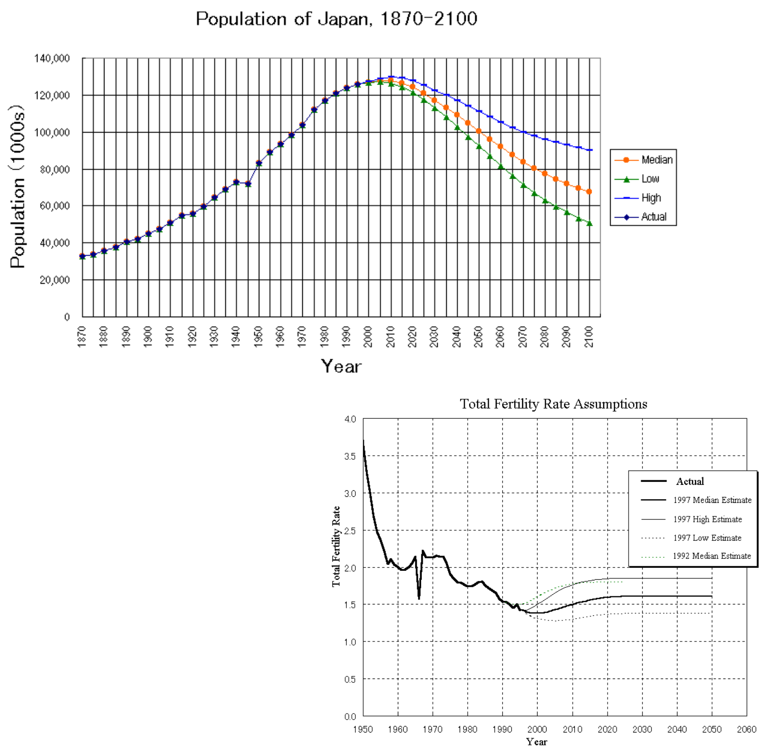


Figure 2: Population scenarios from NSSPPRI (National Social Security and Population Problem Research Institute: www.ipss.go.jp 1997

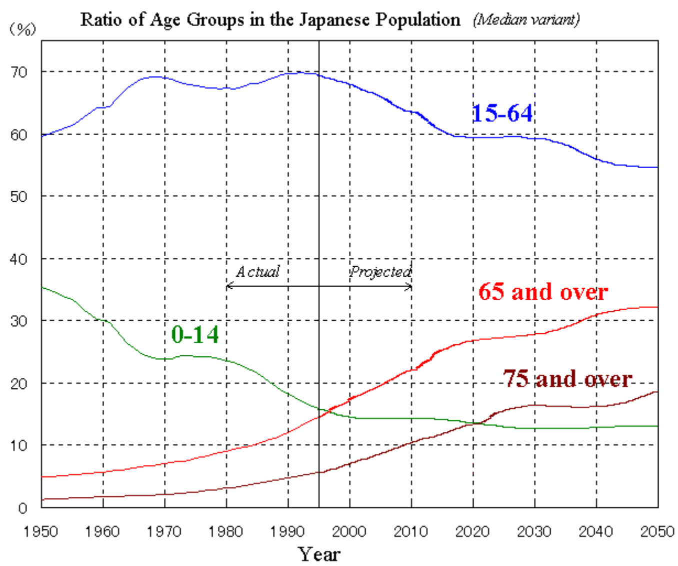


Figure 3: Ratio of age groups: median population scenario from NSSPPRI (National Social Security and Population Problem Research Institute: www.ipss.go.jp 1997

One of the characteristics of the Japanese labor market is the high labor market participation of people over 65 that leads to a mean age at labor force exit for men around 2 years higher than in the US and 5 years higher than in Germany.

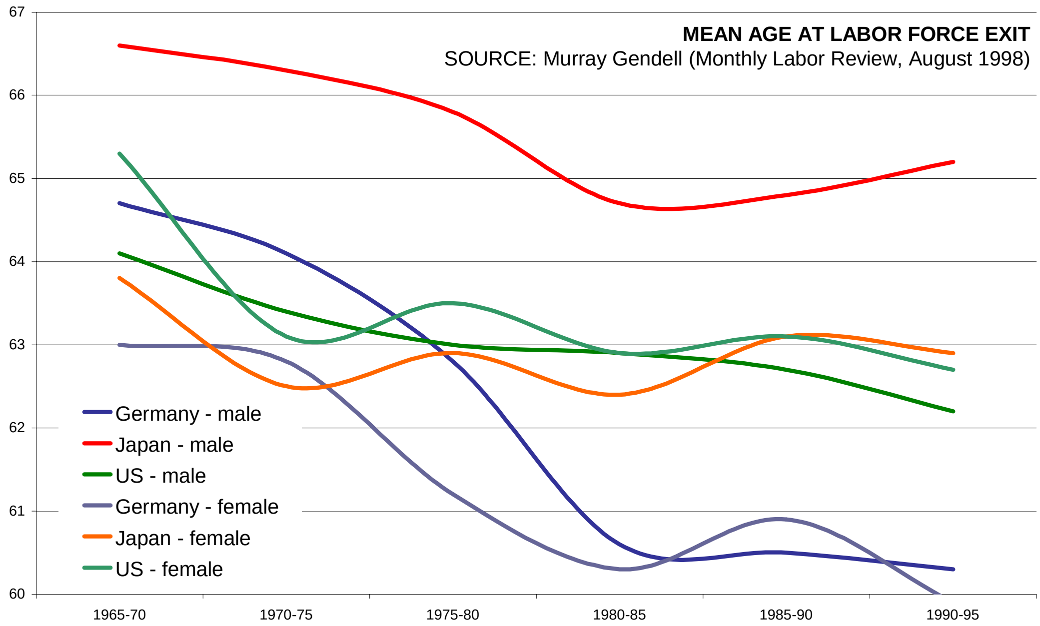


Figure 4: Mean age of labor force exit

Human capital, labor force participation and wage differentials

In the macro-model, age-specific labor force participation rates are exogenous assumptions, as are unemployment rates, currently assumed to be 0. These aggregated numbers have to be disaggregated in a meaningful micro-framework, as labor market participation will highly depend on observable characteristics of the micro-units, namely sex, school enrolment, education level, living arrangement, parity and age of children as well as income or opportunity costs of not working. The macro-model does not introduce human capital, therefore labor input is simply calculated in person-years that, together with total capital, enter a Cobb-Douglas production function determining output and returns for labor (wages) and capital (profits). Total factor productivity is assumed to grow at a fixed exogenously given rate. In its current version, the macro-model assumes that wages follow a logarithmic age-wage path. Age-specific wage rates are directly calculated as a function of the average wage and then re-scaled to ensure that they sum up to total wages as determined in the production function. This procedure does not model different wages as a result of different human capital, and an aging labor force or increased education would not have any effect on the average wage.

By contrast, in the microsimulation framework developed here, human capital proxies are introduced as a function of personal characteristics such as education and work experience. These proxies are used as individual multipliers of hours worked to calculate the "effective labor input" that enters the production function. Wages per "effective labor" unit are determined by a (Cobb-Douglas) production function for total effective labor input and total capital. Microsimulation lends itself to modeling individual human capital growth paths according to individual characteristics and random effects. While average human capital units might be adjusted in the initial year to be unity for the average person-year labor input (or any other meaningful reference labor-input), the average human capital units per person-year will typically rise over time, not only reflecting the increasing average age of the labor force in an aging society, but also increasing average education levels. While this design allows introduction of different education levels and other characteristics as determinants of income, it still implies the strong assumption that equal human capital earns equal wages independent of all other attributes.

For the purpose of this study, i.e. in order to allow for flexible parameterization and the production of both deterministic and stochastic scenarios, the individual human capital evolvement can be specified as a fixed or random walk process. The human capital evolution is assumed to depend only on education level and years worked and depreciation is assumed to be 0. For each education level, a central scenario of (1) a starting value of the human capital proxy for the year entering the labor force, and (2) yearly growth rates subject to total time worked can be specified. To generate a stochastic process, two alternative starting values and vectors of growth rates can be specified. In this way, for given probabilities, the starting values and for each period the yearly growth rate are determined by Monte-Carlo simulation.

Regarding labor market participation, aggregated rates have to be disaggregated in a meaningful micro-framework, as labor market participation will highly depend on various personal and household characteristics. Therefore, even for a given labor market participation rate of e.g. 70% for a given age group, individual participation rates might vary considerable, with the extreme cases that 70% of the members of the age cohort work the whole period, or all members work 70% of the time period. In economic modeling, labor supply is usually assumed to result from some kind of inter-temporal utility maximization of consumption-leisure choices linked with saving decisions and the choice of the retiring age. While the explicit behavioral modeling of labor supply is a central goal of the future model development, at this first stage labor market participation rates are modeled as the outcome of a set of very simple behavioral assumptions, which follow stylized "traditional" patterns of gender roles:

- Male individuals are assumed to enter the labor force when leaving school and leave the labor force when retiring, with given probabilities to retire at a specific age distinguished by education level and age cohort.
- Female individuals are simulated accordingly while childless, and are assumed to stay outside of the labor force for a given fraction of the year giving birth. For the following years probabilities are given to re-enter the labor force, depending on education, parity and birth cohort. The retiring age is determined as for males, using given probabilities for females to retire at a specific age distinguished by education level and age cohort.

In spite of their rather demonstrative nature, the assumptions and the model specifications as stated in this and the previous section already allow to create a starting population "from the scratch", containing all basic demographic characteristics such as age, sex and parity, as well as some economic characteristics such as human capital and labor market participation. Note that, though behavioral assumptions are rather preliminary, the model itself can be parameterized in a way that closely reproduces any demographic and labor supply scenario as currently used in the macro-framework of the IIASA SSR model. It can therefore serve as a starting point for more theory-based modeling as well as for more detailed analyses including distributional aspects and dynamics induced by a changing education composition of the population.

Saving and pension systems

For a given labor supply, saving behavior has still to be modeled that determines capital accumulation over time and therefore - together with labor - the economic output. Saving is of central importance in this kind of analysis, as savings are the "crucial link between decisions today and living standards tomorrow" (Kohl and O'Brien, 1998). Social security systems interact with private saving decisions and put additional complexity into the analysis, as individual contributions to un-funded public pension schemes are transformed into later benefits via an intergenerational contract. With social security contributions usually not accounted for as savings and pension benefits even in funded systems accounted for as income rather than running down assets, measurement problems arise that make it difficult to link micro- and macro-evidence of measured saving rates and therefore to assess life-cycle saving behaviors (Miles 1999). With its fine-grained accounting of age-specific stocks and flows, the IIASA SSR model links household behavior to aggregated numbers following the standard OECD System of National Accounts (SNA). This accounting method can be transformed into the microsimulation framework by further disaggregating the various cohort accounts to individual accounts. Regarding the parameterization of the model this implies that capital stocks of all different types have to be distributed not only between cohorts, but also within cohorts.

The macro-model does not use any explicit decision procedures based on maximizing multi-period objective functions regarding working, consumption and saving decisions or decisions on the distribution of savings to different economic sectors that – in a framework without relative prices – serve a purely accounting purpose. As presently constructed, the IIASA model uses a fixed age-specific coefficient approach to private consumption and labor supply. While it mimics life-cycle models, this is simply because elderly persons, who consume out of annuity income as well as current income, have a lower age-specific saving rate (current income minus consumption relative to current income) than working-age persons. Similarly, the lower labor force participation of older persons due to retirement represents an exogenous assumption, and not the result of an endogenous labor-leisure tradeoff.

Microsimulation modeling might simultaneously maintain the strengths (the accounting treatment, especially the conformity with the SNA), and contribute to overcome the obvious weakness of this model (the mechanical treatment of saving, investment allocation and labor supply decisions). Individual saving rates can be modeled as functions that might take into account different saving motives according to individual and household characteristics. By doing so, the effect of changes of public pension systems on private savings might be assessed both in a consistent theoretical framework and based on empirical evidence, with individual decisions being transformed into macro-outcomes by appropriate aggregation.

At the current state of the pilot microsimulation model developed here, none of these potential strengths have been realized so far, as capital stocks are not tracked individually and only one pension system – a PAYG system in which contribution rates are set every year in order to finance the benefits – is modeled. This imposes further restrictions on saving rates, as only one global saving rate from interest on capital can be set and accumulated capital assets can not be “run down” but have to be implicitly inherited.

2.5 The microSSR software and illustrative results

As part of this pilot study, the prototype of a social security reform microsimulation software microSSR was programmed in C++ that up to now contains a (mostly) demographic module able to generate a synthetic starting population and project its dynamics in a 100+100 years horizon (the first 100 years are used to generate a synthetic population with all individuals born within the simulation). Births, death, education levels, age of leaving school, labor market participation, human capital formation, parental leave and retiring age are determined as outlined in the previous section. As in the macro-model, total output, wages and interest rates are determined by a production function and individuals save part of their wages and interests received, while the government runs a PAYG pension system, collecting contributions and paying benefits according to individual contribution histories. In this way, individual wages and pension benefits as well as individual consumption from these wages and pensions are calculated, while accumulated wealth and consumption from interest can only be calculated for the total population.

Parameters are read in from currently over 30 tables, with all model input and output kept in one EXCEL-file containing various sheets. Graphical output is produced in the form of 3 types of age pyramids, showing total population by sex, total population by occupation, and female population by occupation. Age pyramids can be displayed for any year of the simulation with navigation buttons provided to “browse” through time (see screenshot below). Using standard-Excel format, data can be easily exchanged with various software applications, and the embedded Excel-workbook provides flexible graphical facilities.

In a first stylized scenario, a stationary population of around 200,000 individuals is produced and simulated, resulting in around 2,500 births and deaths per year. The economic simulation and accounting process is simplified, as fixed saving rates from wage income (8%) and of interest income (20%) are assumed. Pension benefits are assumed to be at 60% of the last gross wages – thus, depending on social security contribution rates necessary to balance the PAYG system, replacing around 70% of the last net wage. Pension income is assumed to be fully consumed. No direct or indirect taxes are levied in this simple model, and an initial capital stock of 250,000 “units” is assumed, not further distributed to households. Regarding the production function, a Cobb-Douglas function with a labor coefficient of 0.667 and constant total factor productivity growth of 1% is assumed. Note that the current model specification and the parameterization given below serve purely demonstrative purposes.

Model Parameterization and Baseline Scenario

Mortality rates are read from mortality tables by sex and education level. **Baseline scenario**: mortality patterns by sex, according to Japanese period life-tables for 1950-1995. For the years before 1950 mortality rates from 1950 are applied, mortality patterns are assumed fixed from 1995 on.

Fertility: parity-progression rates and distributions of first births and birth intervals by education and birth cohort are read in from 9 tables. **Baseline scenario**: time- and education-invariant parity distributions as given below:

- No children 15%
- one child 21%
- two children 32%
- three children 16%
- four children 13%
- five and more children 3%

Timing of first birth is determined in years since leaving school, and the same distribution of the time intervals is chosen for the spacing between births:

- 1 or 4 years: 15% (each)
- 2 or 3 years: 30% (each)
- 5 years: 10%
- 6 years: 5%
- 7 or 8 years: 2,5% (each)

In the "Japanese Scenario", only parity progression to first birth is modeled and births are set as totals in order to match Japanese data and projections made outside the model.

Leaving school: time-variant distributions can be given by education level (3 tables). **Baseline scenario**: last year in school

- for education level 0: 15 years
- for education level 1: 18 years
- for education level 2: 23 years

Education level: distributions can be given dependent on mothers' education and birth cohort; currently no distinction is made by sex (3 tables). **Baseline scenario**: distribution invariant from mothers' education and time:

- 60% education level 0
- 30% education level 1
- 10% education level

Labor market participation: individuals are assumed to work from the year after leaving school until retiring if not caring for children (see below). Retiring age distributions can be specified by sex, education and time (6 tables).

Baseline scenario:

- 90% of males retire at 65, 25% of the male population remaining in the labor force retires each following year, all remaining workers retire at 70 years.
- 90% of females retire at 60, 25% of the female population remaining in the labor force retires each following year, all remaining workers retire at 70 years.

Parental leave and staying home with children: time-variant distributions of durations staying outside of the labor force after giving birth can be specified by education level (3 tables). Maximum period staying outside of the labor force is assumed to be 20 years.

Baseline scenario:

- education level 0: 10% of mothers currently staying home re-enter the labor force per year.
- education level 1: 10% of mothers return in the first, 10% of the remaining mothers in the second, and 50% of the remaining mothers return in the third year after birth. Thereafter, 10% of remaining mothers re-enter the labor force per year.
- education level 2: as education level 1, but 75% of women who have not already re-entered the labor force in the first 2 years re-enter in the third year.

Human capital: separate random walks for human capital proxies by education level can be specified by giving three possible starting values per education group (and attached probabilities) and three alternative growth rates for each year worked (3 tables).

Baseline scenario: Central starting value and growth rates (random draw per period).

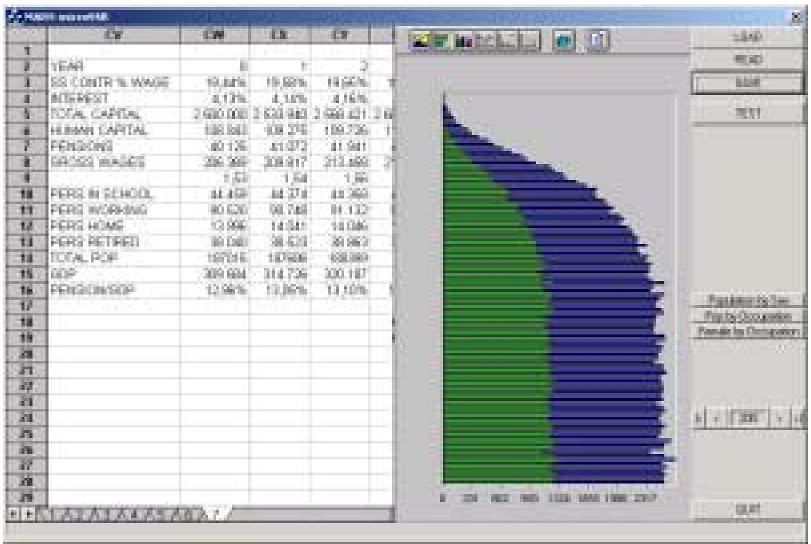
Education 0:				
starting values:	0,6 (25%)	0,8 (50%)	1 (25%)	
growth rates:	0% (25%)	1% (50%)	2% (25%)	years 2 to 20
	-0,5% (25%)	0% (50%)	1% (25%)	afterwards
Education 1:				
starting values:	0,8 (25%)	1 (50%)	1,5 (25%)	
growth rates:	0% (25%)	1% (50%)	3% (25%)	years 2 to 25
	0% (25%)	0,5% (50%)	1% (25%)	afterwards
Education 2:				
Starting values:	1,25 (25%)	2 (50%)	2,5 (25%)	
growth rates:	1% (25%)	2% (50%)	3% (25%)	years 2 to 25
	0% (25%)	0,5% (50%)	1% (25%)	afterwards

The baseline scenario

In the following the baseline scenario will be compared to three alternative scenarios, mainly differing in fertility and education patterns.

- The baseline scenario is one of a stationary population, with initial capital set in a way that puts the economy on a stable growth path with no changes in return to capital and contribution rates.
- The first alternative scenario is a low fertility scenario with all other parameters unchanged.
- The second alternative scenario is based on the low fertility scenario, but with higher rates of people attaining higher education.
- The third alternative scenario is based on the demographic patterns of Japan and the median population scenario (projected births) from NSSPPRI

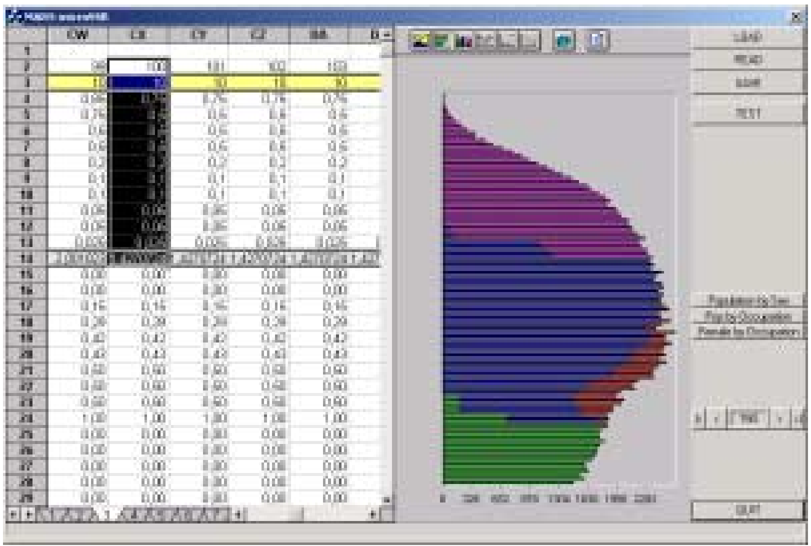
In the baseline scenario, around 19,5% of gross wages have to be contributed to the social security system in order to balance the PAYG system, with pensions being 13% of total GDP. The GDP growth is around 1.5% and the interest rate is around 4.13%. From the total population of around 187,000 persons, 24% are children and/or persons enrolled in schools, 48% work, 7.5% stay home as “housewives”, and 20% are retired. The age-pyramid in the figure below displays the groups disaggregated by age in the year 200, or 100 years after the “setup period” of the population.



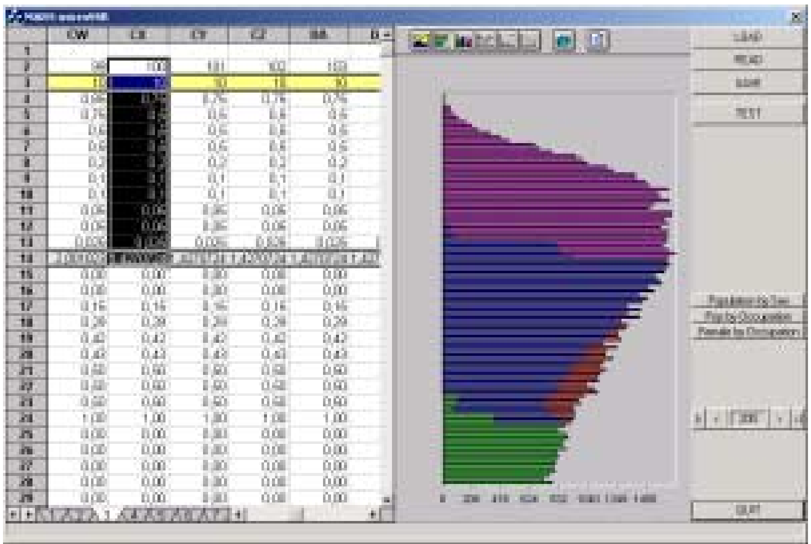
Stationary age pyramid of the baseline scenario. The table displays simulation results for the initial years.

Alternative scenario 1 – "low fertility"

In a first alternative scenario, cohort fertility is lowered from the 100th cohort on to 1.42 children. Starting from the same stationary age distribution as in the baseline scenario, the age pyramid changes for the next 100 years reaching a new stable distribution. The following two screenshots show the resulting age pyramids for the year 150 and the last simulated year 200.



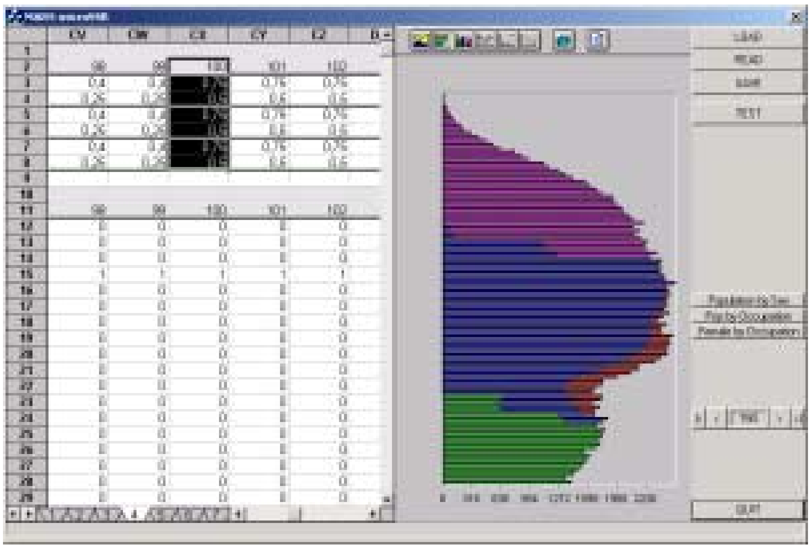
Age pyramid 50 years after change of fertility patterns from replacement to low fertility.



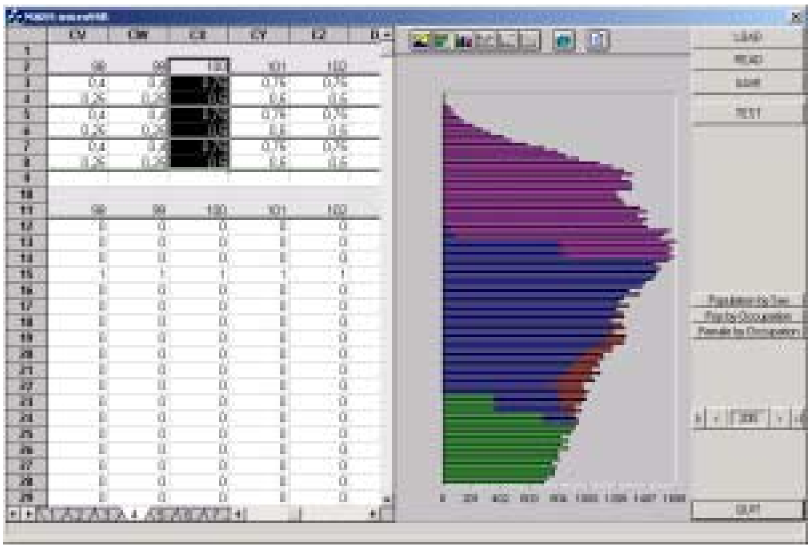
New stable age distribution 100 years after change of fertility patterns from replacement to low fertility.

Alternative scenario 2 – "low fertility & higher education"

In the second alternative scenario, cohort fertility is also lowered from the 100th cohort to 1.42 children. Beginning from the same cohort, education levels are changed, with only 25% staying at level 0 and all others moving to level 1 and 2 with same probabilities. As higher educated women, according to the model, have their children later, this effect can be seen in the shape of the age pyramid. The following two screenshots show the resulting age pyramids for the starting year 150 and the last simulated year 200.



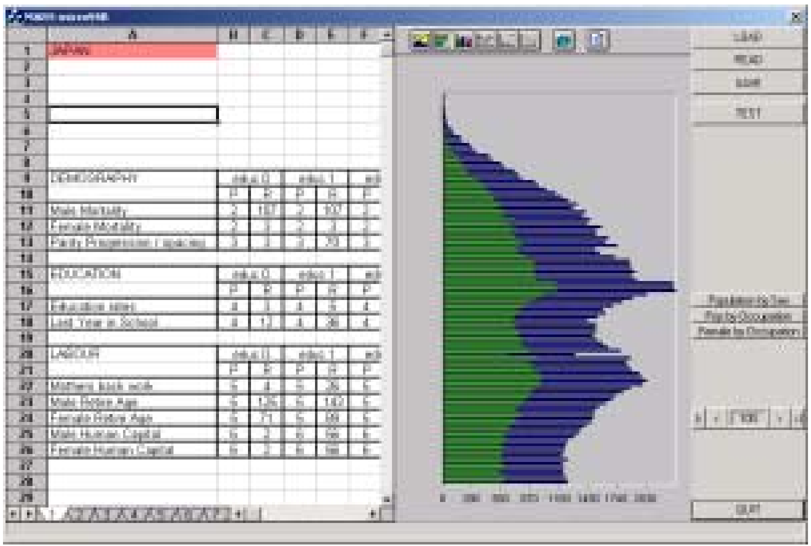
Age pyramid 50 years after change of fertility and education patterns.



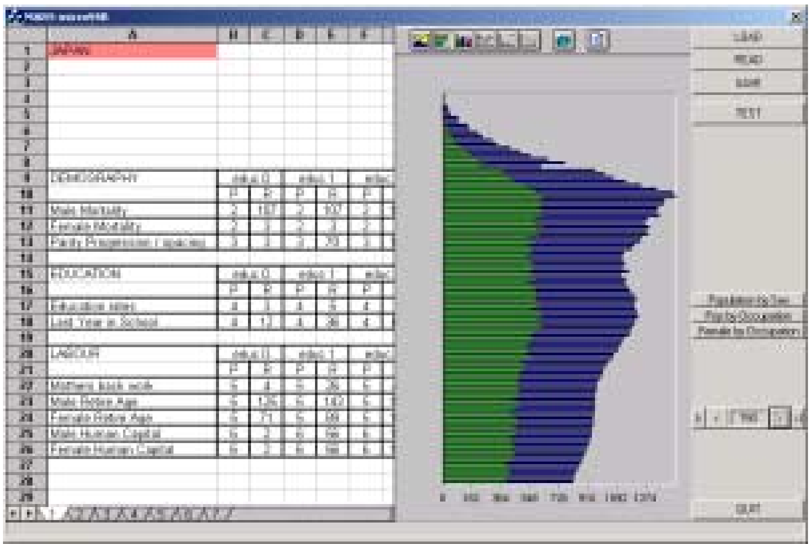
New stable age distribution 100 years after change of fertility and education patterns.

Alternative scenario 3 – "Japan"

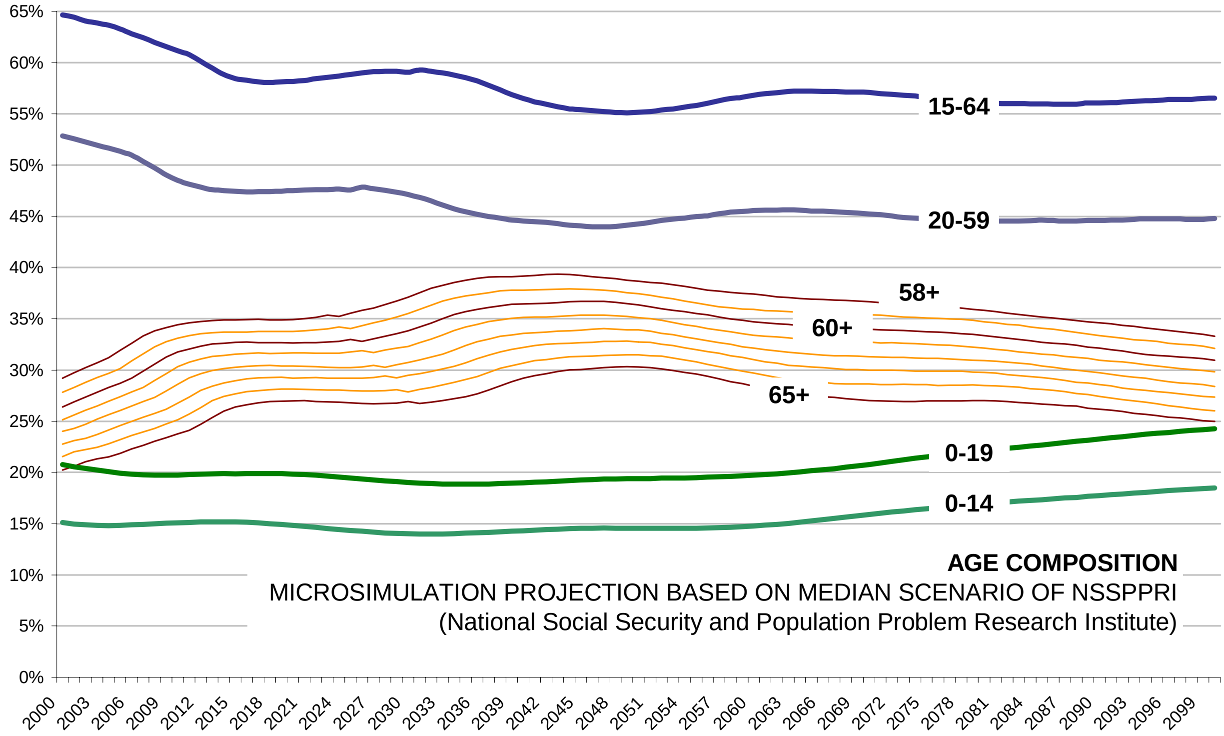
In the third alternative scenario, fertility is not modeled explicitly, but actual and projected births are read in from external sources (the median scenario from NSSPPRI, see above) The age pyramids below therefore reflect actual patterns of Japan in 2000 and projected patterns following the external scenario.



Japanese age pyramide in 2000.



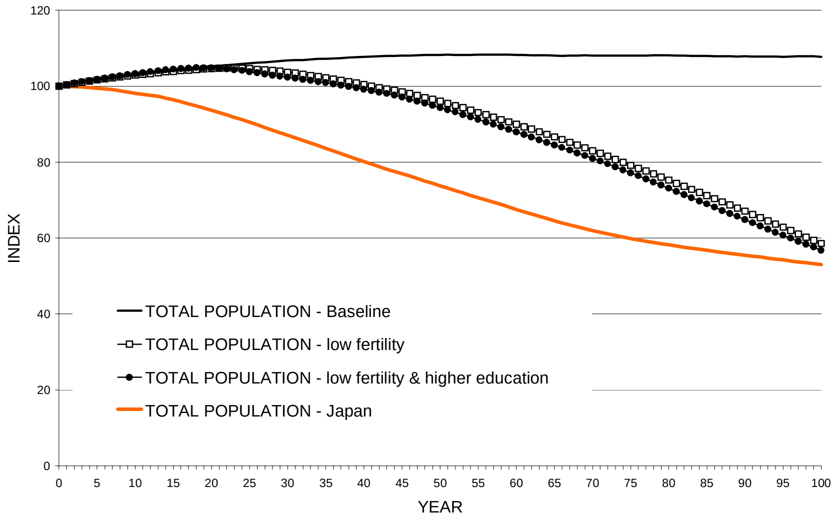
Japanese age pyramide as projected for 2050.



Age composition: projection for Japan

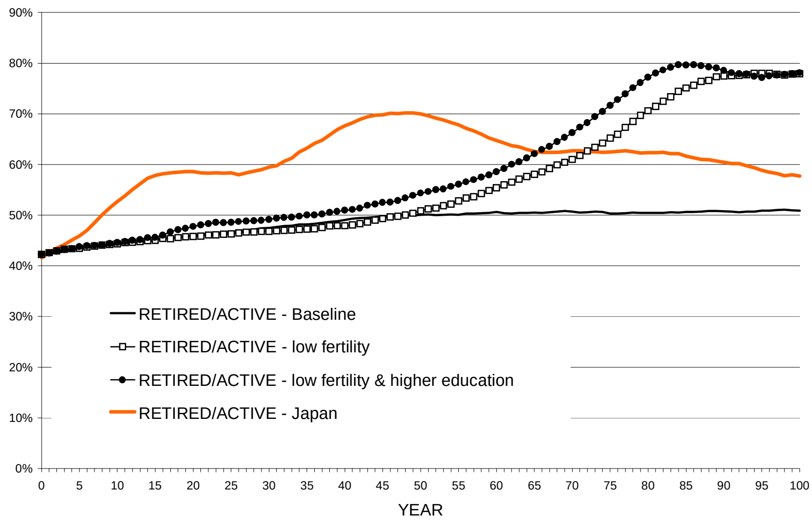
Comparison of scenarios

(1) Demographic changes



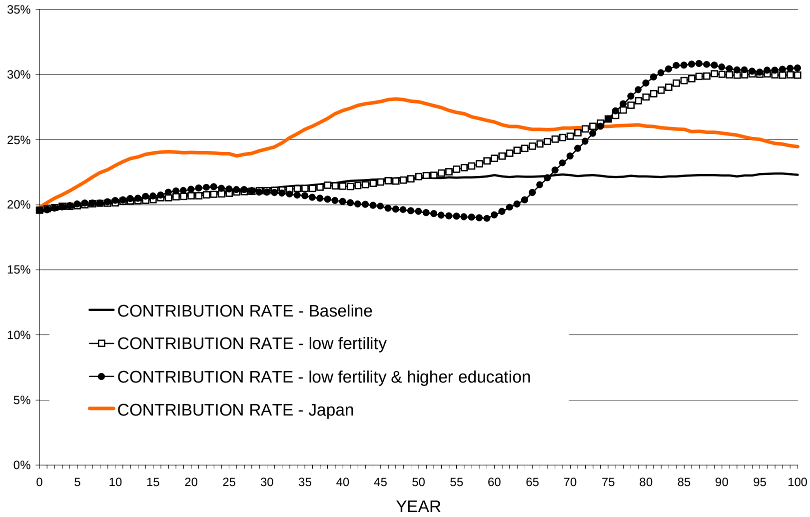
Total population

In all alternative scenarios, the population declines to around half of its initial number 100 years after the drop of fertility from the replacement level to a cohort fertility of 1.42. For the Japanese scenario, total population drops immediately, while in the other scenarios the population initially increases due to mortality changes. As people attaining higher education enter the labor market later in their life course, also the ratio of retired to economically active people departs earlier from the baseline rate in this scenario and - until stabilizing at the same rate in the long run - reaches higher levels due to later births. The Japanese scenario differs considerably from the other stylized figures, with the rate shifting up in two phases until dropping again in the middle of the century (from that on fertility on replacement level is assumed).



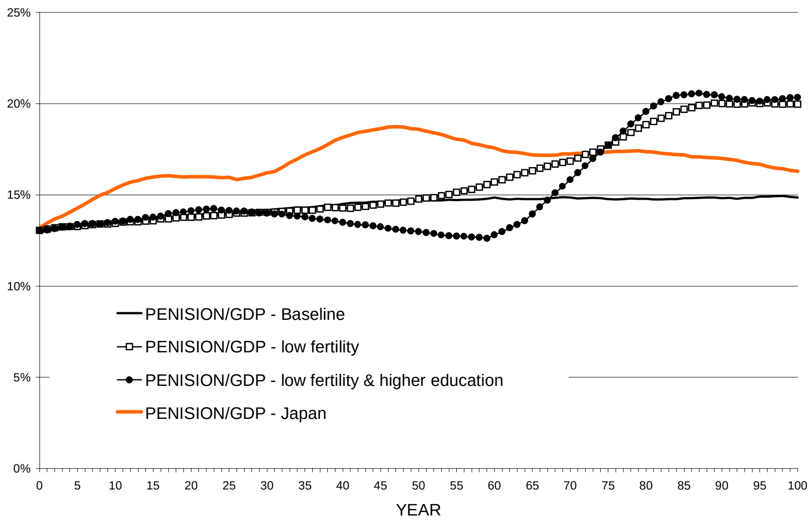
Ratio of retired to economically active people

(2) Pension contributions



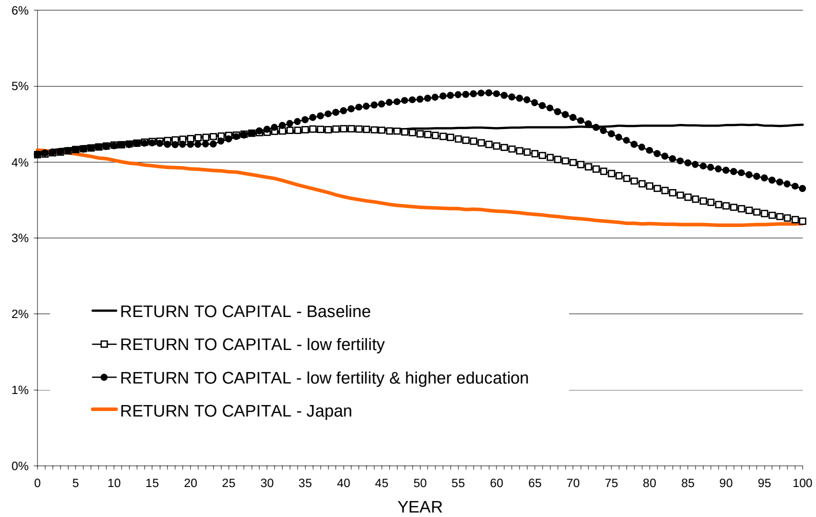
Contribution rates of wages to the pension system

In the baseline scenario of a stationary population, initially 20% of wage income has to be contributed to the PAYG pension system. The low fertility scenario departs from the baseline rate 50 years after cohort fertility drops from the replacement level to 1.42 and stabilizes again at the higher rate of 30%. In the second alternative scenario, the contribution rate temporarily falls with the first cohorts of higher education reaching working age. With these cohorts entering retiring age, contribution rates rise fast from a the minimum of 19% to a new stable level of again 30%. The Japanese Scenario again differs considerably from the others, with contribution rates shifting up in two waves with the highest level reached in the middle of the century. The same pattern is found in pension benefits as percentage of GDP.



Pension benefits as percentage of GDP

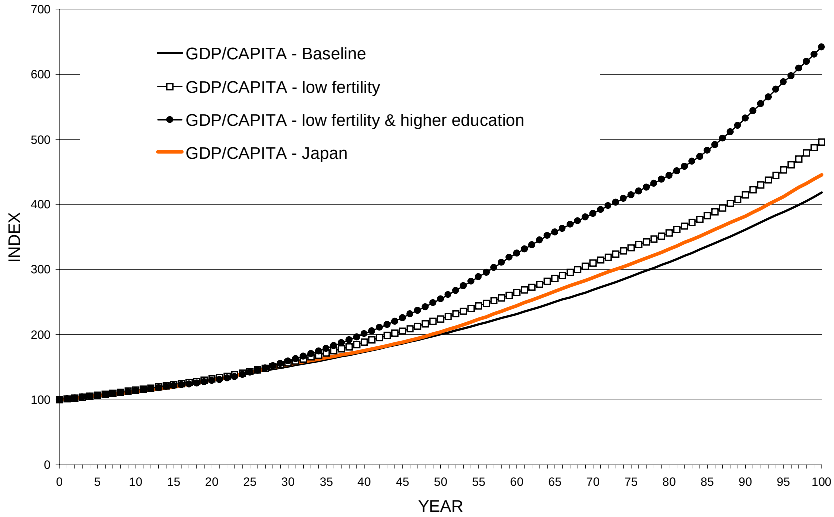
(3) Return to capital



Rate of return to capital

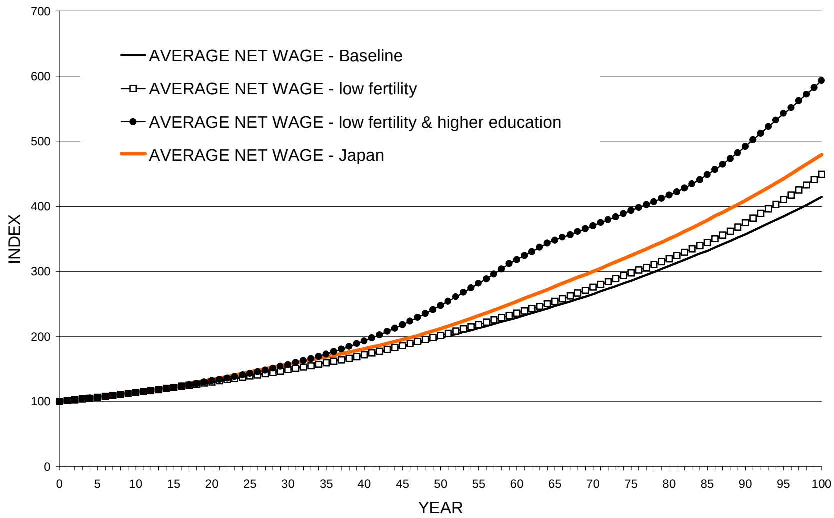
Compared to the baseline scenario, the rate of return to capital falls due to the decreasing labor supply in the low fertility scenario. This effect is temporarily overcompensated in the second alternative scenario, as supply of effective labor initially increases due to higher education. In the Japanese scenario, the rate of return falls immediately as also population and labor supply drop from the first year on.

(4) GDP and wages



GDP/capita

Over the hundred year period, GDP per capita rises about three times in the baseline scenario of a stationary population. Growth is accelerated in per capita terms in the low fertility scenarios, while the Japanese scenario – due to lower returns to capital - is closest to the baseline scenario. Due to higher contribution rates in order to balance the PAYG system, not all growth in GDP/capita is translated into growth of average net wages and the curves for the low fertility scenarios run closer to the baseline case.



Average net wage

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Annex 1: The IIASA Social Security Model: Macro-structure

In this annex, the algebraic structure of the IIASA macromodel is presented and its economic logic is described. For simplicity, a single-region version of the model is presented.

1. Population, labor force, and employment

1.1 Population

Population is divided into age groups $age = \overline{0, MaxAge}$ where *MaxAge* is the beginning year of the terminal age category (for example, $MaxAge = 85$ if the terminal age category is 85+). For this report, we implement the simplest possible approach to population, namely we import a deterministic demographic scenario consisting of population by single-year age groups from another source. Total population is the sum over age groups

$$Pop(t) = \sum_{age=0}^{MaxAge} Pop(male, t, age) + \sum_{age=0}^{MaxAge} Pop(female, t, age)$$

1.2 Labor force and employment

Total labor force is the sum over age groups:

$$LabForce(t) = \sum_{age=15}^{MaxAge} LabForce(t, age)$$

where

$$LabForce(t) = Pop(t, age)\ LabForcePartRate(t, age)$$

Age-specific labor force participation rates are exogenous assumptions, as are unemployment rates:

$$Emp(t) = \sum_{age=15}^{MaxAge} Emp(t, age)$$

$$Emp(t) = LabForce(t, age)\left[1 - UnempRate(t, age)\right]$$

Inclusion of the unemployment rate leaves the way open for applications in which structural unemployment is taken as a function of the long-term trend growth rate of GDP. However, for purposes of this Report, the unemployment rate is assumed to be zero.

2. Capital and the nature of claims

There are four types of capital: residential capital (*KRsdntial*), capital operated by private unincorporated enterprises (*KPvtUnincorpEnt*), capital operated by incorporated enterprises and held on households' behalf by the private pension system (*KPvtPenSys*), and capital operated by incorporated enterprises and held on households’ behalf by other financial institutions (*KOthFinIns*).[2] Also implicitly assigned to *OthFinIns* are households themselves to the extent that they individually hold claims on capital operated by firms (i.e., disintermediated claims). Firms operate capital, either distributing or reinvesting earnings, they do not own shares in other firms. Financial claims on the capital operated by firms are held on behalf of households by institutions (*PvtPenSys* and *OthFinIns*) which collect and distribute dividends. No distinction is made between equity and debt claims.

The private pension system is divided into two components, one of which is a partially funded defined benefit system (*PvtPenSysDB*) and the other of which is a fully funded defined contribution system (*PvtPenSysDC*). Corresponding to each of the four types of capital is an age-specific capital accumulation equation, which tracks the accumulation of assets for each cohort as it ages. There is a structural difference between the dynamics of *KPvtPensSys* and the dynamics of the other three asset classes. Funds flow into *PvtPenSys* only through payroll deductions (including deductions from entrepreneurial income) on behalf of system participants. Dividends earned on assets held by the *PvtPenSys* remain within the system. By contrast, savings of all origins, not just captive retirement-related savings, are invested in *KOthFinIns, KRsdntial, and KPvtUnincorpEnt*. Dividends earned on assets held by *OthFinIns* accrue to households, instead of being retained by the intermediary, and may be allocated to consumption at any point during the life cycle, as may profits accruing to *KPvtUnincorpEnt*. Implicit rents on *KRsdntial* are oonsumed, by assumption, in their entirety; equivalently, all housing is assumed to be owner-occupied. If saved, dividends earned on assets held by *OthFinIns* may remain within *OthFinIns*, or be allocated to residential investment or investment in capital operated by *PvtUnincorpEnt*.

All capital ultimately belongs to households. As described in Section 9, each single-year age-cohort is tracked as it accumulates capital during its working life and draws it down during retirement. Total assets of a cohort in a given year are

$$
\begin{aligned}
&KTot(t,age) = \\
&\qquad KRsdntial(t,age) + KPvtUnincorpEnt(t,age) \\
&\qquad\quad + KPvtPenSysDC(t,age) + KPvtPenSysDB(t,age) \\
&\qquad\quad + KOthFinIns(t,age)
\end{aligned}
$$

or, aggregating together the two components of the private pension system,

[2] From here on, we use "firms" to designated "incorporated enterprises."

$$KTot(t, age) = KPvtPenSys(t, age) + KRsdntial(t, age) + KPvtUnincorpEnt(t, age) + KOthFinIns(t,.age)$$

which expresses a cohort's wealth as the sum of pension- and non-pension wealth.

3. Output and rates of return to factors

3.1 GDP, wage rate, and rate of return to capital

Gross domestic product (*GDP*) is given by a Cobb-Douglas production function

$$GDP(t) = \alpha \left(1 + \mathrm{g}\right)^{t} KTot(t)^{\beta} \, Emp(t)^{1-\beta}$$

where *g*, the rate of total factor productivity growth, is exogenous. Rates of return to factors are neoclassical:

$$R(t) = \beta \left[\frac{GDP(t)}{KTot(t)} \right]$$

$$\overline{WageRate}(t) = \left(1 - \beta\right) \left[\frac{GDP(t)}{Emp(t)} \right]$$

where *R* is the gross profit rate, including depreciation and indirect taxes net of subsidies; and $\overline{WageRate}$ is average (over age groups) employee compensation, including social insurance contributions (contributions to public and private pension schemes).

In order to net depreciation and indirect taxes out of the rate of return to capital, we define

$$r(t) = R(t) - \frac{IndTaxRate(t)\, GDP(t)}{KTot(t)} - DeprRate(t)$$

where *IndTaxRate* is defined with respect to *GDP* and *DeprRate* is the depreciation rate. The advantage of netting out depreciation and indirect taxes immediately is that we can henceforth ignore them in calculating income, outlay, and net savings.

3.2 Age-specific wage rates

In a model with age-structure detail, we require a procedure to ensure that the average wage rate calculated from the marginal productivity condition above equals the average wage rate calculated by summing across age groups. In practice, this means that age-specific wage rates must be adjusted to be consistent with the overall average wage rate. We have approached this problem by defining a scale factor $\sigma(t, age)$ and then calculating age-specific wage rates as

$$WageRate(t, age) = \sigma(t, age)\ \overline{WageRate}(t)$$

$\sigma(t, age)$, a proxy for human capital, is specified to be logarithmic in age and paramaterized so that wages rise rapidly in the twenties and thirties, the average wage over the life cycle is earned at approximately age 45, and there is little increase after 55.

The required consistency condition is

$$\overline{WageRate}(t) = \frac{\sum_{age=15}^{MaxAge} Mult(t, age)\, \sigma(t,age)\, \overline{WageRate}(t) Emp(t, age)}{\sum_{age=15}^{MaxAge} Emp(t, age)}$$

To simplify the problem, let $Mult(t, age)$ to be the age-invariant

$$Mult(t) = \frac{\overline{WageRate}(t) \sum_{age=15}^{MaxAge} Emp(t, age)}{\sum_{age=15}^{MaxAge} \sigma(t, age)\, \overline{WageRate}(t)\ Emp(t, age)} = \frac{\sum_{age=15}^{MaxAge} Emp(t, age)}{\sum_{age=15}^{MaxAge} \sigma(t, age)\ Emp(t, age)}$$

This variable can be interpreted as total "nominal" employment relative to total "effective," i.e. human-capital adjusted employment. Then the identity required for consistency may be rewritten

$$\overline{WageRate}(t) = \frac{\sum_{age=15}^{MaxAge} \left\{ \frac{\sum_{age=15}^{MaxAge} Emp(t, age)}{\sum_{age=15}^{MaxAge} \sigma(t, age)\ Emp(t, age)} \sigma(t,age)\, \overline{WageRate}(t) Emp(t, age) \right\}}{\sum_{age=15}^{MaxAge} Emp(t, age)}$$

Moving age-invariant terms outside the braces,

$$\overline{WageRate}(t) = \frac{\dfrac{\sum_{age=15}^{MaxAge} Emp(t,age)}{\sum_{age=15}^{MaxAge} \sigma(t,age) Emp(t,age)} \overline{WageRate}(t) \sum_{age=15}^{MaxAge} \sigma(t,age) Emp(t,age)}{\sum_{age=15}^{MaxAge} Emp(t,age)}$$

which will clearly always hold true. Therefore, we calculate

$$WageRate(t,age) = Mult(t)\,\sigma(t,age)\,\overline{WageRate}(t)$$

where $Mult(t)$ is as defined above.

4. Income, capital transfers, outlay, and net saving of households

The articulation of income flows elaborated below has two main purposes. The first is to disaggregate income and consumption by age. The second is to make explicit the role of the private pension system in saving and the allocation of capital.

4.1 Income

The sources of household income are wages, imputed rents from residential capital, profits which accrue to capital operated by unincorporated enterprises, dividends distributed from earnings on capital operated by firms, public pension system benefits, private pension benefits, and health and long-term care benefits.

4.1.1. A note on taxation

The taxation of factor incomes in this model follows three simplifying assumptions. First, factor income is taxed once and only once, when it is earned. Thus, dividend income is not taxed because profits have already been taxed at the level of the firm; similarly, there is no capital gains tax when assets are sold because capital gains reflect profits which have already been taxed. Second, no distinction is made from a taxation point of view between different types of capital: profits on capital operated by firms, capital operated by private unincorporated enterprises, and the imputed services of residential housing are all assumed to be taxed at the same rate. Third, tax rates are not indexed by income or age.

4.1.2 Wage income

Disposable wage income is equal to gross wages minus direct taxes minus social insurance contributions to the public PAYG and private pension systems minus contributions to the health and long-term care systems:

$$WageY(t, age) = WageRate(t, age)\ \ Emp(t, age)$$

$$\begin{aligned} &DispWageY(t, age) = \\ &\quad WageY(t, age) - DirTaxWageY(t, age) - ContPubPenSysWageY(t, age, - ContPvtPenSysWageY(t, age) \\ &\quad - ContHlthCareSysWageY(t, age) - ContLngTrmCareSys(t, age) \end{aligned}$$

where $ContPvtPenSysWageY(t, age)$ consists of the sum of contributions to the defined benefit and defined contribution components of the private pension system. Calculation of direct taxes and pension system contributions are described in Sections 6 and 7, respectively, and contributions to the health and long-term care systems are described in Section 8. Note that, even though *PvtPenSys* contributions really represent the acquisition of a financial asset, rather than a current expenditure flow, the System of National Accounts (SNA) nonetheless counts such transactions as a debit in the calculation of disposable income. However, an adjustment is made (see Section 4.1.9) to ensure that the savings associated with such flows are credited to households.

4.1.3 Rental income

Imputed rental income is assumed to be taxed like any other form of income; however, social contributions are assumed to be zero:

$$RntlY(t, age) = r(t)\ KRsdntial(t, age)$$

$$DispRntlY(t, age) = RntlY(t, age) - DirTaxRntlY(t, age)$$

Recall, from Section 3.1, that capital returns are already net of depreciation and indirect taxes.

4.1.4 Entrepreneurial income

Profits from capital operated by unincorporated enterprises are treated the same as wages:

$$EntrY(t, age) = r\ (t)\ KPvtUnincorpEnt(t, age)$$

$$\begin{aligned} &DispEntrY(t, age) = \\ &\quad EntrY(t, age) - DirTaxEntrY(t, age) - ContPubPenSysEntrY(t, age) - Cont\text{P}vtPenSysEntrY(t, age) \\ &\quad - ContHltCareSysEntrY(t, age) - ContLngTrmCareSys(t, age) \end{aligned}$$

4.1.5 Dividend income

The assets held on households' behalf by *PvtPenSys* and *OthFinIns* earn dividends. However, in the first case, dividends are not considered by the SNA to be part of household income; rather, they are considered to represent the acquisition of a financial asset. The adjustment described in Section 4.1.9 will add these dividend earnings captured by the private pension system to household income. Unadjusted household disposable income includes only dividends on assets held by *OthFinIns*:

$$DividY(t, age) = DividDistErngsFirmsKOthFinIns(t, age)$$

The calculation of distributed earnings is given below in Section 5.2.2. Having already been taxed when earned, dividend earnings are not taxed when received by households. Disposable dividend income is thus simply

$$DispDividY(t, age) = DividY(t, age)$$

4.1.6. Interest on government debt

Interest income derived from on government debt is

$$IntGovDebt(t, age) = r(t)\, GovDebt(t, age)$$

and disposable income is

$$DispIntGovDebt(t, age) = IntGovDebt(t, age) - DirTaxIntGovDebt(t, age)$$

4.1.7 Pension income

Pension income comes from three sources: the public PAYG pension system, the private defined contribution pension system, and the private defined benefit pension system. All three systems provide benefits, which are current transfers in the first case and, while representing sales of capital assets in the second two cases, are nonetheless considered for accounting purposes to represent income. The calculation of pension benefits is described in Section 7.

In addition, as described in Section 7.2.2, in any year, some persons will change jobs and a given proportion of these will choose to withdraw their assets from the private pension system rather than rolling them over into new plans. These withdrawals are also treated as income. We assume that withdrawals occur only from the defined contribution pension system. While this point is debatable, however, most countries have in place measures that strongly encourage job-switchers to transfer their defined benefit pension assets into another plan.[3] Combining the two components of the private pension system,

[3] The logic behind this is that, on retirement, the benefit entitlement from the private defined benefit pension scheme is calculated on the basis of years of participation and earnings. If we were to allow withdrawal of assets, it would be necessary to "restart the clock" every time assets were withdrawn, or to link benefits with accumulated assets (as in the defined contribution private pension system) rather than earnings.

$$PensionY(t, age) = BenPubPenSys(t, age) + BenPvtPenSys(t, age) + WthdrwlKPvtPenSys(t, age)$$

Public pension income is subject to taxation because it is a current transfer. Private pension income and early withdrawals from the defined contribution pension system are not taxed because these represent the sale of capital assets whose returns were taxed at the level of the firm. Disposable pension income is therefore

$$\begin{aligned} &DispPensionY(t, age) = \\ &\quad [1 - DirTaxRate(t)]\, BenPubPenSys(t, age) + BenPvtPenSys(t, age) + WthdrwlKPvtPenSys(t, age) \end{aligned}$$

Since private pension system benefits represent the drawing-down of a capital asset, they are included in the adjustment to disposable income described in Section 4.1.11. Early withdrawals (usually in consequence of job change) from the defined contribution private pension system are described in Section 4.2.3; these are also included in the adjustment to disposable income described below.

4.1.8 Health and long-term care benefits

The calculation of health and long-term care benefits is described in Section 8. These are assumed to be untaxed.

4.1.9 Total income

Total income of households is equal to the sum over all income sources.

$$\begin{aligned} TotYHH(t, age) = {} & WageY(t, age) + RntlY(t, age) + EntrY(t, age) + DividY(t, age) + IntGovDebt(t, age) \\ & + PensionY(t, age) + BenHlthCareSys(t, age) + BenLngTrmCareSys(t, age) \end{aligned}$$

4.1.10 Disposable income

Disposable income is analogous:

$$\begin{aligned} &DispYHH(t, age) = \\ &\quad DispWageY(t, age) + DispRntlY(t, age) + DispEntrY(t, age) + DispDividY(\mathrm{t}, age) + DispIntGobDebt(t, age) \\ &\quad + DispPensionY(t, age) + BenHlthCareSys(t, age) + BenLngTrmCareSys(t, age) \end{aligned}$$

4.1.11 Adjusted disposable income.

Adjusted disposable income is equal to disposable income

- plus contributions to *PvtPenSys*,
- plus dividends earned on assets held by *PvtPenSys*,
- minus benefits received from *PvtPenSys*.

- minus early withdrawals from the defined contribution private pension system (see Section 4.2.3)

Thus,

$$
\begin{aligned}
&AdjDispYHH(t,age) = \\
&\qquad DispYHH(t,age) + ContPvtPenSys(t,age) + DividKPvtPenSys(t,age) - BenPvtPenSys(t,age) \\
&\qquad\quad - WthdrwlKPvtPenSysDC(t,age)
\end{aligned}
$$

where the third term represents the sum over defined benefit and defined contribution components of the private pension systems of dividends paid out by firms (given in Sections 7.2.1 and 7.3.1, respectively).

Adjusted disposable income is close to, but not exactly the same as, disposable income plus change in pension wealth. The latter would be equal to

$$
\begin{aligned}
&\Delta KPvtPenSys(t,age) = \\
&\qquad ContPvtPenSys(t,age) + DividKPvtPenSys(t,age) - BenPvtPenSys(t,age) \\
&\qquad\quad - BeqKPvtPenSys(t,age) - WthdrwlKPvtPenSysDC(t,age)
\end{aligned}
$$

where $BeqKPvtPenSys(t,age)$ reflects the fact that upon the death of the claimant, accumulated pension assets are paid out to heirs.[4] There is no accounting for inheritance of pension wealth because paying-out is assumed to take the form of cash allocated either to consumption or to the acquisition of non-pension capital assets. Stated differently, there is no explicit modeling of survivors' benefits, which amount to reassigning title to existing pension assets. In conclusion, we could also write

$$
AdjDispYHH(t,age) = DispYHH(t,age) + \Delta KPvtPenSys(t,age) + BeqPvtPenSys(t,age)
$$

4.2 Capital transfers

Resources available for household consumption take the form of disposable income and the proceeds of transferring claims to capital assets. In this section, the second of these is elaborated.

4.2.1 Annuitization of assets in old age

Starting at the pension eligibility age, households are assumed to divest themselves of non-pension assets in a way calculated to exhaust assets at age 105. This

[4] From the standpoint of calculating individual wealth, the bequest term is irrelevant, because the individual must die in order to bequeath. In calculating cohort wealth, however, bequests must be taken into account.

"annuitization" process -- which we model for simplicity's sake just as a program of asset sales -- is assumed to begin whether households are still in the labor force or not. For $[\bullet] = Rsdntial, PvtUnincorpEnt, OthFinIns$, annuity income is:

$$AnnYK[\bullet](age,t) = \frac{K[\bullet](t,age)}{105 - age} \quad , age \geq \; EligAge$$

If the propensity to consume out of the proceeds of asset sales is unity, there is no bequest motive; if, for example, the propensity is 0.95, elderly households aim to die with 5 percent of their wealth intact, etc. Note that it is assumed that no assets are sold prior to retirement, apart from the special case of assets received via inheritance and the withdrawal of defined contribution pension assets associated with job change, which we discuss in the next sections.

4.2.2 Bequests / inheritance

In all asset classes, age-specific bequests are equal to assets times the proportion of persons in the age group dying. For $[\bullet] = Rsdntial, PvtUnincorpEnt, OthFinIns, PvtPenSysDC, PvtPenSysDB$:

$$BeqK[\bullet](t,age) = K[\bullet](t,age)\left[\frac{Deaths(t,age)}{Pop(t\text{-}1,age\text{-}1)}\right]$$

Without question, defined contribution pension system assets, like non-pension wealth, belong to the individual and are heritable. As we discuss in Section 9.1, the case of assets of the defined benefit pension system is more debatable.

Bequests are received, in the form of inheritance, by the surviving population. For simplicity, we estimate age-specific inheritance simply by dividing total bequests by population age shares, excluding the population under age 15. Total bequests are

$$BeqK[\bullet](t) = \sum_{age=15}^{MaxAge} BeqK[\bullet](t,age)$$

and inheritance (for age groups over 15) is

$$InhK[\bullet](t,t) = \left[\frac{Pop(t,age)}{\sum_{age=15}^{MaxAge-1} Pop(t,age)}\right] \sum_{age=15}^{MaxAge} BeqK[\bullet](t,age)$$

Summing over age groups,

$$InhK[\bullet](t) = \sum_{age=15}^{MaxAge} InhK[\bullet](t, age)$$

This simplification admittedly exaggerates the number of "backwards" bequests (elderly persons inheriting wealth from middle-aged persons, who are in fact more likely to bequeath assets to their children than to their parents). [5]

The assumption is made that, when wealth is inherited, it is converted to cash, some of which is allocated to consumption and the remainder of which is allocated among *ΔKOthFinIns*, *ΔKRsdntial*, and *ΔPvtUnincorpEnt* using the same share coefficients applied to household net saving (see Section 9.3). Note, however, that the portion not consumed does not comprise new household savings; it represents the acquisition of an asset formed as the result of past saving.

Under these assumptions, sales of inherited assets are

$$SaleInhK[\bullet](t, age) = InhK[\bullet](t, age)$$

Consumption out of the proceeds of such sales is described in Section 4.3.4.

4.3 Outlay

4.3.1 Direct taxes.

These are described in Section 6.1.

4.3.2 Social insurance contributions

These are described in Section 7.

4.3.3 Consumption out of income

Average propensities to consume (*AvgPropCons*) out of disposable income streams are exogenous assumptions:

$$ConsWageY(t, age) = DispWageY(t, age)\ AvgPropConsWageY(t, age)$$

$$ConsEntrY(t, age) = DispEntrY(t, age)\ AvgPropConsEntrY(t, age)$$

$$ConsDividY(t, age) = DispDividY(t, age)\ AvgPropConsDividY(t, age)$$

$$ConsBenPubPenSys(t, age) = BenPubPenSys(t, age)\ AvgPropConsBenPubPenSys(t, age)$$

$$ConsBenPvtPenSysDC(t, age) = BenPvtPenSysDC(t, age)\ AvgPropConsBenPvtPenSysDC(t, age)$$

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[5] One expedient way to solve this problem is to assume that only persons under some age, say 65, but this runs the danger of failing to account for significant spousal bequests. Ultimately, a vector of age-specific share coefficients should be applied to allocate bequests from persons of a given age group over heirs by age group.

$$ConsBenPvtPenSysDB(t,age) = BenPvtPenSysDB(t,age)\ Avg\Pr opConsBenPvtPenSysDB(t,age)$$

It is assumed that all imputed housing services received are consumed:

$$ConsRntlY(t,age) = DispRntlY(t,age)$$

4.3.4 Consumption out of the proceeds of asset sales.

It is assumed that consumption out of the proceeds of asset sales takes place in the year of the sale, i.e., households do not hold liquid balances.

4.3.4.1 Consumption out of the proceeds of selling inherited assets

For $[\bullet] = Rsdntial, PvtUnincorpEnt, OthFinIns, PvtPenSysDC, PvtPenSysDB$ consumption out of the sales of inherited assets is

$$ConsSaleInhK[\bullet](t,age) = SaleInhK[\bullet](t,age)\ ConsShareSaleInhK[\bullet](t,age)$$

and the sharing-out of what is not consumed between *ΔKOthFinIns*, *ΔKRsdntial,* and *ΔKPvtUnincorpEnt* is described in Section 9.3 below. We use a mnemonic corresponding to "consumption share" instead of *AvgPropCons* because average propensity to consume is properly considered with reference to income.

4.3.4.2 Consumption out of retirement annuity income.

Consumption in old age financed by the sale of assets accumulated during working life is treated in the same way. Because private pension system benefits are classified for purposes of the SNA as income, rather than capital transfers, this component has already been described above. For the remaining components $[\bullet] = Rsdntial, PvtUnincorpEnt, OthFinIns$:

$$ConsAnnYK[\bullet](t,age) = AnnYK[\bullet](t,age)\ ConsShareAnnYK[\bullet](t,age)$$

As mentioned above in discussion private pension system benefits, if there is no bequest motive, the consumption shares are assumed to be unity. However, this assumption can be generalized to allow for bequests. In this case, the complement of the consumption share is simply the proportion of wealth upon retirement which households wish to bequeath.

4.3.4.3 Consumption out of the proceeds of selling defined contribution pension assets withdrawn in consequence of job change.

The final component of consumption is:

$$ConsWthdrwlKPvtPenSysDC(t, age) = WthdrwlKPvtPenSysDC(t, age)\, ConsShareWthdrwlKPvtPenSys(t, age)$$

Early withdrawals from the privare defined-benefit pension system are assumed to be zero.

4.3.5 Consumption of health and long-term care

Determination of age-specific consumption of health and long-term care services is described in Section 8.

4.4 Net savings of households

Recapitulating, disposable and adjusted disposable household incomes are

$$\begin{aligned} &DispYHH(t, age) = \\ &\quad DispWageY(t, age) + DispRntlY(t, age) + DispEntrY(t, age) + DispDividY(\mathrm{t}, age) + DispPensionY(t, age) \\ &\quad + BenHlthCareSys(t,age) + BenLngTrmCareSys(t, age) \end{aligned}$$

$$AdjDispYHH(t,age) = DispYHH(t,age) + ContPvtPenSys(t,age) + DividPvtPenSys(t,age) - BenPvtPenSys(t,age)$$

and total consumption is

$$\begin{aligned} &PvtCons(t, age) = \\ &\quad ConsDispWageY(t, age) + ConsDispRntlY(t, age) + ConsDispEntrY(t, age) \\ &\qquad + ConsDispDvidY(\mathrm{t}, age) + ConsDispPensionY(t, age) \\ &\qquad + ConsSaleInhKRsdntial(t, age) + ConsSaleInhKPvtUnincorpEnt(t, age) \\ &\qquad + ConsSaleInhKOthFinIns(t, age) + ConsSaleInhKPvtPenSysDC(t, age) \\ &\qquad + ConsAnnYKRsdntial(t, age) + ConsAnnKYPvtUnincorpEnt(t, age) + ConsAnnYKOthFinIns(t, age) \\ &\qquad + ConsWthdrlKPvtPenSysDC(t, age) \\ &\qquad + ConsHlthCare(t, age) + ConsLngTrmCare(t, age) \end{aligned}$$

The first two lines on the right-hand side give consumption out of income (including pension income), the second two lines give consumption financed by the sale of inherited assets, the fifth line gives consumption out of annuity income, the sixth line covers consumption which occurs when a worker changes jobs and elects to withdraw defined contribution pension assets, and the last line gives consumption of health and long-term care services.

Household net saving is the difference between disposable income and consumption:

$$NetSvngHH(t, age) = DispYHH(t, age) - PvtCons(t, age)$$

and adjusted net savings includes savings captured by the private pension system:

$$AdjNetSvngHH(t, age) = AdjDispYHH(t, age) - PvtCons(t, age)$$

or, expressing in terms of unadjusted disposable income and change in pension wealth (see Section 4.1.9),

$$AdjNetSvngHH(t, age) = DispYHH(t, age) + \Delta KPvtPenSys(t, age) + BeqKPvtPenSys(t, age) - PvtCons(t, age)$$

In performing the consistency check in Section 10 below, we will use this identity in the form

$$NetSvngHH(t, age) = AdjNetSvngHH(t, age) - \Delta KPvtPenSys(t, age) + BeqKPvtPenSys(t, age)$$

5. Income, outlay, and net savings of firms

Firms operate capital, earn profits and pay out direct taxes and dividends.

5.1 Income

Earnings of firms consist of earnings on capital owned by the three institutional claimants $[\bullet] = PvtPenSysDC, PvtPenSysDB, OthFinIns$:

$$ErngsFirmsK[\bullet](t, age) = r(t)\, K[\bullet](t, age)$$

$$ErngsFirmsK[\bullet](age) = \sum_{\mathrm{age}=0}^{\mathrm{MaxAge}} ErngsFirmsK[\bullet](t, age)$$

Recall that depreciation and indirect taxes have already been netted out.

5.2 Outlay

5.2.1 Direct taxes

Taxes on profits are described in Section 6.1.

5.2.2 Dividends

Dividend distributions are made out of pre-tax earnings, and the proportion of earnings distributed is independent of the claimant by assumption. For the three claimants $[\bullet] = PvtPenSysDC, PvtPenSysDB, OthFinIns$:

$$DividDistErngsFirmsK[\bullet](t, age) = DividDistShare(t)\; ErngsFirmsK\;[\bullet](t, age)$$

$$DividDistErngsFirmsK[\bullet](t) = \sum_{age=0}^{MaxAge} DividDistErngsFirmsK[\bullet](t, age)$$

where the share of earnings distributed as dividends is an exogenous variable.

5.3 Net savings of firms

Net savings (retained earnings) of firms are

$$NetSvngErngsFirmsK[\bullet](t, age) = ErngsFirms\ K[\bullet](t, age)$$
$$- DirTaxErngsFirmsK[\bullet](t, age) - DividDistErngsFirmsK[\bullet](t, age)$$

The sum over claimants is total net savings of firms:

$$NetSvngFirms(t, age) = \sum_{[\bullet]} NetSvngErngsFirmsK[\bullet](t)$$

and the sum over age groups gives total corporate savings:

$$NetSvngFirms(t) = \sum_{\text{age}=0}^{\text{MaxAge}} NetSvngFirms(t, age)$$

6. Income, outlay, and net savings of government

The government sector is rudimentary. Government consumes an exogenous share of GDP, collects taxes and social security contributions and pays social security benefits.

6.1 Income

Government revenues are

$$GovRvn(t) = IndTax(t) + DirTax(t) + ContPubPenSyst(t)$$

where

$$IndTax(t) = IndTaxRate(t)\ GDP(t)$$

$$DirTax(t) =$$
$$\sum_{age=15}^{MaxAge} DirTaxWageY(t, age) + \sum_{age=15}^{MaxAge} DirTaxEntrY(t, age) + \sum_{age=15}^{MaxAge} DirTaxRntlY(t, age)$$
$$+ \sum_{age=EligAge}^{MaxAge} DirTaxBenPubPenSys(t, age) + \sum_{age=EligAge}^{MaxAge} IntGovDebt(t, age)$$
$$+ \sum_{age=0}^{\text{MaxAge}} \sum_{[\bullet]} DirTaxErngsFirmsK[\bullet](t, age)$$

where $[\bullet] = PvtPenSysDC, PvtPenSysDB, OthFinIns$ and the direct tax streams are

$$DirTaxWageY(t, age) = DirTaxRate(t)\, WageY(t, age)$$

$$DirTaxRntlY(t, age) = DirTaxRate(t)\, RntlY(t, age)$$

$$DirTaxEntrY(t, age) = DirTaxRate(t)\, EntrY(t, age)$$

$$DirTaxBenPubPenSys(t, age) = DirTaxRate(t)\, BenPubPenSys(t, age)$$
$$DirTaxIntGovDebt(t, age) = DirTaxRate(t)\, IntGovDebt(t, age)$$
$$DirTaxErngsFirmsK[\bullet](t, age) = DirTaxRate(t)\, ErngsFirmsK[\bullet](t)$$

Contributions to the public pension system are described in Section 7.1.1.

6.2 Outlay

Government expenditure is:

$$GovExp(t) = GovCons(t) + BenPubPenSys(t) + IntGovDebt(t)$$

where government consumption is taken simply as a fixed share of GDP:

$$GovCons(t) = GovConsShare(t)\, GDP(t)$$

Benefits paid out by the public pension system are described in Section 7.1.2. Interest on government debt is the sum over age groups

$$IntGovDebt(t) = \sum_{age=15}^{MqxAge} r(t)\, GovDebt(t, age0$$

6.3 Net savings of government

Government net savings are

$$NetSvngGov(t) = GovRvn(t) - GovExp(t)$$

Net savings of government are allocated across age groups using shares drawn from the age-distribution of wealth:

$$NetSvngGov(t, age) = \frac{KTot(t, age)}{\sum_{\hat{a}ge=0}^{MaxAge} KTot(t, age)} NetSvngGov(t)$$

6.4 Government debt

Government debt is cumulated

$$GovDebt(t) = GovDebt(-1) + NetSvngGov(t)$$

and shared down over age groups

$$GovDebt(t,age) = \frac{KTot(t,age)}{\sum_{\hat{a}ge=0}^{MaxAge} KTot(t,age)} GovDebt(t)$$

7. Pension system

7.1 Public defined benefit PAYG pension system

7.1.1 Income

Contributions to the public pension system out of wages are

$$ContPubPenSysWageY(t,age) = \left[PartSharePubPenSys\ ContRatePubPenSys(t)\ TaxableWageYShare\right] WageY(t,age)$$

where $PartSharePubPenSys$ is the proportion of the workforce participating in the public pension system and $TaxableWageYShare$ is the proportion of the wage bill which is subject to social security taxation. For simplicity, both of these parameters are assumed to be age-independent. The social security contribution rate $ContRatePubPenSys(t)$ is also assumed to be age-independent. For each year it is calculated to ensure that total contributions to the system are equal total payments of benefits. No distinction is made between employees' and employers' contributions. Social security contributions out of entrepreneurial income are calculated similarly:

$$ContPubPenSysEntrY(t,age) = \left[PartSharePubPenSys\ ContRatePubPenSys(t)\ TaxableShareEntrY\right] EntrY(t,age).$$

Contribution rates out of wage and entrepreneurial income are assumed to be the same.

Total public pension system revenues out of each income stream are

$$ContPubPenSysWageY(t) = \sum_{age=15}^{MaxAge} ContPubPenSys\,WageY(t,age)$$

$$ContPubPenSysEntrY(t) = \sum_{age=15}^{MaxAge} ContPubPenSysEntrY(t,age)$$

and the system total is:

$$ContPubPenSys(t) = ContPubPenSysWageY(t) + ContPubPenSysEntrY(t)$$

7.1.2 Outlay

The public pension system is assumed to be a DB system financed on a Pay As You Go (PAYG) basis. Let $BenEntPubPenSys(t, age, RtrmntDuration)$ be the social security benefit entitlement for the average person aged age who retired $RtrmntDuration$ years ago, where we assume that $BenEntPubPenSys(t, age, 0) = 0$. The pension for persons entering retirement is computed according to the formula:

$$BenEntPubPenSys(t, age, 1) = WageRate(t-1, age-1) RplcmntRatePubPenSys(t)$$

where

$$RplcmntRatePubPenSys(t, age) = AvgYearPartPubPenSys(t, age) AccrualRatePubPenSys(t),$$

i.e. the replacement ratio is equal to the accrual rate $AccrualRatePubPenSys(t)$ (an exogenous assumption) times the average number of years of labor force participation. The major simplification made is to calculate average years spent in the labor force on a period basis, in the year in which retirement occurs, instead of on a cohort basis over the life of the worker. For a worker retiring at age *age* in year *t*, then:

$$AverYearPartPubPenSys(t, age) = \sum_{j=15}^{age} LabForcePartRate(t, j)$$

Once persons have retired, their pension is indexed to the average wage rate. For people who were already retired at (*t-1*), the pension is

$$\begin{aligned} &BenEntPubPenSys(t, age, RtrmntDuration) = \\ &\quad BenEntPubPenSys(t-1, age-1, RtrmntDuration-1) \left[1 + IndexRate(t) \frac{\overline{Wage}(t) - Wage(t-1)}{\overline{Wage}(t-1)} \right] \end{aligned}$$

where $IndexRate$ is the rate of indexation of pensions to the average wage rate $\overline{Wage}(t)$. When $IndexRate = 1$ pensions are fully indexed to wages, when $IndexRate = 0$ there is no indexation.

Social security system benefits paid out by age group of recipient are equal to the age- and retirement-duration specific entitlement times the number of recipients:

$$BenPubPenSys(t, age) = \sum_{RtrmntDuration=0}^{MaxAge-EligAge+1} BenEntPubPenSys(t, age, RtrmntDuration)\, Pop(t, age, RtrmntDuration)$$

where, making the simplifying assumption that once retired, persons stay retired,

$$\begin{aligned} &Pop(t, age, RtrmntDuration) = \\ &\quad Pop(t, age) \begin{bmatrix} LabForcePartRate(t - RtrmntDuration, age - RtrmntDuration) \\ - LabForcePartRate(t - RtrmntDuration + 1, age - RtrmntDuration + 1) \end{bmatrix} \end{aligned}$$

for $age = \overline{1, age - RtrmntAge + 1}$. System-wide expenditures are equal to the summation over age groups

$$BenPubPenSys(t) = \sum_{age=EligAge}^{MaxAge} BenPubPenSys(t, age)\, Pop(t, age)\,.$$

7.1.3 System balance

In a classic PAYG system (for example, the German system), total contributions equal total benefits; there is neither accumulation of a return-generating surplus nor a deficit to be financed out of general government revenue. The default model solution option is one in which the required contribution rate is calculated by setting contributions equal to expenditures.

7.2 Private defined contribution pension system

7.2.1 Income

Income of the private defined contribution pension system is comprised of (1) current contributions (zero for persons who have retired), and (2) receipt of dividends. (1) is the sum over contributions out of wage and entrepreneurial income, each consisting of the share of the workforce participating times the proportion of total income contributed:

$$\begin{aligned}&ContPvtPenSysDCWageY(t, age) = \\ &\qquad [PartSharePvtPenSaysDC\ \ ContRatePvtPenSysDCWageY(t, age)]\ WageY(t, age)\end{aligned}$$

$$\begin{aligned}&ContPvtPenSysDCEntrY(t, age) = \\ &\qquad [PartSharePvtPenSaysDC\ \ ContRatePvtPenSysDCEntrY(t, age)]\ EntrY(t, age)\end{aligned}$$

In the case of the private defined contribution pension system (and the defined benefit pension system as well) there is no term analogous to $TaxableWageYShare$. Total contributions and dividend earnings are

$$ContPvtPenSysDC(t, age) = ContPvtPenSysDCWageY(t, age) + ContPvtPenSysDCEntrY(t, age)$$

and

$$DividPvtPenSysDC(t) = \sum_{age=15}^{MaxAge} DivDistErngsFirmsKPvtPenSysDC(t, age)$$

where the paying-out of dividends was described in Section 5.2.2.

7.2.2 Outlay

Expenditures of the private defined contribution pension are (1) benefits paid out (zero for persons still in the labor force), (2) payout to heirs of the pension assets of system participants who die, and (3) withdrawal of assets by job-switchers who choose not to roll over their pension wealth into another plan. (1) is analogous to the "annuitization" of non-pension capital assets described in Section 4.2.1 above, with the difference that only those who have left the labor force receive pension benefits:

$$BenPvtPenSysDC(t, age) = [1 - LabForcePartRate(t, age)] \frac{KPvtPenSysDC(t, age)}{105 - age}$$

where $age \geq EligAge$ and 105 is the maximum age to which a person expects to live. (2) was described above in Section 4.2.2. (3) is calculated using an exogenously assumed withdrawal rate reflecting both the number of job-changes and the proportion who choose not to roll over their assets:

$$WthdrwlPvtPenSysDC(t, age) = WthdrwlRatePvtPenSysDC\ KPvtPenSysDC(t, age)$$

If, for example, 10 percent of system participants change jobs every year and half choose to withdraw their assets, we would have $WthdrwlRatePvtPenSysDC = 0.05$.

7.3 Private defined benefit pension system

7.3.1 Income

In contrast to the public DB pension system, in the private DB pension system only wage income is taxed, in addition to which, all wage income is subject to contributions. Therefore,

$$\begin{aligned} &ContPvtPenSysDBWageY(t, age) = \\ &\qquad [PartSharePvtPenSysDB\ \ ContRatePvtPenSysDB(t, age)]\ \ WageY(t, age) \end{aligned}$$

The DB private system does not cover entrepreneurs, so

$$ContPvtPenSysDB(t, age) = ContPvtPenSysDBWageY(t, age)$$

In the case of the public pension system, the contribution rate is calculated to balance the system. In the case of the private DB system, the same calculation is performed, but then the resulting contribution rate is adjusted upward or downward (on an ad hoc basis) to ensure that system reserves neither increase or decrease unreasonably.

Age-specific dividends are

$$DividKPvtPenSysDB(t, age) = DividDistErngsFirmsKPvtPenSysDB(t, age)$$

The total over age groups is

$$DividPvtPenSysDB(t) = \sum_{age=15}^{MaxAge} DividKPvtPenSysDB(t, age)$$

7.3.2 Outlay

The average private DB pension entitlement for a newly-retired person is calculated in the same way as the average initial public pension entitlement:

$$BenEntPvtPenSysDB(t, age, 1) = WageRate(t-1, age-1) RplcmntRatePvtPenSysDB(t)$$

where

$$RplcmntRatioPvtPenSys(t, age) = AverYearPartPvtPenSys(t, age) AccrualRatePvtPenSys(t) \, ,$$

For a worker retiring at age *age* in year *t*, as in the public pension system,

$$AverYearPartPubPenSys(t, age) = \sum_{j=15}^{age} LabForcePartRate(t, j)$$

As in the public pension system, the pensions of already-retired persons are indexed to wages. For people who were already retired at (*t-1*), the pension is

$$BenEntPvtPenSys(t, age, RtrmntDuration) =$$
$$BenEntPvtPenSys(t-1, age-1, RtrmntDuration-1)\left[1 + IndexRatePvtPenSys(t)\frac{\overline{Wage}(t) - \overline{Wage}(t-1)}{\overline{Wage}(t-1)}\right]$$

where $IndexRatePvtPenSys$ is the rate of indexation of pensions to the average wage rate $\overline{Wage}(t)$ ..

Total benefits paid out within a given cohort are

$$BenPvtPenSysDB(t, age) =$$
$$\sum_{RetDur=0}^{MaxAge-EligAge+1} BenEntPvtPenSysDB(t, age, RtrmntDuration)\; Pop(t, age, RtrmntDuration)$$

We assume that $RtrmntDuration$ is the same for the public PAYG and private DB pension systems. The total benefits paid out are the sum over cohorts:

$$BenPvtPenSysDB(t) = \sum_{age=EligAge}^{MaxAge} BenPvtPenSysDB(t, age)\; Pop(t, age)$$

7.3.3 System balance

Firms have the freedom to accumulate or run down assets in their defined benefit pension schemes so long as they are in compliance with prevailing actuarial standards. *Grosso modo*, however, when the ratio of beneficiaries to contributors rises, firms are forced to increase contribution rates. Therefore, we calculate the private defined benefit pension system contribution rate to balance the system

7.4 Total private pension system contributions and benefits

Private pension system totals are

$$ContPvtPenSysWageY(t) = ContPvtPenSysDCWageY(t) + ContPvtPenSysDBWageY(t)$$

$$ContPvtPenSysEntrY(t) = ContPvtPenSysDCEntrY(t) + ContPvtPenSysDBEntrY(t)$$

$$DividKPvtPenSys(t) = DividKPvtPenSysDC(t) + DividKPvtPenSysDB(t)$$

$$BenPvtPenSys(t) = BenPvtPenSysDB(t) + BenPvtPenSysDC(t)$$

8. Health and long-term care systems

8.1 Income

The health care and long-term care systems, assumed to run on a PAYG basis like the public pension system, are structurally identical. For $\bullet = Hlth, LngTrm$ we have contributions out of wage and entrepreneurial income:

$$Cont[\bullet]CareSysWageY(t) = ContRate[\bullet]CareSys(t, age)WageY(t, age).$$

$$Cont[\bullet]CareSysEntrY(t) = ContRate[\bullet]CareSys(t, age)EntrY(t, age)$$

Total contributions are

$$Cont[\bullet]CareSys(t) = Cont[\bullet]CareSysWageY(t, age) + Cont[\bullet]CareSysEntrY(t, age).$$

As in the case of the public pension system, each of the two contribution rates is calculated to balance its corresponding system.

8.2 Outlay

There are two sources of growth in health- and long-term care expenditure, one related to demographic change (both the size of the population and its age structure) and the second related to technological change, development of new treatments, labor costs, age-specific coverage and service utilization rates, etc. We treat the second, "underlying" rate of growth as an exogenous assumption and concentrate our modeling strategy on the first.

Let $h(age)$ be the relative cost (i.e., expenditure per capita) of health care at age *age* with the numéraire being, say, the cost of health care between age 0 and 1; let $l(age)$ be the analogous index for long-term care. Then indices of demographically-induced growth in consumption of health and long-term care are

$$ConsHlthCareIndex(t) = \frac{\sum_{age=1}^{MaxAge(t)} Pop(age,t)\ h(age,t)}{\sum_{age=1}^{MaxAge(t)} Pop(age,0)\ h(age,0)}$$

$$ConsLngTrmCareIndex(t) = \frac{\sum_{age=1}^{MaxAge(t)} Pop(age,t)\ l(age,t)}{\sum_{age=1}^{MaxAge(t)} Pop(age,0)\ l(age,0)}$$

where 0 represents the base year. These indices contain both volume effects related to population growth and composition effects related to the age structure of the population.

Then total health- and long-term expenditures are equal to themselves lagged one year times growth in the index times the underlying growth rate:

$$Cons[\bullet]Care(t) = Cons[\bullet]Care(t-1) \text{ x } \left[1 + UnderRateCons[\bullet]Care(t-1,t)\right]$$
$$\text{x} \left[\frac{Cons[\bullet]CareIndex(t)}{Cons[\bullet]CareIndex(t-1)}\right]$$

There is some basis in history for assuming that the underlying rate of growth of expenditure in both sectors is simply equal to the rate of GDP growth.

Total expenditure is shared down by age group:

$$Cons[\bullet]Care(age,t) = Cons[\bullet]Care(t) \;\; Cons[\bullet]AgeShare(age,t)$$

whre age shares are implicit in the expenditure indices introduced above:

$$ConsHlthCareAgeShare(age,t) = \frac{h(age)Pop(age,t)}{\sum_{age} h(age)Pop(age,t)}$$

$$ConsLngTrmCareAgeShare(age,t) = \frac{l(age)Pop(age,t)}{\sum_{age} l(age)Pop(age,t)}$$

Note that we implicitly assume that the underlying growth rates of health- and long-term care expenditure are age-neutral.

We split age-specific spending into components covered by social insurance and private co-payments, assuming that these exogenous shares are constant over the age spectrum; this residual is then equal to system benefits :

$$Bens[\bullet]CareSys(age,t) = (1 - [\bullet]CareCoPayShare(age,t))Cons[\bullet]Care(age,t)$$

9. The life-cycle dynamics of capital accumulation

Corresponding to each of the types of capital *KPvtPenSysDC*, *KPvtPenSysDB*, *KRsdntial*, *KPvtUnincorpEnt* and *KOthFinIns* is an age-specific capital accumulation identity.

9.1 Defined contribution private pension system

Change in age-specific private defined contribution pension wealth is

$$\begin{aligned}\Delta KPvtPenSysDC(t, age) = {} & ContPvtPenSysDC(t, age) + DividKPvtPenSysDC(t, age) - \\ & - BenPvtPenSysDC(t, age) - BeqKPvtPenSysDC(t, age) - WthdrwlKPvtPenSysDC(t, age)\end{aligned}$$

The most important characteristic of the private defined contribution pension system is that there is a fixed relationship between the amount a cohort pays in during its working life and the amount it receives after retirement. For an individual cohort born in year $t = 0$ whose last members die out in year $t = 105$ lifetime pension contributions plus lifetime earnings on pension assets minus lifetime pension benefits received equals bequest of pension wealth. Expressing this differently,

$$\sum_{t=0}^{105} \sum_{age=15}^{105} \Delta KPvtPenSysDC(t,age) =$$

$$\sum_{t=0}^{105} \sum_{age=15}^{105} \begin{bmatrix} ContPvtPenSysDC(t,age) \\ \quad + DividPvtPenSysDC(t,age) \\ \quad - BenPvtPenSysDC(t,age) - BeqKPvtPenSysDC(t,age) \\ \quad - WthdrwlKPvtPenSysDC(t,age) \end{bmatrix} = 0$$

a. Defined benefit private pension system

Contributions to the private defined contribution pension system purchase an asset, which is owned by the system participant who made the contribution. Contributions into the private defined benefit pension system, by contrast, purchase a claim on a future pension to be paid by the firm, which is in turn backed by an asset acquired by the firm. The question of whether assets backing a defined benefit pension scheme belong to system participants or to the firm is a complicated one, and legal regimes differ from country to country. For accounting purposes, however, we treat assets of the defined benefit pension system the same way we treat assets of the defined contribution pension scheme.

$$\begin{aligned}&\Delta KPvtPenSysDB(t, age) = \\ &\quad ContPvtPenSysDB(t, age) + DividPvtPenSysDB(t, age) - BenPvtPenSysDB(t, age) \\ &\quad\quad - BeqKPvtPenSysDB(t, age)\end{aligned}$$

For very aged cohorts, unlike in the case of the defined contribution pension scheme, defined benefit pension wealth can be negative, i.e., persons will have received more in assets than they accumulated during their working lives.. In this case, there is a negative "inheritance" upon death.

b. Other assets

For $[\bullet] = Rsdntial, PvtUnincorpEnt, OthFinIns$, the age-specific accumulation equations are

$$\begin{aligned}&\Delta K[\bullet](t, age) = \\ &\quad K[\bullet]Share(t)\left[NetSvngHH(t, age) + NetSvngFirms(t, age) + NetSvngGovt(t, age)\right] \\ &\quad\quad - AnnYK[\bullet](t, age) + K[\cdot]Share(t)\sum_{[\bullet]} AnnYK[\bullet] \\ &\quad\quad - BeqK[\bullet](t, age) + InhK[\bullet](t, age) - SaleInhK[\bullet](t, age) + K[\bullet]Share(t)\sum_{[\bullet]} SaleInhK[\bullet] \\ &\quad\quad + K[\bullet]Share(t)[SaleInhKPvtPenSysDC(t, age) + SaleInhKPvtPenSysDB(t, age)]\end{aligned}$$

The components of change are, in order:

- In the first line on the right-hand side of the identity, a share variable $K[\bullet]Share(t)$ summing to unity across the three forms of non-pension wealth is used to apportion unadjusted household net savings plus the imputed age-specific savings of firms and government.between $\Delta KRsdntial$, $\Delta KPvtUnincorpEnt$, and $\Delta KOthFinIns$.[6]
- The second line on the right-hand side is of relevance only for elderly households. The first term subtracts dissaving in the form of annuitization of assets, as described in Section 4.2.1. The second term, when combined with the consumption from annuity income which is implicit in net household savings in the first line, has the effect of distributing savings from annuity income between the non-pension asset classes.
- The third line on the right-hand side subtracts net bequests (the first two terms) and, analogously to the second line, distributes that portion of inherited wealth not converted into consumption among asset classes. Consumption financed by the sale of inherited assets is not accounted for here because, like consumption from annuity income, it has already been subtracted off in calculating net household savings in the first line.
- The fourth line on the right-hand side distributes inheritance of pension assets between the non-pension asset classes.[7] Again, associated consumption has already been accounted for when net household savings in the first line is calculated.

10. Macroeconomic identities

10.1. Gross domestic product

Gross domestic product (GDP) is the sum of wages, net profits, indirect taxes, and depreciation:

$$\begin{aligned}
&GDP(t) = \\
&\quad WageY(t) + RntlY(t) + EntrY(t) \\
&\qquad + ErngsFirmsKPvtPenSysDB(t) + ErngsFirmsKPvtPenSysDC(t) + ErngsFirmsKOthFinIns(t) \\
&\qquad + \left[\frac{IndTaxRate(t)\,GDP(t)}{KTot(t)} + DeprRate(t)\right] KTot(t)
\end{aligned}$$

Since

$$\begin{aligned}
&KTot(t) = \\
&\quad KRsdntial(t) + KPvtUnincorpEnt(t) \\
&\qquad + KPvtPenSysDB(t) + KPvtPenSysDC(t) \\
&\qquad + KOthFinIns(t)
\end{aligned}$$

it is clear without further checking that GDP thus expressed will be equal to GDP calculated using the production function in Section 3.1.

---

[6] Allocation shares are not indexed by age for computational simplicity.

[7] Note that early withdrawals from the private defined contribution pension system, as well as consumption financed by such withdrawals, are included in net houshold savings in the first line.

10.2 National disposable income

National disposable income is GDP adjusted for depreciation:

$$NatDispY(t) = GDP(t) - DeprRate(t)KTot(t)$$

10.3 Net national savings

Net national savings are equal to national disposable income minus consumption:

$$NetNatSvng(t) = NatDispY(t) - PvtCons(t) - GovCons(t)$$

We show in the next section that net national savings thus calculated are equal to the sum of net savings of households, firms, and government.

11. Accounting consistency checks

We apply two accounting consistency checks, first to confirm that the sum of net savings over age groups equals total capital formation, and second to confirm that net savings calculated by summing across households, firms and government equal net savings calculated by subtracting consumption from GDP.

11.1 Net savings equals capital formation

In order to stress the consistency linkage between the pension system and the macroeconomic framework we show here that total net capital formation is equal to total net savings. First, adding across the accumulation equations given in Section 8.3 for the three non-pension forms of wealth, and remembering that the $K[\bullet]Share(t)$ variables sum to unity across the three non-pension asset classes,

$$\begin{aligned}
&\Delta KRsdntial(t,age) + \Delta KPvtUnincorpEnt(t,age) + \Delta KOthFinIns(t,age) = \\
&\quad [NetSvngHH(t,age) + NetSvngFirms(t,age) + NetSvngGovt(t,age)] \\
&\qquad - AnnYKRsdntial(t,age) - AnnYKPvtUnincorpEnt(t,age) - AnnYKOthFinIns(t,age) \\
&\qquad + AnnYKRsdntial(t,age) + AnnYKPvtUnincorpEnt(t,age) + AnnYKOthFinIns(t,age) \\
&\qquad - BeqKRsdntial(t,age) - BeqKPvtUnincorpEnt(t,age) - BeqKOthFinIns(t,age) \\
&\qquad + InhKRsdntial(t,age) + InhKPvtUnincorpEnt(t,age) + InhKOthFinIns(t,age) \\
&\qquad - SaleInhKRsdntial(t,age) - SaleInhKPvtUnincorpEnt(t,age) - SaleInhKOthFinIns(t,age) \\
&\qquad + SaleInhKRsdntial(t,age) + SaleInhKPvtUnincorpEnt(t,age) + SaleInhKOthFinIns(t,age) \\
&\qquad + SaleInhKPvtPenSys(t,age)
\end{aligned}$$

Making cancellations and remembering that

$$InhKPvtPenSys(t,age) = SaleKPvtPenSys(t,age)$$

we arrive at

$$
\begin{aligned}
&\Delta KRsdntial(t,age)+\Delta KPvtUnincorpEnt(t,age)+\Delta KOthFinIns(t,age)=\\
&\qquad\left[NetSvngHH(t,age)+NetSvngFirms(t,age)+NetSvngGovt(t,age)\right]\\
&\qquad\quad-BeqKRsdntial(t,age)-BeqKPvtUnincorpEnt(t,age)-BeqKOthFinIns(t,age)\\
&\qquad\quad+InhKRsdntial(t,age)+InhKPvtUnincorpEnt(t,age)+InhKOthFinIns(t,age)\\
&\qquad\quad+InhKPvtPenSys(t,age)
\end{aligned}
$$

Adding pension wealth, change in total wealth is

$$
\begin{aligned}
&\Delta KTot(t,age)=\\
&\qquad\Delta KPvtPenSys(t,age)+\left[NetSvngHH(t,age)+NetSvngFirms(t,age)+NetSvngGovt(t,age)\right]\\
&\qquad\quad-BeqKRsdntial(t,age)-BeqKPvtUnincorpEnt(t,age)-BeqKOthFinIns(t,age)\\
&\qquad\quad+InhKRsdntial(t,age)+InhKPvtUnincorpEnt(t,age)+InhKOthFinIns(t,age)\\
&\qquad\quad+InhKPvtPenSys(t,age,)
\end{aligned}
$$

Based on the definition of adjusted net household savings given above in Section 4.4,

$$NetSvngHH(t,age)=AdjNetSvngHH(t,age)-\Delta KPvtPenSys(t,age)-BeqKPvtPenSys(t,age)$$

so

$$
\begin{aligned}
&\Delta KTot(t,age)=\\
&\qquad\Delta KPvtPenSys(t,age)\\
&\qquad\quad+\begin{bmatrix}AdjNetSvgHH(t,age)-\Delta KPvtPenSys(t,age)-BeqKPvtPenSys(t,age)\\ \quad+NetSvngFirms(t,age)+NetSvngGovt(t,age)\end{bmatrix}\\
&\qquad\quad-BeqKRsdntial(t,age)-BeqKPvtUnincorpEnt(t,age)-BeqKOthFinIns(t,age)\\
&\qquad\quad+InhKRsdntial(t,age)+InhKPvtUnincorpEnt(t,age)+InhKOthFinIns(t,age)\\
&\qquad\quad+InhKPvtPenSys(t,age)
\end{aligned}
$$

$\Delta KPvtPenSys(age,t)$ cancels, leaving the result

$$
\begin{aligned}
&\Delta KTot(t,age)=\\
&\qquad\left[AdjNetSvgHH(t,age)+NetSvngFirms(t,age)+NetSvngGovt(t,age)\right]\\
&\qquad\quad-BeqKRsdntial(t,age)-BeqKPvtUnincorpEnt(t,age)-BeqKPvtPenSys(t,age)-BeqKOthFinIns(t,age)\\
&\qquad\quad+InhKRsdntial(t,age)+InhKPvtUnincorpEnt(t,age)+InhKOthFinIns(t,age)+InhKPvtPenSys(t,age)
\end{aligned}
$$

In other words, change in wealth for members of a given cohort is equal to

- their net saving, adjusted to include include net saving through the private pension system,
- plus their imputed share of the net savings of firms and government,
- plus the sum across all asset classes (pension- and non-pension alike) of inheritance minus bequests.

Summing over age groups, inheritance and bequests cancel out, leaving

$$\Delta KTot(t)=AdjNetSvngHH(t)+NetSvngFirms(t)+NetSvngGovt(t)$$

which is the desired result.

11.2 Net savings calculated "bottom up" equal net savings calculated "top down"

In this section, we wish to confirm that net national savings calculated “top down” as national disposable income minus consumption is equal to net national savings calculated "bottom up" by summing net savings across households, firms, and government.

11.2.1 "Bottom up"

We start by summing across sectors:

11.2.1.1 Households

From Section 4.4,

$$
\begin{aligned}
&AdjDispYHH(t) = \\
&\quad DispWageY(t) + DispRntlY(t) + DispEntrY(t) + DispDividY(t) + DispPensionY \\
&\quad\quad + ContPvtPenSys(t) - BenPvtPenSys(t) + DividKPvtPenSys(t) - WthdrwlKPvtPenSysDC(t, age)
\end{aligned}
$$

Expanding this expression and subtracting off consumption, we obtain adjusted net saving:

$$
\begin{aligned}
&AdjNetSvngHH(t) = \\
&\quad WageY(t) - DirTaxWageY(t) - ContPubPenSysWageY(t) - ContPvtPenSysWageY(t) \\
&\quad\quad + RntlY(t) - DirTaxRntlY(t) \\
&\quad\quad + EntrY(t) - DirTaxEntrY(t) - ContPubPenSysEntrY(t) - ContPvtPenSysEntrY(t) \\
&\quad\quad + DividY(t) \\
&\quad\quad + BenPubPenSys(t) - DirTaxBenPubPenSys(t) + BenPvtPenSys(t, age) + WthdrwlKPvtPenSysDC(t, age) \\
&\quad\quad + ContPvtPenSys(t) + DividKPvtPenSys(t) - BenPvtPenSys(t,age) - WthdrwlKPvtPenSys(t,age) \\
&\quad\quad - PvtCons(t)
\end{aligned}
$$

Making cancellations and substituting dividends paid out by firms for dividends received by households, the expression is written as

$$
\begin{aligned}
&AdjNetSvngHH(t) = \\
&\quad WageY(t) - DirTaxWageY(t) - ContPubPenSysWageY(t) \\
&\quad\quad + RntlY(t) - DirTaxRntlY(t) \\
&\quad\quad + EntrY(t) - DirTaxEntY(t) - ContPubPenSysEntrY(t) \\
&\quad\quad + DividDistErngsFirmsKOthFinIns(t) \\
&\quad\quad + BenPubPenSys(t) - DirTaxBenPubPenSys(t) \\
&\quad\quad + DividDistErngsFirmsKPvtPenSys(t) \\
&\quad\quad - PvtCons(t)
\end{aligned}
$$

11.2.1.2 Firms

$$
NetSvngFirms(t) = NetSvngErngsFirmsKPvtPenSys(t) + NetSvngErngsFirmsKOthFinIns(t)
$$

which expands to

$$
\begin{aligned}
&NetSvngFirms(t) = \\
&\quad ErngsFirms\ KPvtPenSys(t) - DirTaxErngsFirmsKPvtPenSys(t) - DividDistErngsFirmsKPvtPenSys(t) \\
&\quad\quad + ErngsFirmsKOthFinIns(t) - DirTaxErngsFirmsKOthFinIns(t) - DividDistErngsFirmsKOthFinIns(t)
\end{aligned}
$$

11.2.1.3 Government

$$NetSvngGov(t) =$$

$$
\begin{aligned}
&IndTax(t) + DirTax(t) + ContPubPenSys(t) \\
&- GovCons(t) - BenPubPenSys(t)
\end{aligned}
$$

which expands to

$$
\begin{aligned}
&NetSvngGov(t) = \\
&\quad IndTax(t) \\
&\quad\quad + DirTaxWageY(age,t) + DirTaxEntrY(age,t) + DirTaxRntlY(age,t) + DirTaxBenPubPenSys(t) \\
&\quad\quad + DirTaxErngsFirmsKPvtPenSys(t) + DirTaxErngsFirmsKOthFinIns(t) \\
&\quad\quad + ContPubPenSysWageY(t) + ContPubPenSysEntrY(t) \\
&\quad\quad - GovCons(t) - BenPubPenSys(t)
\end{aligned}
$$

11.2.1.4 Total

Adding across sectors and making cancellations, net national savings are

$$
\begin{aligned}
&NetNatSvng(t) = \\
&\quad WageY(t) + EntrY(t) + \mathrm{Re}ntalY(t) \\
&\quad\quad + ErngsFirmsKPvtPenSys(t) + ErngsFirmsKOthFinIns(t) \\
&\quad\quad + IndTax(t) - PvtCons(t) - GovCons(t)
\end{aligned}
$$

11.2.2 "Top down"

Net national savings are given by the expression

$$NetNatSvng(t) = NatDispY(t) - PvtCons(t) - GovCons(t)$$

First, we express *NatDispY* in terms of *GDP*

$$NatDispY(t) = GDP(t) - DeprRate(t)\ KTot(t)$$

and then expand GDP using the expression from Section 9.1 above:

$$
\begin{aligned}
&NetNatSvng(t) = \\
&\quad WageY(t) + EntrY(t) + \mathrm{Re}ntalY(t) \\
&\quad\quad + ErngsFirmsKPvtPenSys(t) + ErngsFirmsKOthFinIns(t) \\
&\quad\quad + IndTax(t) + DeprRate(t)\ KTot \\
&\quad\quad - DeprRate(t)\ KTot(t) - PvtCons(t) - GovCons(t)
\end{aligned}
$$

Depreciation cancels, leaving

$$
\begin{aligned}
&NetNatSvng(t) = \\
&\qquad WageY(t) + EntrY(t) + RntlY(t) \\
&\qquad\quad + ErngsFirmsKPvtPenSys(t) + ErngsFirmsKOthFinIns(t) \\
&\qquad\quad + IndTax(t) - PvtCons(t) - GovCons(t)
\end{aligned}
$$

This is the same as the expression at the end of Section 11.2.1

Annex 2: Initialization assumptions

Table A3.2.1: Macroeconomic Assumptions

	GDP per capita ($US, base year)	Capital : GDP (base year)	Rate of total factor productivity growth (%, time invariant)	Capital coefficient in Cobb-Douglas production function (time invariant)
Japan	36078	2.6	1	0.33
US	29149	2.5	1	0.33
Other industrial countries	25779	2.5	1	0.33
LDC's	2170	1.6	1	0.33
	Government consumption (base year, % of GDP)	**Government debt (% of GDP, target level, time invariant)**	**Direct tax rate (%, time invariant)**	**Indirect tax rate (%, time invariant)**
Japan	12.7	30	10	5
US	17.0	30	15	5
Other industrial countries	16.7	30	15	5
LDC's	10.7	30	15	7
	Depreciation rate (%, time invariant)	**Share of overseas investments consisting of FDI (%, time invariant)**	**Share of earnings on FDI repatriated (%, time invariant)**	**Share of earning distributed as dividends (%, time invariant)**
Japan	5	50	75	15
US	4	50	75	15
Other industrial countries	4	50	75	15
LDC's	6	50	75	15

Table A3.2.2: Consumption Rates
(% consumed, time- and age-invariant)

	Japan	US	Other industrial countries	LDC's
Disposable income				
Wages	0.9	0.95	0.9	0.95
Rentalal income	1	1	1	1
Entrepreneurial income	0.6	0.6	0.6	0.6
Dividends	0.5	0.5	0.5	0.6
Interest on gov. debt	0.6	0.6	0.6	0.7
Pvt. DC pension benefits	0.6	0.6	0.6	0.7
Pvt. DB pension benefits	0.6	0.6	0.6	0.7
Public pension benefits	0.9	0.9	0.9	0.95
Other				
Annuity income	0.6	0.6	0.6	0.7
Inheritance	0.1	0.1	0.1	0.1

Table A3.2.3: Age-specific Labor Force Participation Rates (time-invariant)

1995 -- 1999		Japan	US	Other industrial countries	LDCs
	15--19	38.05	38.05	38.05	50.95
	20--24	78.50	78.50	78.50	72.35
	25--29	83.90	83.90	83.90	75.70
	30--34	84.15	84.15	84.15	76.20
	35--39	84.30	84.30	84.30	77.55
	40--44	83.05	83.05	83.05	76.00
	45--49	80.45	80.45	80.45	73.25
	50--54	75.55	75.55	75.55	67.60
	55--59	57.10	57.10	57.10	56.00
	60--64	32.55	32.55	32.55	44.75
	65--69	8.25	8.25	8.25	24.85
	70+	0.00	0.00	0.00	0.00

2000 -- 2009		Japan	US	Other industrial countries	LDCs
	15--19	36.00	36.00	36.00	40.70
	20--24	80.05	80.05	80.05	69.65
	25--29	86.20	86.20	86.20	74.35
	30--34	85.95	85.95	85.95	75.25
	35--39	86.45	86.45	86.45	75.25
	40--44	85.20	85.20	85.20	75.20
	45--49	82.45	82.45	82.45	74.85
	50--54	76.45	76.45	76.45	69.45
	55--59	55.80	55.80	55.80	50.80
	60--64	31.00	31.00	31.00	35.15
	65--69	5.70	5.70	5.70	15.00
	70+	0.00	0.00	0.00	0.00

2010--2050		Japan	US	Other industrial countries	LDCs
	15--19	35.95	35.95	35.95	39.25
	20--24	80.45	80.45	80.45	69.90
	25--29	86.25	86.25	86.25	75.10
	30--34	85.95	85.95	85.95	76.05
	35--39	86.60	86.60	86.60	75.70
	40--44	86.00	86.00	86.00	75.00
	45--49	83.25	83.25	83.25	73.65
	50--54	77.30	77.30	77.30	69.30
	55--59	56.10	56.10	56.10	50.05
	60--64	31.50	31.50	31.50	33.80
	65--69	5.35	5.35	5.35	13.50
	70+	0.00	0.00	0.00	0.00

Table A3.2.4: Pension, Health, and Long-term Care System Parameters (time- and age-invariant)

Public Pension System

	Share of workers participating in earnings-related scheme (%)	Share of income subject to contributions (%)	Accrual rate (%)	Benefit Indexation (to average wage rate, %)	Share of workers participating in flat pension scheme (%)	Flat pension : average wage (%)
Japan	65	80	1	20	20	10
US	75	80	1	50	0	0
Other industrial countries	75	80	1	50	0	0
LDCs	75	80	1	50	0	0

	Private DB Pension System				***Private DC Pension System***				
	Share of workers participatiing (%)	Share of income subject to contributions (%)	Accrual rate (%)	Benefit Indexation (to wage, %)	Share of workers participating (%)	Share of income subject to contributions (%)	Contribution rate (%)	Annuity termination age	Early withdrawals (% of assets)
Japan	15	100	1	10	30	100	5	80	3
US	20	100	1	10	30	100	5	80	3
Other industrial countries	20	100	1	10	50	100	5	80	3
LDCs	10	100	1	10	20	100	5	80	3

Health care system

	Patient co-payments (% of total expenditure)
Japan	30
US	30
Other industrial countries	30
LDCs	30

Long-term care system

	Patient co-payments (% of total expenditure)
Japan	30
US	30
Other industrial	30
LDCs	60

Table A3.2.5: Allocation of net savings to capital formation (time- and age-invariant)

	1995	2025	2050
Japan			
Residential	0.3	0.3	0.3
Non-residential	0.7	0.7	0.7
of which			
Capital operated by private unincorported enterprises	0.33	0.33	0.33
Capital operated by firms	0.67	0.67	0.67
US			
Residential	0.3	0.3	0.3
Non-residential	0.7	0.7	0.7
of which			
Capital operated by private unincorported enterprises	0.33	0.33	0.33
Capital operated by firms	0.67	0.67	0.67
Other industrial countries			
Residential	0.3	0.3	0.3
Non-residential	0.7	0.7	0.7
of which			
Capital operated by private unincorported enterprises	0.33	0.33	0.33
Capital operated by firms	0.67	0.67	0.67
LDCs			
Residential	0.3	0.3	0.3
Non-residential	0.7	0.7	0.7
of which			
Capital operated by private unincorported enterprises	0.33	0.33	0.33
Capital operated by firms	0.67	0.67	0.67

Table A3.2.6: International portfolio allocation shares
(allocation of net new capital formation, by region, age-invariant)

	Private pension system (DB and DC)			*Other financial institutions*		
	1995	**2025**	**2050**	**1995**	**2025**	**2050**
Japan						
Domestic	0.90	0.90	0.90	0.85	0.85	0.85
US	0.03	0.03	0.03	0.04	0.02	0.02
Other industrial countries	0.03	0.03	0.03	0.08	0.03	0.03
LDC's	0.03	0.03	0.03	0.03	0.10	0.10
US						
Japan	0.03	0.03	0.03	0.01	0.01	0.01
Domestic	0.90	0.90	0.90	0.89	0.89	0.90
Other industrial countries	0.03	0.03	0.03	0.04	0.04	0.01
LDC's	0.03	0.03	0.03	0.06	0.06	0.08
Other industrial countries						
Japan	0.01	0.01	0.01	0.01	0.01	0.01
US	0.03	0.03	0.03	0.02	0.02	0.02
Domestic	0.95	0.95	0.95	0.95	0.95	0.95
LDC's	0.01	0.01	0.01	0.02	0.02	0.02
LDC						
Japan	0.01	0.01	0.01	0.01	0.01	0.01
US	0.01	0.01	0.01	0.01	0.01	0.01
Other industrial countries	0.02	0.02	0.02	0.02	0.02	0.02
Domestic	0.96	0.96	0.96	0.96	0.96	0.96

Annex 3: Comparison of IIASA Model Baseline Projections for Japan, 1995-2050, with the Projections of Other Models

Introduction

The baseline scenario presented in this paper foresees that rapid aging of the Japanese population over the next half-century will be accompanied by a slowing of economic growth, a decline in saving rates, and a decline in the current account balance. The model also projects that contribution rates for public pensions and the health and long-term care systems will rise substantially and that these expenditures will account for a significantly increasing share of total product. As a way of assessing the "reasonableness' of the projections of the base case of the IIASA model, the projections of that model have been compared with the projections of other models. Regrettably, not all of the recent studies using models of the Japanese population and economy have published the values corresponding to their reference scenario.[8] However, enough information exists to make possible a number of comparisons in respect to nine variables, two demographic, five economic and two dealing with the social security system. The list of variables is shown below.

Number	Variable Name
Demographic Variables	
I	Population Growth Rates
II	The Old Age Dependency Ratio
Economic Variables	
III	Rate of Growth of Per Capita Output
IV	The Capital-Output Ratio
V	The Rate of Return to Capital
VI	Net National Savings as a Percent of GDP
VII	Change in Net Foreign Assets as a Percent of GDP
Social Security Variables	

[8] Hviding and Louvir (1998) present in their Table 4 the effects of policy reform on the national saving rate. In table 5 they show its effect on the real return on capital. In table 6, they show its effect on per capita GDP. However all these figures were given as per cent differences from the baseline values and the baseline values were not given so that it was impossible to compare their projections with those of the IIASA model. Turner et al. (1998) provide some very limited information about their reference scenario. Table 5 (page 49) provides projections of the growth rates of GDP per capita (adjusted for changes in the terms of trade). Table A2 (page 105) gives initial values for the capital output ratio, the real interest rate and the trade balance as a percent of GDP. However all projections for 2010, 2020, 2030,2040 and 2050 are presented in tables as differences from the reference case. Since the reference case projections are not given the projections of Turner et. al. cannot be systematically compared with those of the IIASA model. McMorrow and Roeger (1999) present their demographic projections in Table 1 (page 9). However their economic projections are generally expressed as changes relative to the baseline.

VIII	Public Pension Contribution as a Percent of Covered Wages
IX	Public Pension Contributions as a Percent of GDP

I. Population Growth Rates

Model scenarios will inevitably be sensitive to the underlying demographic assumptions. The IIASA model scenario is based on the assumption of a continuing decline in population growth rates over the next half-century, with negative population growth after 2010. The projections underlying the scenarios of Mason et al. (1994), [9] Ogawa (1995) Turner et al. (1998) and Takayama (1995)[10] are quite similar to those of the IIASA model. They also follow a downward course but in the IIASA baseline scenario, the growth rate turns negative sooner and the declines are greater.

Table 1
Rate of Population Growth, Japan, 1995-2050

YEAR	IIASA	TURNER	MASON	OGAWA	TAKA-YAMA
1995	0.18	0.2	0.44	0.28	0.31
2000	0.17	0.2	0.48	0.28	0.3
2010	-0.1	0	0.09	0.05	0.23
2020	-0.4	-0.4	-0.25	-0.04	
2030	-0.53	-0.4			-0.29
2040	-0.59	-0.4			
2050	-0.65	-0.4			

II. The Old Age Dependency Ratio

IIASA uses as its Old-Age Dependency Ratio the ratio of persons aged 60 and above to the number of persons aged 15 through 59. The old-age dependency rates for Japan calculated from the study by Chauveau and Loufir (YEAR)[11] are very close to the IIASA rates. The models of McMorrow[12] and Roeger (1999) and Turner et al. (1998) project old age depend-

[9] Mason et. al. gave figures for total population in five-year intervals. Growth rates were calculated from that data (Mason et al., Table 7.8) The Mason et al. projections were based on a 1986 projection by Ogawa et al. (1986) and thus did not fully capture the effect of the continuing decline in Japanese fertility that occurred during the 1990s. (Mason et. al, pp. 176)

[10] Takayama's projections were based on Ministry of Health projections published in 1992 (Table 6)

[11] Chauveau and Loufir disaggregated the Japanese population into three age groups (under 20), (20 through 59) and (60 and over). To make their distribution comparable to the IIASA distribution, 25 percent of their under 20 group was added to their (20 through 59) age group.

[12] McMorrow and Roeger disaggregated the Japanese population into three age groups (under 15), (15 through 64) and (65 and over). To make their distribution comparable to the IIASA distribution, 10 percent of their (15 through 64) group was subtracted from that group and added to their (65 and over) age group.

ency for the years 1995 and 2000 and 2050. Although the trends in their numbers match those of the IIASA figures, their old age dependency ratios are consistently lower.

Table 2
Population 60+/ Population 15-59 (%)

YEAR	IIASA	CHAU-VEAU	MCMOR-ROW	TURNER
1995	30.43	32.24	25.54	21.28
2000	35.21	37.15	29.59	21.28
2010	49.98	52.18		
2020	57.1	59.08		
2030	61.9	64.14		
2040	73.15	72.3		
2050	74.87	70.06	65.35	55.56

Five studies made projections of the old age dependency ratio using other definitions of that ratio but did not include the data needed to make them comparable to the IIASA figures. Takayama (1995) defined the old age ratio as the population aged 65 and older divided by the population aged 20-64. Horioka (1989) did the same as did Heller (1989). Meredith (1995) defined the old age ratio as the population aged 65 and older divided by the population aged 15-64.

Though the dependency ratios in those studies are not directly comparable to the IIASA figures, they are presented in Table 3 below. Since the dependent group in the five studies is smaller than in the IIASA study, the old age dependency ratios are smaller. However, the Takayama, Ogawa and Horioka projections are very closely aligned with each other and follow the same trend as the IIASA projections.

Table 3
Alternative Measures of the Old-Age Dependency Ratio, Japan, 1995-2050

YEAR	IIASA	TAKA-YAMA	HORIOKA	OGAWA	HELLER	MEREDITH
1995	30.43	23.13	22.9	20.9		20
2000	35.21	27.42	27.1	25	24.15	
2010	49.98	36.98	35.8	34.4	31.48	
2020	57.1	47.84	44	44.9	40	40
2030	61.9	47.71	42.6			
2040	73.15	53.95	46.4			
2050	74.87	55.73	44.3			

III. Rate of Growth of Per Capita Output

Only Mason et al. (1994) and Turner et al. (1998) present their results for the growth rate of per capita output. Both projections differ markedly from the IIASA projection. Mason et al. foresee a growth rate that is initially higher than the IIASA figure in 1995 and then generally increases over the next two decades. The initially higher rate may reflect the fact that the Mason et al. study is based on a higher projected population growth rate, although it is not clear why this should lead the authors to project an acceleration of GDP growth in the later years of their projection. While Turner et al. foresee a lower growth rate of per capita GDP than does IIASA model, they also project that the growth rate of per capita output will decline over time.

Table 4
The Growth Rate of Per Capita GDP, Japan, 1995-2050

YEAR	IIASA	MASON	TURNER
1995	2.05	3.17	
2000	1.66	3.13	1.5
2010	2.29	3.84	1.3
2020	1.38	4.1	1.3
2030	1.25		1.4
2040	1.15		0.8
2050	1.47		0.9

IV. The Capital-Output Ratio

The IIASA model projects that Japan's capital-output ratio will rise modestly over the next 50 years, from roughly 2.5 to 3.5. This implies a gradual decline in the average productivity of the capital stock, presumably in response to a rising capital-labor ratio (see Table 5) The model of Mason et al. (1994) presents data on "Private Capital"[13] and hence it is not surprising that their figures for the initial capital-output ratio would be below those assumed by IIASA.[14] It is striking that, in spite of continued accumulation, their capital-output ratio declines significantly over the projection period, implying a continuing improvement in the productivity of capital. Turner et al. have only a point estimate of the capital output ratio in the year 2000. Not shown are Ando et al.'s (1995) estimates of the ratio of net worth to income. They are 6.74 for 2020 and 8.73 for 2050.[15]

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[13] Thus investments in public capital such as roads, bridges, port facilities, and school buildings are omitted from this measure.

[14] Hayashi (1991) also estimated an extremely low capital-output ratio for Japan .For the year 1989; his capital-output ratio was 1.54.

[15] The fact that the net worth to income ratio is two or more times the capital output ratio may be due to the exclusion of land wealth and foreign assets from the numerator of the IIASA projection of the capital-output ratio.

Table 5
The Capital-Output Ratio, Japan, 1995-2050

YEAR	IIASA	MASON	TURNER
1995	2.59	2.06	
2000	2.69	2.13	2.2
2010	2.98	1.73	
2020	3.19	1.15	
2030	3.3		
2040	3.45		
2050	3.46		

V. The Rate of Return to Capital

There is a great divergence of opinion on the long-run future course of the rate of return to capital in Japan. The IIASA model projects steady decline. The model of Mason et al. (1994) projects a decline in the rate of return on capital until 2010, followed by an increase over the following decade. Yashiro and Oishi (1997)[16] project a sharp rise in the rate of return between 1995 and 2000 followed by a decline through mid-century. Their projection roughly tracks that of IIASA model. Though the rate of return projection for 2000 of Turner et al. (5 percent) is close to IIASA's figure[17], they project a rise in the rate of return to capital to 5.4 by 2050, a figure almost twice that of the IIASA model projection. Thus, Turner et al. and IIASA differ both in estimated level and projected trend.

Table 6
Rate of Return to Capital, Japan, 1995 to 2050 (Percent)

YEAR	IIASA	MASON	YASHIRO	TURNER
1995	5.82	6.3	3.1	
2000	5.42	4.6	4.6	5
2010	4.39	3.6		
2020	3.73	4.4		
2030	3.33			
2040	2.81			
2050	2.58		3	5.42

[16] Yashiro and Oichi project long-term interest rates (Table 1, page 38).

[17] The figures presented by Turner et al. refer to the real interest rate rather than the return to capital. In a state of macroeconomic equilibrium, the two figures should be equal.

VI. Net National Savings as a Percent of GDP

It is widely believed that the aging of the Japanese population will bring with it a decline in the proportion of income that is saved. Though this decline is reflected in the IIASA projections, it is considerably less rapid than is forecast by most models of the Japanese economy. The study by Kato (1998) uses the National Savings concept as does the IIASA model and the projected savings rates are similar. Kato's rate begins in 1995 at a slightly lower level than the rate derived from the IIASA model. From that point on, Kato's savings rates fall more rapidly than those generated by the IIASA model.

The savings rates projected by Horioka (1989)[18] show a higher initial value and a more rapid rate of decline than Kato's rates, thus, a much more rapid rate of decline than those projected by the IIASA model. The steep decline reflects Horioka's view that life cycle considerations are the major determinant of savings rates in Japan. Like Horioka, Higgins also projects a substantially negative savings rate by 2010 (-6.2%). Furthermore he projects that the savings rate will fall to –9.3% by 2025 (not shown in Table 7).

Table 7

Net National Saving[19] as a Percent of GDP, Japan, 1995-2050 (Percent)

YEAR	IIASA	KATO	HO-RIOKA	HIG-GINS
1995	7.09	6.21	10.9	
2000	7.56	5.17	6.7	
2010	6.25	3.16	-3.8	-6.2
2020	4.36	0.53	-12.2	
2030	3.68	-0.7	-10.2	
2040	2.76	-1.72	-15.4	
2050	1.45	-2.65	-12.9	

A second group of national savings rate projections differs from both the IIASA and the other projections shown above in that they project considerably greater rates of saving, presumably due to definitional differences. The values estimated by Roseveare et al.(1996) are nearly double the early-year estimates in the IIASA scenario. Mason estimates saving rates similar to Roseveare's for 1995 and 2000 and does not project a decline in savings rates in later years.

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[18] Horioka's savings rates refer to personal saving calculated on the basis of replacement cost rather than historical cost. Horioka (1991) projected savings rates for 1995, 2000 and 2010 that were almost identical with his (1989) projections.

[19] The IIASA model relates Net National Saving with GDP. A case could be made for relating Net National Savings, a figure that does not include depreciation allowances, to NNP or NDP.

The savings rates projected by Ando et al. (1995) for the years 2020 and 2050 indicate an absolute decline of about 5 percentage points over the intervening 30 years. This is consistent with the IIASA model. But the savings rates projected by Ando et al. are about 10 percentage points higher than the IIASA rates. The largest savings rates were found in Yashiro and Oishi (1997).[20] Though much larger than the other savings rate projections, their projections reflected the same downward trend as Ando.

Table 8
National Savings Rates, Japan, 1995-2050

YEAR	IIASA	ROSEVEARE	MASON	OGAWA	YASHIRO	ANDO
1995	7.09	15	16.8	21.96	30.8	
2000	7.56	15.2	16.7	18.63	33.2	
2010	6.25		18	12.08		
2020	4.36		18.8	8.12		15.1
2030	3.68					
2040	2.76					
2050	1.45				2.9	10.3

VII. Change in Net Foreign Assets as a Percent of GDP

The IIASA model projects that change in the ratio of net foreign assets to GDP[21] will be extremely small though positive in 1995 (0.22%) and 2000 (0.32%), then decline slightly over the projection period, ending up at –0.16% of GDP. Turner et al. projected that in the years 2000 to 2015, the current account surplus of Japan will remain stable at 2 percent of its GDP. However, Turner et al. project that by 2050 Japan's stock of external assets will be exhausted, a projection consistent with the IIASA model's projection of a decline in external assets from 2030 on. The projections of Horioka (1996) are not very different from those of Turner et al. He projects that the ratio of the current account to GDP will be zero in 2020 and fall to –1.0 percent in 2025 (not shown in Table 9). Meredith (1995) projected that the current account to GDP ratio will fall to minus 15 percent in 2020 and Yashiro (1997) projected that the current account balance would be –5.7% by 2025 and would be – 17% by 2050. These results are considerably more dramatic than those of the IIASA scenario.

[20] Their concept was "Aggregate Savings" which corresponds to Gross Investment does not make allowance for depreciation (page 11.)

[21] The change in net foreign assets should be approximately equal to the balance on Current Account or the negative of the balance on the Capital Account..

Table 9
Change in Net Foreign Assets as a Percent of GDP, Japan, 1995-2050

YEAR	IIASA	HIG-GINS	MERE-DITH	YASHIRO	HO-RIOKA	TURNER
1995	0.22			1.9		2
2000	0.32		2	0.04		2
2010	0.21	-1.25				2
2020	0.03		-15		0	2
2030	0.04					
2040	-0.09					
2050	-0.16			-17		

VIII. Public Pension Contribution as a Percent of Covered Wages

The IIASA model projects that the proportion of covered wages contributed to public pensions will increase by about 50 per cent between 2000 and 2050. This corresponds almost exactly to the projection of the Ogawa (1995) model.

Table 10
Public Pension Contributions as a Percent of Covered Wages, Japan, 1995-2020

YEAR	IIASA	OGAWA
1995	18.22	18.9
2000	20.3	18.9
2010	26.69	22.01
2020	30.91	30.91

IX. Public Pension Contributions as a Percent of GDP

The IIASA model projects that as a result of population aging the proportion of Japanese GDP contributed to the public pension funds will grow from 6.83 percent in 1995 to 14.74 percent in 2050. The Roseveare et al. projections closely track those made by the IIASA model.[22] The same is true for the Auerbach et al. projections for 2010 and 2030. However Auerbach et al. project a reversal of the trend and a sharp decline in the contributions as a percent of GDP in 2050. Takayama projects rapidly rising share of output going to public pensions. His projection for 2025 (not shown) for the proportion of national income going to public pensions[23] is 15 percent, which is greater than the IIASA projection for 2050.

[22] Roseveare et al. and Takayama projected public pension *expenditures* as a percent of GDP, rather than contributions

[23] Takayama projected public pension expenditures as a percent of NI, rather than GDP.

Table 11
Public Pension Plan Contributions as a Percentage of Output, Japan, 1995-2050

YEAR	IIASA	ROSEVEARE	AUER-BACH	TAKAYAMA
1995	6.83	6.6		8.6
2000	7.55	7.5		10.5
2010	9.79	9.6	9.7	14
2020	11.24	12.4		
2030	12.22	13.4	11.4	
2040	14.27	14.9		
2050	14.74	16.5	7.5	

Conclusion

This note compared the IIASA model's projections of Japan's population, economy and pension system with those of 18 other individual researchers and research teams. What is most striking is that there is no unanimity among the projections. In general, the IIASA economic projection foresees greater slowing of per capita economic growth. This is probably a reflection of its more extreme demographic projection. On the other hand, the IIASA model projects that the aging of the population will have a smaller effect on the savings rate and the current account balance than most other models. The fixed coefficients approach to age-specific consumption rates and the ad hoc nature of the foreign sector may account for these differences, but a closer analysis would be required to state this with confidence.

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