

## Inverted Index Algorithm and Compression

## Inverted Index

- Regardless of the retrieval strategy we need a data structure to efficiently store:
  - For each term in the document collection
    - The list of documents that contain the term
  - For each occurrence of a term in a document
    - The frequency the term appears in the document (tf)
    - The position in the document for which the term appears (only needed if proximity queries will be supported).
      - » Position may be expressed as section, paragraph, sentence, location within sentence ,

## Inverted Index Construction: Periodic write to disk

```

For each document d in the collection
  Begin
    numSubSet = 1
    While memory exists:
      For each term t in document d
        Find term t in the term dictionary
        If term t exists, add a node to its posting list
        Otherwise, add term t to the term dictionary
      Write SubSet of Inverted index to disk
      numSubSet = numSubSet + 1
    Free memory
  End
For I = 1 to numSubSet
  Merge SubSet I with Inverted Index
  
```

## Compression of Inverted Index

- I/O to read a posting list is reduced if the inverted index takes less storage
- Stop words eliminate about half the size of an inverted index. “the” occurs in 7 percent of English text.
- Other compression
  - Posting List
  - Term Dictionary
- Half of terms occur only once (*hapax legomena*) so they only have one entry in their posting list
- Problem is some terms have very long posting lists -- in Excite's search engine 1997 occurs 7 million times.

## Things to Compress

- Term name in the term list
- Term Frequency in each posting list entry
- Document Identifier in each posting list entry

## Data Compression

- Applied to posting lists
  - term:  $(d_1, tf_1), (d_2, tf_2), \dots (d_n, tf_n)$
- Documents are ordered, so each  $d_i$  is replaced by the interval difference, namely,  $d_i - d_{i-1}$
- Numbers are encoded using fewer bits for smaller, common numbers
- Index is reduced to 10-15% of database size

## Compressing *tf*: Elias Encoding

$X$	$Z$	
1	0	<u>To represent a value <math>X</math>:</u>
2	10 0	• $\lfloor \log_2 X \rfloor$ ones representing the highest power of 2 not exceeding $X$
3	10 1	• a 0 marker
4	110 00	• $\lfloor \log_2 X \rfloor$ bits representing to represent the remainder $X - 2^{\lfloor \log_2 X \rfloor}$ in binary.
5	110 01	
6	110 10	
7	110 11	
8	1110 000	
63	111110 11111	• The smaller the integer, the fewer the bits used to represent the value. Most <i>tf</i> 's are relatively small.

## Elias Code

1 = 0	• 3 parts, not byte aligned
2 = 1 0 0	1. n ones, one for each bit in part 3
3 = 1 0 1	2. a 0 to mark the end of part 1.
4 = 11 0 00	3. the next n numbers in binary
5 = 11 0 01	
6 = 11 0 10	
7 = 11 0 11	
8 = 111 0 000	
9 = 111 0 001	

For 63, its  $2^5 = 32 + 31$  in binary (11111)  
**11111 0 11111**

Instead of two bytes for the *tf* we now are using only a few bits.

## Variable Length Compress Used for Document Identifier

- Document identifiers (the difference) may not all be small
- A generalization of Elias is to develop a vector  $V$  with the powers of some integer in its component.
- Examples
  - $V <1,2,4,8,16,32>$
  - $V <2,4,8,16,32,64>$ , etc.

## Variable Length Encoding (cont.)

- Choose Vector  $V$
- For an integer  $x$  to be compressed, find  $k$  such that sum of the vector components is greater than or equal to  $x$ .
- Encode  $k-1$  in unary.
- Now subtract the sum of the first  $k-1$  components of  $V$  from  $x$ . The difference is  $d$ .
- Encode a 0 stop bit
- Encode  $d$  in binary.

## Variable Length Encoding (cont.)

- Formula to find  $k$  is...

$$\sum_{i=1}^{k-1} V_i < x \leq \sum_{i=1}^k V_i$$

- remainder =  $d = x - \sum_{i=1}^{k-1} V_i - 1$
- Now the result will be made of 3 parts
  - Encode  $k-1$  in unary.
  - Encode a 0 stop bit
  - Encode the remainder  $d$  in  $\lceil \log_2 V_k \rceil$  bits.

## Variable Length Encoding (Example 1)

- For  $x = 7$
- Using Vector  $<1,2,4,8,16>$ , it requires the sum of  $<1,2,4>$  to exceed  $x$ . Hence the index  $k$  is 3 and  $k-1$  is 2. Encode 2 in unary.
- The remainder is  $7 - (1+2) - 1 = 3$ , encode this in binary after the stop bit.
- To encode  $x$  use **11011**

## Example 2

- To encode 9 with vector of  $\langle 1, 2, 4, 8, 16, 32 \rangle$ 
  - If  $k=3$ :  $1+2+4=7$
  - And if  $k=4$ :  $1+2+4+8=15$ 
    - We select  $k=4$  ... So encode  $(k-1)$  in unary (which is  $111$ )
  - Encode the stop bit 0
  - Encode  $r = 9 - 7 = 2$ , encode this in binary as  $010$  (we encode in 3 bits as  $\lceil \log_2 8 \rceil = 3$ )
  - So we have  $1110001$  (seven bits)
- To encode 9 with new vector that starts with 2 of  $\langle 2, 4, 8, 16, 32 \rangle$ 
  - If  $k=3$ 
    - we get the equation as:  $(2+4)=6 < 9 \leq 14 = (2+4+8)$
    - we select  $k=3$  ... so encode  $(k-1)$  in unary (which is  $11$ )
    - Encode the stop bit 0
    - Encode  $r = 9 - 6 = 3$ , encode this in binary as  $010$  (we encode in 3 bits as  $\lceil \log_2 8 \rceil = 3$ )
    - So we have  $110010$  (six bits)

## Changing V

- If  $V$  contains larger values, fewer bits will be needed to represent larger values.
- A constant  $b$  can be varied such that  $V$  is  $b, 2b, 4b, 8b, 16b, 32b, 64b$ .
- $b$  can be varied for *each posting list*
- Use the median of the document identifier differences for each posting list.
- Requires knowledge of how large a posting list, but you know this in the final stages of index development.

## Example 3

- Suppose a posting list had:
 

term -->	$d_4$	$d_{10}$	$d_{20}$	$d_{30}$	$d_{35}$
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- Differences are 6, 10, 10, 5 so median is  $10 = b$
- $V$  is now  $\langle 10, 20, 30, 40 \rangle$
- To encode the differences we have:
 

$4_{10}$	$6_{10}$	$10_{10}$	$10_{10}$	$5_{10}$
00011	00101	01001	01001	00100
- Note: We never needed *any* leading bits. With a vector of  $\langle 1, 2, 4, 8, 16 \rangle$  we would have had:
 

$4_{10}$	$6_{10}$	$10_{10}$	$10_{10}$	$5_{10}$
11000	11010	1110010	1110010	11001

Variable length we used 25 bits. Regular Elias we used 29 bits.

## Byte-Aligned codes

<b>0-63</b>	00xxxxxx
<b>64-16K</b>	01xxxxxx xxxxxxxx
<b>16K-4M</b>	10xxxxxx xxxxxxxx xxxxxxxx
<b>4M-1G</b>	11xxxxxx xxxxxxxx xxxxxxxx xxxxxxxx
<b>0</b>	00000000
<b>1</b>	00000001
<b>...</b>	...
<b>63</b>	00111111
<b>64</b>	01000000 00000000
<b>65</b>	01000000 00000001

The hope here is that the document distance between posting list nodes will be small.

## Compression Summary

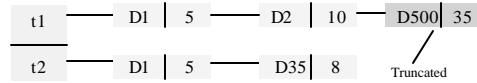
- Pro
  - Can reduce I/O for query of inverted index.
  - Reduce storage requirements of inverted index.
- Con
  - Takes longer to build the inverted index.
  - Software becomes *much* more complicated.
  - Uncompress required at query time – note that this time is usually offset by dramatic reduction in I/O.

## Top Docs

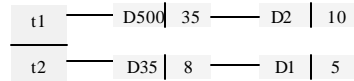
- Other structures may be built at index creation to optimize performance.
- Instead of retrieving the whole posting list, we might want to only retrieve the top  $x$  documents where the documents are ranked by weight.
- A separate structure with sorted, truncated posting lists may be produced.

## Inverted Index and TopDoc

### Inverted Index



### TopDoc (D = 2)



## Top Doc Summary

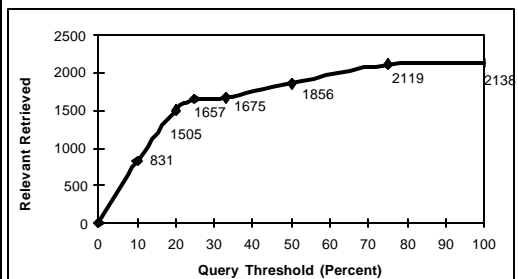
- Pro
  - Avoids need to retrieve the entire posting list
  - Dramatic savings on efficiency for large posting lists
- Con
  - Not feasible for Boolean queries
  - Can miss some relevant documents due to truncation

## Query Threshold

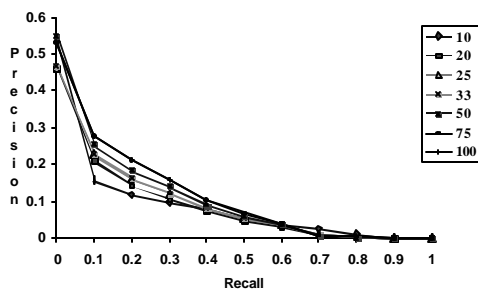
Consider a query with terms  $t_1, t_2, t_3, \dots, t_n$ . Sort the terms by their frequency across the collection (least frequent terms appear first). Define a threshold as the percentage of terms taken in the original query in a newly created reduced query.

term1	threshold = 20%
term2	
term3	
term4	
term5	threshold = 50%
term6	
term7	
term8	
term9	threshold = 80%
term10	

## Relevant Retrieved for Varying Query Thresholds



## Precision/Recall



## Threshold Summary

- Pro
  - Avoids large posting lists
  - Dramatic savings on efficiency when large posting list is not retrieved
  - Effectiveness does not degrade (as long as we do not threshold too much) because we are omitting only those terms with long posting lists
- Con
  - Still can have some very long posting lists