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| **Technical report** |
| **Optimum Portfolio Allocation Based on ETFs**  **Using Modern Portfolio Theory** |
| **by** |
|  |
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**for**

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**Key words**

Mean-variance analysis; minimum

variance portfolio; portfolio allocation,

diversiﬁcation and optimization; eﬃcient frontier;

measures of risk; ETF’s; rate of return

1. **Introduction**

In a survey of investment professionals, conducted in March 2008, 67% called ETFs the most innovative investment vehicle of the last two decades and 60% reported that ETFs have fundamentally changed the way they construct investment portfolios (*Knowledge@Wharton, 2008*).

This paper addresses optimum allocation based on certain ETFs covering world financial markets and sectors. The selection of assets or equities is not just a problem of finding attractive investments. Designing the correct portfolio of assets cannot be done by human intuition alone and requires modern, powerful and reliable mathematical programs using optimizers. Any investor would like to have the highest return possible from an investment. However, this has to be counterbalanced by the amount of risk the investor is able or desires to take. The expected return and the risk measured by the variance (or the standard deviation, which is the square-root of the variance) are the two main characteristics of a portfolio. Unfortunately, equities with high expected returns usually correlate with high risk.

The behavior of a portfolio can be quite diﬀerent from the behavior of individual components of the portfolio. The risk of a properly constructed portfolio from equities in leading markets could be half the sum of the risks of individual assets in the portfolio. This is due to complex correlation patterns between individual assets or equities. A good portfolio optimizer can exploit the correlations, the expected returns, the risk (variance) and user constraints to obtain an optimized risk-adjusted return. SAS optimization routines can deliver optimized and diversiﬁed portfolios to match investor expectations of return and risk.

The mathematical problem of portfolio optimization was initiated by Professor Harry Markowitz in the fifties and he was rewarded with a Nobel Prize in Economics in 1990 which he shared with Professors William Sharpe and Merton Miller. For details, see *Pioneers’ of Financial Economics*.

Portfolio optimization is often called mean-variance (MV) optimization. The term mean refers to the mean or the expected return of the investment and the variance is the measure of the risk associated with the portfolio. The mathematical problem can be formulated in many ways but the principal problem that this paper is about is to minimize risk for a speciﬁed expected return.

The above problem could have linear or nonlinear constraints, equality and inequality constraints.

When market conditions or the investors risk preferences change, it is advisable to rebalance the portfolio. The transactions costs associated with purchasing a new portfolio or rebalancing a portfolio could represent a signiﬁcant cost to the investor.

1. **Definitions**

Definitions of some key words in Finance are given below. Source: Wikipedia

A **portfolio** is a collection of investments held by an institution or an individual. Holding a portfolio is a part of an investment and risk-limiting strategy called diversification. By owning several assets, certain types of risk (in particular specific risk) can be reduced. The assets in the portfolio could include bank accounts, stocks, bonds, options, warrants, gold certificates, real estate, futures contracts, production facilities, or any other item that is expected to retain its value.

**Modern portfolio theory** (**MPT**) refers to investment which attempts to maximize portfolio expected return for a given amount of portfolio risk, or equivalently minimize risk for a given level of expected return, by carefully choosing the proportions of various assets. It is indeed a mathematical formulation of the concept of diversification in investing, with the aim of selecting a collection of investment assets that has collectively lower risk than any individual asset. That is possible and can be seen intuitively because different types of assets often change in value in opposite ways. For example, when prices in the stock market fall, prices in the bond market often increase, and vice versa. A collection of both types of assets can therefore have lower overall risk than either individually. But diversification lowers risk even if assets' returns are not negatively correlated—indeed, even if they are positively correlated.

An **exchange-traded fund** (**ETF**) is an investment fund traded on stock exchanges, much like stocks. An ETF holds assets such as stocks, commodities, or bonds and trades at approximately the same price as the net asset value of its underlying assets over the course of the trading day. Most ETFs track an index, such as the S&P 500 or MSCI EAFE. ETFs may be attractive as investments because of their low costs, tax efficiency, and stock-like features. ETFs are the most popular type of exchange-traded product.

**Standard deviation** is a representation of the **risk** associated with a given security (stocks, bonds, property, etc.), or the risk of a portfolio of securities (actively managed mutual funds, index mutual funds, or ETFs). **Risk** is an important factor in determining how to efficiently manage a portfolio of investments because it determines the variation in returns on the asset and/or portfolio and gives investors a mathematical basis for investment decisions (known as mean-variance optimization).

**The Relative Risk (RR)** is calculated by: where Pi and Pmin denote portfolios.

1. **Objective**

The objective of this study is to build and allocate a diversified portfolio with the goal of minimizing its risk and maximizing its expected return. We use the Markowitz model which is an optimal model for balancing the return and risk portfolio. The model minimizes the variance of the portfolio’s total return, subject to the constraints that:

1. the expected growth of the portfolio reaches some target level .
2. you do not invest more capital than you have.
3. **Data**

This study analyzed ETF data available from a public source “Yahoo Finance”. The timeline covers from Feb 2005 to Jan 2011. The periods are Daily (1492 days), Weekly (309 weeks) and Monthly (76 months).

**Structure and Types of ETFs:**

The available ETFs could be broken up into the following categories and some subcategories:

Domestic Equity: Large Caps, Mid Caps, Small Caps.

Sectors: Energy, Health Care, Financials, Telecom., Materials, Industrials, Information Technologies.

International: Emerging Markets, Europe, Asia, Americas, Global.

Commodities: Gold, Metals, US Oil Fund, etc..

Bonds: Short Term Corporate, Intermediate Term Corporate, Long Term Corporate, Investment Grade, High Yield, US Government, International.

Currency

Real Estate

**Variables:**

The source of the definition of these variables is from *Wikipedia*.

Open: The price at the start of trading on a security exchange.

High/Low: The highest and lowest price at which a stock has traded during its date.

Close: The price at the end of a trading session.

Volume: The number of shares or contracts traded in a security or an entire market during a given period of time.

Adj Close: A stock's closing price on any given day of trading that has been amended to include any distributions and corporate actions that occurred at any time prior to the next day's open. It is often used when examining historical returns or performing a detailed analysis on historical returns.

Rate of Return (ROR): The rate of return, also known as return on investment (ROI), rate of profit, is the ratio of money gained or lost (whether realized or unrealized) on an investment relative to the amount of money invested.

**Selection of ETFs:**

Table 1 below shows the few ETFs that were used for analysis. We selected 14 ETFs among 30 downloaded because we eliminated several that could not support a 6-year price or performance history. Their prices were collected (Daily, Weekly, Monthly) from February 2005 to January 2011.

Table 1

*Selected ETFs for Analysis*

|  |  |  |
| --- | --- | --- |
| Symbol | Name | Description |
| IYY | iShares Dow Jones U.S. Index | U.S. equity broad market |
| SPY | SPDRs | Weighted index of 500 common stock prices in US companies. |
| EFA | iShares MSCI EAFE Index | Securities in the European, Australian and Far Eastern markets |
| IAU | iShares COMEX Gold Trust | Price of gold bullion |
| EEM | iShares MSCI Emerging Markets Index | Equity market performance in the global emerging markets |
| AGG | iShares Barclays Aggregate Bond | United States investment grade bond market |
| IJH | iShares S&P MidCap 400 Index | U.S. equity mid-cap market |
| IJR | iShares S&P SmallCap 600 Index | U.S. equity small-cap market |
| XLE | Energy Select Sector SPDR | U.S. equity Energy Sector |
| EWZ | iShares MSCI Brazil Index | Exchange traded-fund of Brazilian stocks. |
| EWH | iShares MSCI Hong Kong Index | Exchange traded-fund of Hong Kong stocks. |
| EZU | iShares MSCI EMU Index | traded securities in the European Monetary Union (EMU) markets |
| EWJ | iShares MSCI Japan Index Fund | Exchange traded-fund of Japanese stocks |
| IYR | iShares Dow Jones US Real Estate | U.S. Real Estate |

1. **Rate of Return**

**Single-Period Rate of Return:**

rarim= \*100

The arithmetic return is considered to calculate the investment return. It is given by:

*where Vi andVf are the* initial and final values of an investment. The return data were in terms of percentage return. Indeed three periods (daily, weekly, and monthly) returns were analyzed.

**Data Summary and Graphical Description:**

**Case study example of** “iShares Barclays Aggregate Bond (AGG)”

After plotting numerous graphical summaries of the rate of return and volume, we illustrate a case example using AGG. Then, we draw a general summary of the rate of return and volume.



Figure 1: *Descriptive Statistics of the Rate of Return*

The rate of return of Bond is normally distributed as we can expect. The maximum return reached up to 3.87% while the minimum or most negative return was -6.84%. The mean and the median are pretty close in term of percentage.

We are interested in finding about the distribution of volume and how it affects the rate of return. Thus, scatter plots of Volume, Volume vs. Rate of Return will follow below.



Figure 2: *Descriptive Statistics of Volume*

The volume is highly skewed and distributed. Its mean is greater than it median.

turn …………………………………………………………………1 turn …………………………………………………………………1 

Figure 3: *Scatter Plot of the Rate of Return vs. Volume*

The rate of return is mainly concentrated over the range of -1% and 1% with few returns reaching up to -2% and 2%. The mean return is close to zero, though it is positive. Two of the returns, one each on negative and positive direction, are extreme and thus seem to be outliers.

**General summary of the rate of return and volume**

* Very high volume is likely associated with high positive or high negative rate of return.
* Very low volume shows likely association with very low positive or negative rate of return.
* Volume hardly explains the variability in the overall rate of return.

1. **Data Analysis**

**Correlation Structure**

### Pearson’s correlation definition

Linear relationships between variables x and y can be quantified using the Pearson Product-Moment Correlation Coefficient: *ρxy* where Sx and Sy are the standard deviations of x and y respectively. The value of this statistic is always between -1 and 1; and if x and y are unrelated it will equal zero.

**Why correlation coefficient is important?**

An investor can reduce portfolio risk simply by holding investments which are not or less correlated. In other words, investors can reduce their exposure to individual asset risk by holding a diversified portfolio of assets. Diversification will allow for the same portfolio return with reduced risk. If all the assets of a portfolio have a correlation of 1, i.e., perfect correlation, the portfolio volatility (standard deviation) will be equal to the weighted sum of the individual asset volatilities. Hence the portfolio variance will be equal to the square of the total weighted sum of the individual asset volatilities. If all the assets have a correlation of 0, i.e., perfectly uncorrelated, the portfolio variance is the sum of the individual asset weights squared times the individual asset variance (and volatility is the square root of this sum). If correlation is less than zero, i.e., the assets are inversely correlated, the portfolio variance and hence volatility will be less than if the correlation is 0.

**Correlation Tables**

Below are the correlation tables from Daily, Weekly and Monthly rate of return with a brief summary.

Table 2

*Daily Return Correlation Table*

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | SPY | IJH | IJR | IYY | XLE | EWZ | EWJ | EWH | EEM | EZU | EFA | AGG | IAU | IYR |
| SPY | 1 | 0.95 | 0.91 | 0.98 | 0.81 | 0.82 | 0.80 | 0.83 | 0.89 | 0.89 | 0.90 | 0.05 | 0.07 | 0.81 |
| IJH | 0.95 | 1.00 | 0.96 | 0.97 | 0.79 | 0.80 | 0.76 | 0.79 | 0.86 | 0.85 | 0.87 | -0.09 | 0.10 | 0.85 |
| IJR | 0.91 | 0.96 | 1.00 | 0.93 | 0.72 | 0.74 | 0.71 | 0.75 | 0.81 | 0.80 | 0.82 | -0.15 | 0.06 | 0.84 |
| IYY | 0.98 | 0.97 | 0.93 | 1.00 | 0.80 | 0.81 | 0.78 | 0.82 | 0.89 | 0.89 | 0.91 | -0.09 | 0.08 | 0.84 |
| XLE | 0.81 | 0.79 | 0.72 | 0.80 | 1.00 | 0.80 | 0.68 | 0.71 | 0.80 | 0.77 | 0.80 | -0.03 | 0.26 | 0.58 |
| EWZ | 0.82 | 0.80 | 0.74 | 0.81 | 0.80 | 1.00 | 0.72 | 0.77 | 0.90 | 0.80 | 0.83 | -0.01 | 0.23 | 0.61 |
| EWJ | 0.80 | 0.76 | 0.71 | 0.78 | 0.68 | 0.72 | 1.00 | 0.76 | 0.80 | 0.79 | 0.87 | 0.04 | 0.15 | 0.61 |
| EWH | 0.83 | 0.79 | 0.75 | 0.82 | 0.71 | 0.77 | 0.76 | 1.00 | 0.89 | 0.78 | 0.83 | -0.10 | 0.10 | 0.68 |
| EEM | 0.89 | 0.86 | 0.81 | 0.89 | 0.80 | 0.90 | 0.80 | 0.89 | 1.00 | 0.87 | 0.90 | -0.08 | 0.19 | 0.71 |
| EZU | 0.89 | 0.85 | 0.80 | 0.89 | 0.77 | 0.80 | 0.79 | 0.78 | 0.87 | 1.00 | 0.97 | -0.02 | 0.20 | 0.72 |
| EFA | 0.92 | 0.87 | 0.82 | 0.91 | 0.80 | 0.83 | 0.87 | 0.83 | 0.90 | 0.97 | 1.00 | 0.00 | 0.20 | 0.72 |
| AGG | -0.05 | -0.09 | -0.15 | -0.09 | -0.03 | -0.01 | 0.04 | -0.10 | -0.08 | -0.02 | 0.00 | 1.00 | 0.09 | -0.14 |
| IAU | 0.07 | 0.10 | 0.06 | 0.08 | 0.26 | 0.23 | 0.15 | 0.10 | 0.19 | 0.20 | 0.20 | 0.09 | 1.00 | 0.02 |
| IYR | 0.81 | 0.85 | 0.84 | 0.84 | 0.58 | 0.61 | 0.61 | 0.68 | 0.71 | 0.72 | 0.72 | -0.14 | 0.02 | 1.00 |

Table 3

*Weekly Return Correlation Table*

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | SPY | IJH | IJR | IYY | XLE | EWZ | EWJ | EWH | EEM | EZU | EFA | AGG | IAU | IYR |
| SPY | 1.00 | 0.95 | 0.92 | 0.99 | 0.79 | 0.80 | 0.75 | 0.77 | 0.85 | 0.88 | 0.90 | 0.25 | 0.01 | 0.79 |
| IJH | 0.95 | 1.00 | 0.97 | 0.97 | 0.79 | 0.81 | 0.70 | 0.73 | 0.86 | 0.85 | 0.87 | 0.14 | 0.07 | 0.84 |
| IJR | 0.92 | 0.97 | 1.00 | 0.94 | 0.73 | 0.75 | 0.67 | 0.71 | 0.83 | 0.82 | 0.83 | 0.09 | 0.05 | 0.85 |
| IYY | 0.99 | 0.97 | 0.94 | 1.00 | 0.78 | 0.79 | 0.73 | 0.75 | 0.85 | 0.87 | 0.89 | 0.19 | 0.02 | 0.81 |
| XLE | 0.79 | 0.79 | 0.73 | 0.78 | 1.00 | 0.81 | 0.67 | 0.63 | 0.79 | 0.77 | 0.80 | 0.21 | 0.30 | 0.54 |
| EWZ | 0.80 | 0.81 | 0.75 | 0.79 | 0.81 | 1.00 | 0.69 | 0.74 | 0.91 | 0.81 | 0.85 | 0.16 | 0.25 | 0.60 |
| EWJ | 0.75 | 0.70 | 0.67 | 0.73 | 0.67 | 0.69 | 1.00 | 0.70 | 0.75 | 0.76 | 0.85 | 0.28 | 0.11 | 0.53 |
| EWH | 0.77 | 0.73 | 0.71 | 0.75 | 0.63 | 0.74 | 0.70 | 1.00 | 0.81 | 0.76 | 0.80 | 0.15 | 0.12 | 0.62 |
| EEM | 0.85 | 0.86 | 0.83 | 0.85 | 0.79 | 0.91 | 0.75 | 0.81 | 1.00 | 0.87 | 0.90 | 0.12 | 0.21 | 0.69 |
| EZU | 0.88 | 0.85 | 0.82 | 0.87 | 0.77 | 0.81 | 0.76 | 0.76 | 0.87 | 1.00 | 0.97 | 0.20 | 0.18 | 0.69 |
| EFA | 0.90 | 0.87 | 0.83 | 0.89 | 0.80 | 0.85 | 0.85 | 0.80 | 0.90 | 0.97 | 1.00 | 0.25 | 0.19 | 0.69 |
| AGG | 0.25 | 0.14 | 0.09 | 0.19 | 0.21 | 0.16 | 0.28 | 0.15 | 0.12 | 0.20 | 0.25 | 1.00 | -0.06 | 0.02 |
| IAU | 0.01 | 0.07 | 0.05 | 0.02 | 0.30 | 0.25 | 0.11 | 0.12 | 0.21 | 0.18 | 0.19 | -0.06 | 1.00 | 0.03 |
| IYR | 0.79 | 0.84 | 0.85 | 0.81 | 0.54 | 0.60 | 0.53 | 0.62 | 0.69 | 0.69 | 0.69 | 0.02 | 0.03 | 1.00 |

Table 4

*Monthly Return Correlation Table*

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | SPY | IJH | IJR | IYY | XLE | EWZ | EWJ | EWH | EEM | EZU | EFA | AGG | IAU | IYR |
| SPY | 1.00 | 0.95 | 0.92 | 1.00 | 0.68 | 0.73 | 0.75 | 0.76 | 0.85 | 0.90 | 0.91 | 0.14 | 0.06 | 0.81 |
| IJH | 0.95 | 1.00 | 0.97 | 0.97 | 0.69 | 0.73 | 0.75 | 0.72 | 0.84 | 0.84 | 0.86 | 0.11 | 0.07 | 0.83 |
| IJR | 0.92 | 0.97 | 1.00 | 0.94 | 0.62 | 0.64 | 0.71 | 0.65 | 0.78 | 0.80 | 0.81 | 0.07 | -0.01 | 0.86 |
| IYY | 1.00 | 0.97 | 0.94 | 1.00 | 0.68 | 0.74 | 0.76 | 0.76 | 0.86 | 0.90 | 0.91 | 0.14 | 0.06 | 0.82 |
| XLE | 0.68 | 0.69 | 0.62 | 0.68 | 1.00 | 0.78 | 0.57 | 0.62 | 0.75 | 0.64 | 0.68 | -0.06 | 0.28 | 0.43 |
| EWZ | 0.73 | 0.73 | 0.64 | 0.74 | 0.78 | 1.00 | 0.66 | 0.82 | 0.91 | 0.75 | 0.79 | 0.10 | 0.36 | 0.52 |
| EWJ | 0.75 | 0.75 | 0.71 | 0.76 | 0.57 | 0.66 | 1.00 | 0.64 | 0.77 | 0.80 | 0.87 | 0.28 | 0.18 | 0.65 |
| EWH | 0.76 | 0.72 | 0.65 | 0.76 | 0.62 | 0.82 | 0.64 | 1.00 | 0.88 | 0.78 | 0.82 | 0.22 | 0.28 | 0.58 |
| EEM | 0.85 | 0.84 | 0.78 | 0.86 | 0.75 | 0.91 | 0.77 | 0.88 | 1.00 | 0.88 | 0.91 | 0.19 | 0.29 | 0.67 |
| EZU | 0.90 | 0.84 | 0.80 | 0.90 | 0.64 | 0.75 | 0.80 | 0.78 | 0.88 | 1.00 | 0.98 | 0.29 | 0.14 | 0.75 |
| EFA | 0.91 | 0.86 | 0.81 | 0.91 | 0.68 | 0.79 | 0.87 | 0.82 | 0.91 | 0.98 | 1.00 | 0.26 | 0.18 | 0.74 |
| AGG | 0.14 | 0.11 | 0.07 | 0.14 | -0.06 | 0.10 | 0.28 | 0.22 | 0.19 | 0.29 | 0.26 | 1.00 | 0.32 | 0.25 |
| IAU | 0.06 | 0.07 | -0.01 | 0.06 | 0.28 | 0.36 | 0.18 | 0.28 | 0.29 | 0.14 | 0.18 | 0.32 | 1.00 | 0.07 |
| IYR | 0.81 | 0.83 | 0.86 | 0.82 | 0.43 | 0.52 | 0.65 | 0.58 | 0.67 | 0.75 | 0.74 | 0.25 | 0.07 | 1.00 |

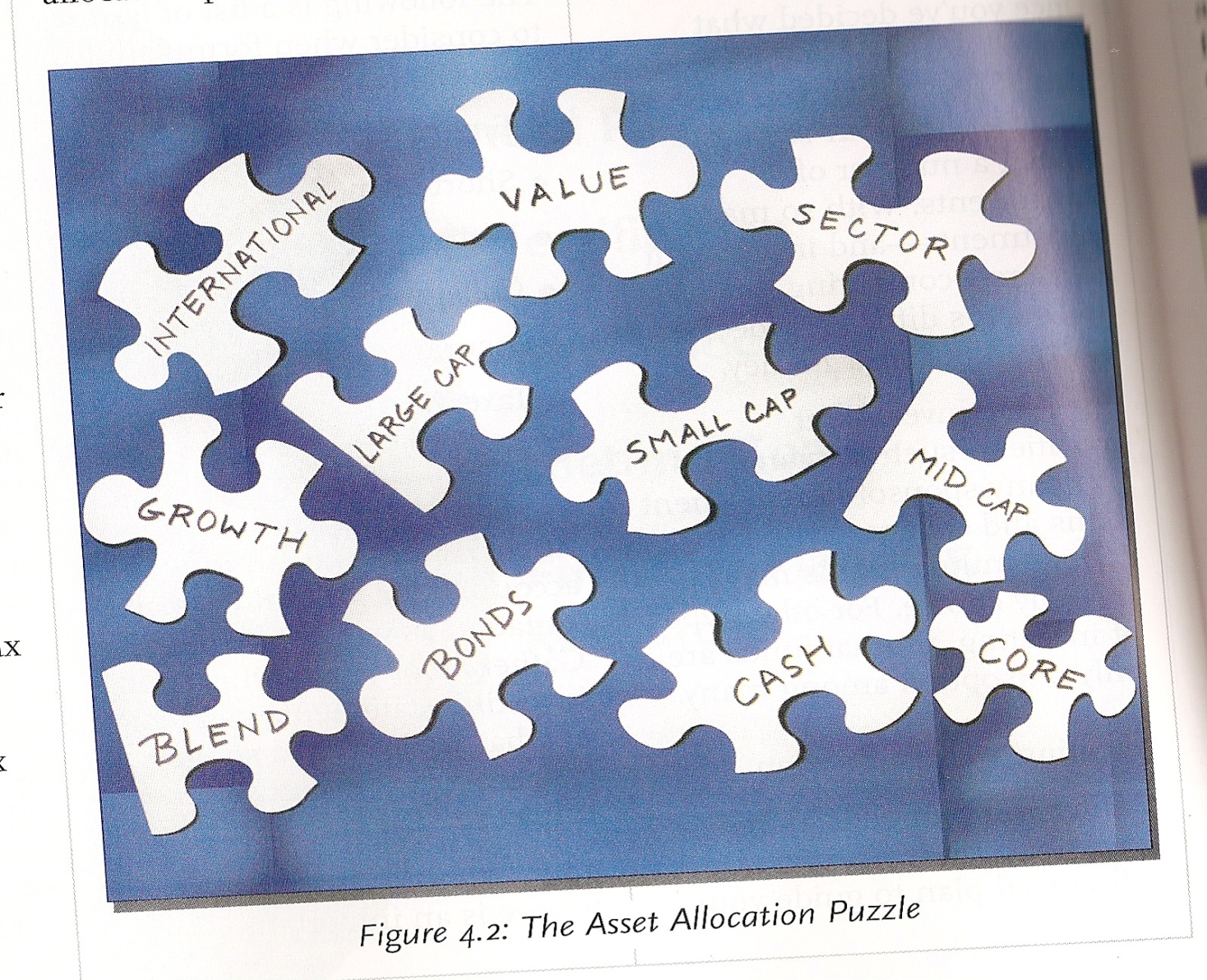
Summary: From these three correlation tables above, we observe that the rate of return of Gold (IAU) and the Aggregate Bond (AGG) share low correlation with the equity, international and real estate ETF’s which rate of return, in turn, show strong correlations among themselves. Based on these three tables, one can expect to obtain more allocations of Gold and Bond.

1. **Portfolio Allocation**

Investors need a financial strategy to guide their investments journey. The best mix of investments for your portfolio depends on your risk tolerance, your time horizon and your expectation for return.

**Diversification**

Diversification cushions you to a certain degree from the ups and downs of various market segments. Studies show that investors with a diversified portfolio of stocks and bonds – or of funds invested in stocks and bonds – maximize their investing returns. A diversified portfolio experiences less volatility than a concentrated portfolio. So one must decide how the various pieces of the asset allocation puzzle fit together.



*Source: Better Investing Educational Series*

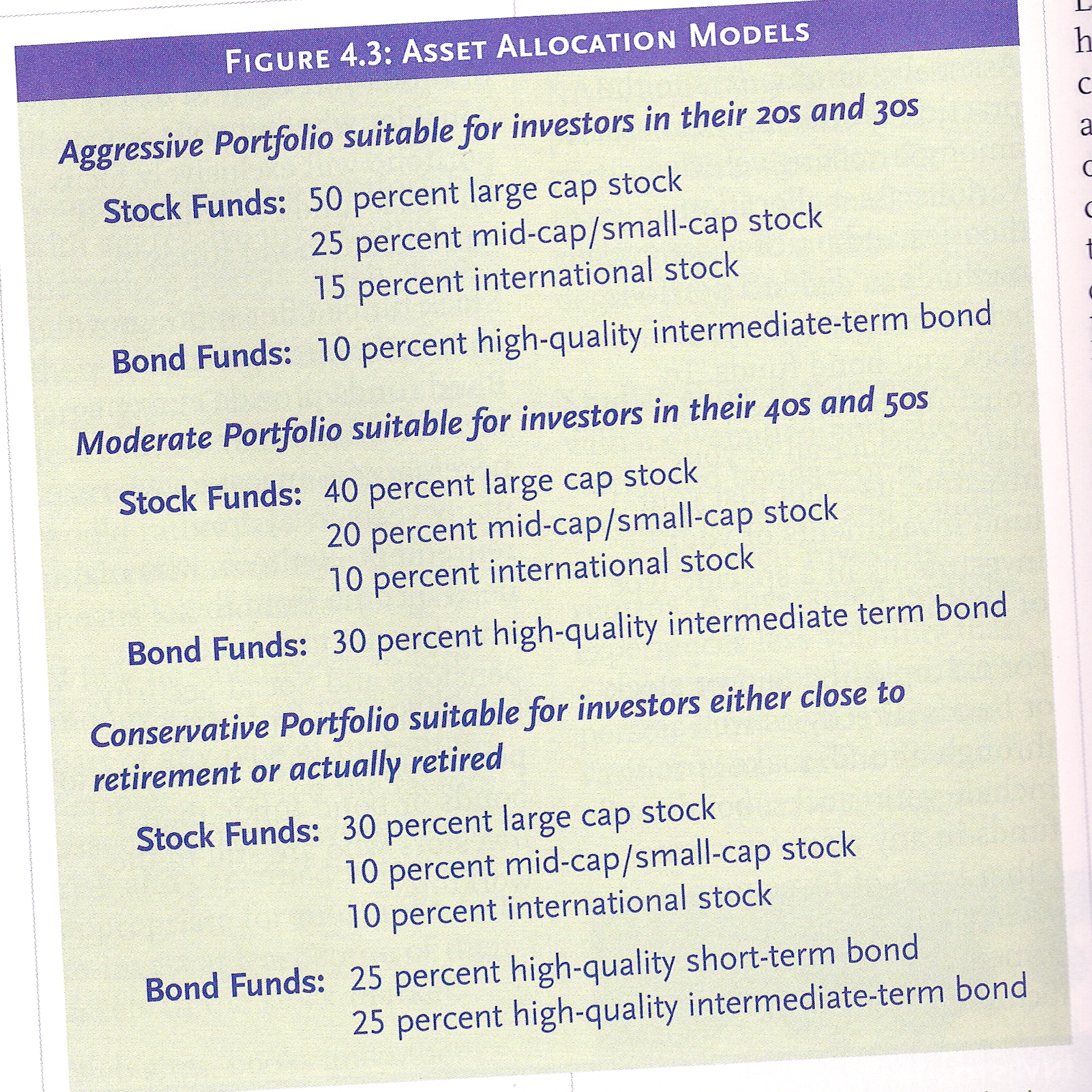
Diversification works because stocks and bonds- and indeed different types of stocks and bonds- perform differently at different times. For example, when stocks perform well, often bonds do not. And even when stocks are generally performing well, not every company stock price goes up. A good example is the bull market of the late 1990s. At that time, virtually any company that related to technology experienced incredible returns, but not related to non-technology sectors.

**Asset Allocation**

Asset allocation involves dividing assets among different asset classes for investment. Various assets allocation theories and practices suggest methods to divide a portfolio between different types of stock and bond funds.

Figure 4.3 lists certain asset allocation models commonly employed in portfolio construction

*Source: Better Investing Educational Series*



In constructing an asset allocation plan, consider all investments in a specific type of account.

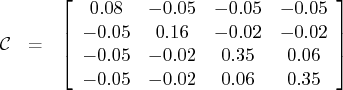
Note: The asset allocation examples in Figure 4.3 illustrate how an investor’s profile could change over time. The sample portfolios are included as an illustration of how a portfolio can be managed, and are not an investment recommendation.

**Portfolio Optimization**

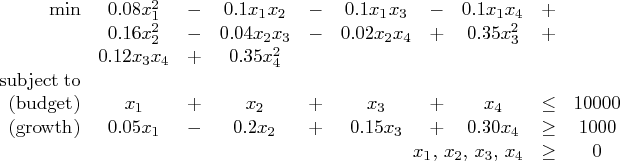
For a portfolio optimization as stated in the objective, the two competing goals of investment are (1) long-term growth of capital and (2) low risk. A good portfolio grows steadily without wild fluctuations in value. The Markowitz model is an optimization model for balancing the return and risk of a portfolio. The decision variables are the amounts invested in each asset. The objective is to minimize the variance of the portfolio's total return, subject to the constraints that (1) the expected growth of the portfolio reaches at least some target level and (2) you do not invest more capital than you have.

Let x_1,  ... , x_nbe the amount invested in each asset, **B** be the amount of capital you have, **R** be the random vector of asset returns over some period, and **r**=E(R) be the expected value of **R**. Let **G** be the minimum growth you hope to obtain, and **C** be the covariance matrix of **R**. The objective function is **Var**, which can be equivalently denoted as **x TC x.**

Assume, for example, n= 4. Let B= $10,000, G= $1000, r =, and C is as given:



The QP (quadratic programming) formulation can be written as



=

We use SAS software to solve the problem. Note that we have two constraints based on budget and growth.

Source: SAS Documentation

1. **Application**

Using the 14 ETFs, we will construct multiple portfolios based on daily, weekly, monthly returns and their expected growth.

Let B=$10,000, n=14, r=E(R) = expected of value of the rate of return, **G** is the desired growth in percentage and **C** is the covariance.

To illustrate the portfolio allocation based on daily, weekly and monthly return, we choose different growths (**G**) desired. The results will follow below.

**Asset Allocation from Daily Rate of Return**

In this case, the matrix **C** is as shown next.

Matrix 1

*Daily Return Covariance Matrix*

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | SPY | IJH | IJR | IYY | XLE | EWZ | EWJ | EWH | EEM | EZU | EFA | AGG | IAU | IYR |
| SPY | 2.14 | -0.20 | -0.16 | -0.21 | -0.46 | -0.28 | -0.21 | -0.21 | -0.15 | -0.22 | -0.23 | -0.07 | -0.19 | -0.18 |
| IJH | 2.22 | 2.57 | 2.66 | 2.20 | 2.82 | 3.71 | 1.95 | 2.59 | 3.47 | 2.64 | 2.43 | -0.05 | 0.22 | 3.61 |
| IJR | 2.28 | 2.66 | 2.97 | 2.28 | 2.77 | 3.69 | 1.98 | 2.65 | 3.50 | 2.67 | 2.45 | -0.10 | 0.15 | 3.87 |
| IYY | 2.03 | 2.20 | 2.28 | 2.01 | 2.53 | 3.34 | 1.78 | 2.39 | 3.15 | 2.44 | 2.24 | -0.05 | 0.15 | 3.17 |
| XLE | 2.63 | 2.82 | 2.77 | 2.53 | 4.96 | 5.13 | 2.44 | 3.21 | 4.46 | 3.30 | 3.08 | -0.03 | 0.81 | 3.46 |
| EWZ | 3.45 | 3.71 | 3.69 | 3.34 | 5.13 | 8.32 | 3.36 | 4.53 | 6.51 | 4.47 | 4.18 | -0.01 | 0.93 | 4.73 |
| EWJ | 1.88 | 1.95 | 1.98 | 1.78 | 2.44 | 3.36 | 2.59 | 2.51 | 3.25 | 2.47 | 2.45 | 0.02 | 0.34 | 2.63 |
| EWH | 2.48 | 2.59 | 2.65 | 2.39 | 3.21 | 4.53 | 2.51 | 4.16 | 4.53 | 3.07 | 2.93 | -0.08 | 0.28 | 3.69 |
| EEM | 3.26 | 3.47 | 3.50 | 3.15 | 4.46 | 6.51 | 3.25 | 4.53 | 6.27 | 4.20 | 3.93 | -0.08 | 0.64 | 4.74 |
| EZU | 2.52 | 2.64 | 2.67 | 2.44 | 3.30 | 4.47 | 2.47 | 3.07 | 4.20 | 3.74 | 3.25 | -0.01 | 0.53 | 3.69 |
| EFA | 2.34 | 2.43 | 2.45 | 2.24 | 3.08 | 4.18 | 2.45 | 2.93 | 3.93 | 3.25 | 3.02 | 0.00 | 0.47 | 3.34 |
| AGG | -0.03 | -0.05 | -0.10 | -0.05 | -0.03 | -0.01 | 0.02 | -0.08 | -0.08 | -0.01 | 0.00 | 0.15 | 0.05 | -0.15 |
| IAU | 0.13 | 0.22 | 0.15 | 0.15 | 0.81 | 0.93 | 0.34 | 0.28 | 0.64 | 0.53 | 0.47 | 0.05 | 1.89 | 0.09 |
| IYR | 3.15 | 3.61 | 3.87 | 3.17 | 3.46 | 4.73 | 2.63 | 3.69 | 4.74 | 3.69 | 3.34 | -0.15 | 0.09 | 7.11 |

Table 5

*Asset Allocation Table- Daily Return Case*

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | **Budget=B=$10,000** | |  |  |  |  |  |
| **Growth%** |  | **0.02** | **0.025** | **0.03** | **0.035** | **0.04** | **0.05** | **0.07** | **0.09** |
| **ETFs** | **r %** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** |
| **SPY** | 0.02 | $875.21 | $868.75 | $872.51 | $897.19 | $922.19 | $946.52 | $974.01 | $1,001.51 |
| **IJH** | 0.04 |  |  |  |  |  |  |  |  |
| **IJR** | 0.04 | $356.92 | $561.09 | $514.88 | $247.62 |  |  |  |  |
| **IYY** | 0.02 | $401.43 | $104.83 |  |  |  |  |  |  |
| **XLE** | 0.07 |  |  |  |  |  |  |  |  |
| **EWZ** | 0.14 |  |  |  | $250.76 | $520.81 | $912.57 | $1,527.23 | $2,141.88 |
| **EWJ** | 0.02 |  |  |  |  |  |  |  |  |
| **EWH** | 0.06 |  | $38.88 | $227.15 | $175.88 | $91.08 |  |  |  |
| **EEM** | 0.09 |  |  |  |  |  |  |  |  |
| **EZU** | 0.03 |  |  |  |  |  |  |  |  |
| **EFA** | 0.03 |  |  |  |  |  |  |  |  |
| **AGG** | 0.02 | $7,968.42 | $7,894.69 | $7,233.79 | $6,864.52 | $6,509.98 | $5,419.60 | $2,974.00 | $528.41 |
| **IAU** | 0.09 | $398.02 | $531.76 | $1,151.66 | $1,525.01 | $1,879.54 | $2,721.31 | $4,524.76 | $6,328.20 |
| **IYR** | 0.05 |  |  |  | $39.02 | $76.41 |  |  |  |
|  | **Std(Pi)** | 3313.813 | 3322.777 | 3502.013 | 3881.245 | 4366.850 | 5573.383 | 8523.635 | 11742.389 |
|  | **RR** | **1.000** | **1.003** | **1.057** | **1.171** | **1.318** | **1.682** | **2.572** | **3.543** |

Allocations from SPY, Gold (IAU) and Bond (AGG) appear in all of the portfolios. It is because of their negative to low correlation structure. As we increase growth rate, less money will be allocated to the Bond ETF and more money to the Gold. This is because of lower expected return of AGG and higher expected return of IAU.

Note that the risk (STD (Pi)) of portfolio, say i, and its relative risk (RR) increase as the expected growth increases. However, Portfolio 3 has 50% more rate of return compared to the first one with only 6% increase in its relative risk. Hence, this may be the best portfolio with the least possible risk. However, a further increase in expected return rate leads to portfolios with increasing relative risks as noted earlier. For example, if one desires for the expected return rate to be .04 vs. 0.02, which is a 100% increase in growth, then it comes with an increase of 32% risk over that of Portfolio 1.

The following pie chart depicts Portfolio 3 asset allocation.

Observing Portfolio 3 as a better one, the minimum variance that earns an expected return of at least .03% is AGG=72%, IAU=12%, SPY=9%, IJR=5% and EWH=2%. Some assets get nothing because their covariance with the other assets is not sufficiently negative for it to bring any diversification benefits.

**Allocation Based on Weekly Rate of Return**

Matrix 2

*Weekly Return Covariance Matrix*

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | SPY | IJH | IJR | IYY | XLE | EWZ | EWJ | EWH | EEM | EZU | EFA | AGG | IAU | IYR |
| SPY | 8.40 | 9.24 | 9.51 | 8.12 | 9.67 | 13.11 | 6.44 | 7.64 | 10.82 | 9.50 | 8.57 | 0.64 | 0.06 | 10.76 |
| IJH | 9.24 | 11.23 | 11.57 | 9.18 | 11.12 | 15.32 | 6.99 | 8.44 | 12.61 | 10.68 | 9.53 | 0.43 | 0.71 | 13.16 |
| IJR | 9.51 | 11.57 | 12.61 | 9.47 | 11.02 | 15.11 | 7.05 | 8.71 | 12.98 | 10.89 | 9.66 | 0.29 | 0.52 | 14.16 |
| IYY | 8.12 | 9.18 | 9.47 | 7.99 | 9.37 | 12.71 | 6.14 | 7.33 | 10.54 | 9.22 | 8.25 | 0.48 | 0.16 | 10.81 |
| XLE | 9.67 | 11.12 | 11.02 | 9.37 | 17.83 | 19.37 | 8.35 | 9.21 | 14.62 | 12.14 | 11.11 | 0.78 | 3.63 | 10.72 |
| EWZ | 13.11 | 15.32 | 15.11 | 12.71 | 19.37 | 32.06 | 11.51 | 14.36 | 22.64 | 17.18 | 15.65 | 0.82 | 4.04 | 15.96 |
| EWJ | 6.44 | 6.99 | 7.05 | 6.14 | 8.35 | 11.51 | 8.78 | 7.10 | 9.83 | 8.37 | 8.20 | 0.74 | 0.91 | 7.37 |
| EWH | 7.64 | 8.44 | 8.71 | 7.33 | 9.21 | 14.36 | 7.10 | 11.82 | 12.25 | 9.72 | 8.97 | 0.46 | 1.19 | 10.01 |
| EEM | 10.82 | 12.61 | 12.98 | 10.54 | 14.62 | 22.64 | 9.83 | 12.25 | 19.31 | 14.35 | 12.95 | 0.47 | 2.71 | 14.27 |
| EZU | 9.50 | 10.68 | 10.89 | 9.22 | 12.14 | 17.18 | 8.37 | 9.72 | 14.35 | 13.95 | 11.89 | 0.66 | 1.95 | 12.14 |
| EFA | 8.57 | 9.53 | 9.66 | 8.25 | 11.11 | 15.65 | 8.20 | 8.97 | 12.95 | 11.89 | 10.69 | 0.73 | 1.76 | 10.65 |
| AGG | 0.64 | 0.43 | 0.29 | 0.48 | 0.78 | 0.82 | 0.74 | 0.46 | 0.47 | 0.66 | 0.73 | 0.80 | -0.15 | 0.09 |
| IAU | 0.06 | 0.71 | 0.52 | 0.16 | 3.63 | 4.04 | 0.91 | 1.19 | 2.71 | 1.95 | 1.76 | -0.15 | 8.30 | 0.46 |
| IYR | 10.76 | 13.16 | 14.16 | 10.81 | 10.72 | 15.96 | 7.37 | 10.01 | 14.27 | 12.14 | 10.65 | 0.09 | 0.46 | 22.04 |

Table 6

*Asset Allocation Table- Weekly Return Case*

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **Budget=B=$10,000** |  |  |  | |  | |  |  |  |
|  | **Growth%** | **0.1** | **0.15** | **0.2** | **0.25** | | **0.3** | | **0.4** | **0.5** | **0.6** |
|  |  |  |  |  | **Allocation on i=1 to 8 Portfolios** | | | | |  |  |
| **ETF's** | **r %** | **1** | **2** | **3** | **4** | **5** | | **6** | | **7** | **8** |
| SPY | 0.1 |  |  |  |  |  | |  | |  |  |
| IJH | 0.18 |  |  |  |  |  | |  | |  |  |
| IJR | 0.16 | $60.92 |  |  |  |  | |  | |  |  |
| IYY | 0.1 |  |  |  |  |  | |  | |  |  |
| XLE | 0.3 |  |  |  |  |  | |  | |  |  |
| EWZ | 0.62 |  |  | $414.62 | $891.84 | $1,285.40 | | $2,072.40 | | $4,000.00 | $9,000.00 |
| EWJ | 0.07 |  |  |  |  |  | |  | |  |  |
| EWH | 0.27 |  | $187.43 | $205.43 |  |  | |  | |  |  |
| EEM | 0.36 |  |  |  |  |  | |  | |  |  |
| EZU | 0.1 |  |  |  |  |  | |  | |  |  |
| EFA | 0.12 |  |  |  |  |  | |  | |  |  |
| AGG | **0.09** | $8,749.15 | $7,942.39 | $6,824.57 | $5,692.03 | $4,415.40 | | $1,862.10 | |  |  |
| IAU | **0.42** | $972.45 | $1,666.59 | $2,555.37 | $3,416.13 | $4,299.30 | | $6,065.60 | | $6,000.00 | $1,000.00 |
| IYR | 0.17 | $217.48 | $203.59 |  |  |  | |  | |  |  |
|  | **Std(Pi)** | 8301.018 | 8623.0264 | 10508.252 | 13243.43 | 16439.298 | | 23459.429 | | 31712.458 | 34641.016 |
|  | **RR** | 1.000 | 1.039 | 1.266 | 1.595 | 1.980 | | 2.826 | | 3.820 | 4.173 |

Gold (IAU) and Bond (AGG) ETFs appear to be nearly in all of the portfolios except 2 for the Bond in weekly case. Again, it is because of their negative to low correlation structure. As we increase expected return or growth rate, less money will be allocated to the Bond ETF and more money to the Gold except for the last 2 portfolios. This is due to lower expected return of AGG and higher expected return of IAU. Observe that the risk of portfolio, say i, and its relative risk increase as the expected growth increases. Portfolio 2 has 25% more rate of return compared to the first one with only 4% increase in its relative risk. Therefore, this may be the best portfolio with the least possible risk. However, a further increase in expected return rate leads to portfolios with increasing relative risks as noted earlier.

The pie chart below represents Portfolio 2 weekly asset allocation.

The minimum variance for weekly portfolio 2 that earns an expected return of at least .015% is EWH= **$187.43**, AGG2**=$7,942.39**, IYR= **$203.59**, IAU**=$1666.59**; the other ETFs which account about 71% of the assets get no allocations because their covariance with the other assets is not sufficiently negative for them to bring any diversification benefits.

**Asset Allocation from Monthly Rate of Return**

Matrix 3

*Monthly Return Covariance Matrix*

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Monthly | SPY | IJH | IJR | IYY | XLE | EWZ | EWJ | EWH | EEM | EZU | EFA | AGG | IAU | IYR |
| SPY | 22.60 | 25.60 | 26.52 | 23.13 | 22.96 | 35.92 | 18.18 | 24.66 | 32.08 | 31.98 | 26.44 | 0.85 | 1.56 | 32.50 |
| IJH | 25.60 | 32.07 | 33.25 | 26.70 | 27.68 | 42.91 | 21.45 | 27.77 | 37.51 | 35.41 | 29.54 | 0.81 | 2.09 | 39.90 |
| IJR | 26.52 | 33.25 | 36.96 | 27.75 | 26.73 | 40.15 | 22.03 | 26.92 | 37.58 | 36.44 | 30.09 | 0.54 | -0.30 | 43.98 |
| IYY | 23.13 | 26.70 | 27.75 | 23.82 | 23.62 | 37.16 | 18.79 | 25.34 | 33.08 | 32.74 | 27.10 | 0.86 | 1.53 | 33.94 |
| XLE | 22.96 | 27.68 | 26.73 | 23.62 | 50.74 | 57.67 | 20.71 | 30.21 | 42.07 | 34.08 | 29.75 | -0.53 | 10.65 | 25.66 |
| EWZ | 35.92 | 42.91 | 40.15 | 37.16 | 57.67 | 106.72 | 34.38 | 57.49 | 74.39 | 57.82 | 49.85 | 1.31 | 20.08 | 45.10 |
| EWJ | 18.18 | 21.45 | 22.03 | 18.79 | 20.71 | 34.38 | 25.74 | 22.02 | 31.08 | 30.19 | 27.00 | 1.85 | 4.87 | 27.72 |
| EWH | 24.66 | 27.77 | 26.92 | 25.34 | 30.21 | 57.49 | 22.02 | 46.16 | 47.09 | 39.72 | 33.97 | 1.91 | 10.14 | 33.21 |
| EEM | 32.08 | 37.51 | 37.58 | 33.08 | 42.07 | 74.39 | 31.08 | 47.09 | 62.64 | 52.16 | 44.13 | 1.97 | 12.35 | 44.82 |
| EZU | 31.98 | 35.41 | 36.44 | 32.74 | 34.08 | 57.82 | 30.19 | 39.72 | 52.16 | 55.60 | 44.57 | 2.74 | 5.72 | 47.02 |
| EFA | 26.44 | 29.54 | 30.09 | 27.10 | 29.75 | 49.85 | 27.00 | 33.97 | 44.13 | 44.57 | 37.21 | 2.04 | 5.90 | 38.38 |
| AGG | 0.85 | 0.81 | 0.54 | 0.86 | -0.53 | 1.31 | 1.85 | 1.91 | 1.97 | 2.74 | 2.04 | 1.64 | 2.21 | 2.76 |
| IAU | 1.56 | 2.09 | -0.30 | 1.53 | 10.65 | 20.08 | 4.87 | 10.14 | 12.35 | 5.72 | 5.90 | 2.21 | 28.52 | 2.95 |
| IYR | 32.50 | 39.90 | 43.98 | 33.94 | 25.66 | 45.10 | 27.72 | 33.21 | 44.82 | 47.02 | 38.38 | 2.76 | 2.95 | 71.46 |

Table

*Asset Allocation - Monthly Return Case*

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | **B=$10,000** |  |  |  |  |  |  |  |
|  | **Growth%** | **0.4** | **0.5** | **0.6** | **0.7** | **0.8** | **0.9** | **1** | **1.5** | **1.7** |
|  |  |  |  |  | **Portofolio i** |  |  |  |  |  |
| **ETF's** | **r %** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** |
| SPY | 0.37 |  |  |  |  |  |  |  |  |  |
| IJH | 0.73 |  |  |  |  |  |  |  |  |  |
| IJR | 0.63 |  |  |  |  |  |  |  |  |  |
| IYY | 0.42 |  |  |  |  |  |  |  |  |  |
| XLE | 1.2 | $406.06 | $580.68 | $383.33 | $164.15 |  |  |  |  |  |
| EWZ | 2.57 |  |  | $246.67 | $512.90 | $752.64 | $913.36 | $1,074.1 | $1,877.7 | $2,199.1 |
| EWJ | 0.25 |  |  |  |  |  |  |  |  |  |
| EWH | 1.14 |  |  |  |  |  |  |  |  |  |
| EEM | 1.45 |  |  |  |  |  |  |  |  |  |
| EZU | 0.43 |  |  |  |  |  |  |  |  |  |
| EFA | 0.52 |  |  |  |  |  |  |  |  |  |
| AGG | 0.4 | $9,593.9 | $9,010.57 | $8,485.99 | $7,982.73 | $7,440.67 | $6,782.82 | $6,125 | $2,835.7 | $1,520.0 |
| IAU | 1.71 |  | $408.74 | $884.01 | $1,340.21 | $1,806.69 | $2,303.82 | $2,801 | $5,286.6 | $6,280.9 |
| IYR | 0.69 |  |  |  |  |  |  |  |  |  |
|  | Std(Pi) | 12457.465 | 13069.585 | 14710.934 | 16846.473 | 19314.295 | 22037.633 | 24945.827 | 40809.351 | 47441.072 |
|  | RR | 1.000 | 1.049 | 1.181 | 1.352 | 1.550 | 1.769 | 2.002 | 3.276 | 3.808 |

Similarly to Daily and Weekly allocation description, Portfolio 2 is considered to be the best one because it has 50% more rate of return compared to Portfolio 1 with only 5% increase in its relative risk. However, we observe only three allocations per Portfolio in Monthly case except for Portfolio 3. Brazil (EWZ) and Gold (IAU) ETFs have the highest average return of 2.57% and 1.71%, respectively. The following pie chart represents Portfolio 2 monthly asset allocation.

For this portfolio, the minimum variance from Monthly that earns an expected return of at least .05% is IAU= $408.74, XLE =$580.68 and AGG= $9010.57. About 90% of the assets were allocated into the Bond (AGG). The other ETFs got no allocation because their covariance with the other assets is not sufficiently negative for them to bring any diversification benefits.

1. **Conclusion**

The data used during this project contained ETFs that are mostly correlated with exception of few ETFs such as Bond (AGG), Gold (IAU) that show low or no correlation with the others. In addition, the average rate of return of individual ETF ranged from 0.02% to 2.57%. Thus the expected growths desired by one will range within the mean of the rate of return. For instance, it was not possible for an investor to desire a return of 3% with these 14 ETFs. Thus, the asset allocations were affected by correlation structure mostly because some ETFS show no allocation.

Let’s look at some samples of correlation matchups that can help one see the difference.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| High positive correlations   |  |  |  | | --- | --- | --- | | DIS | [**+ 0.93**](http://www.macroaxis.com/invest/market/DIS--compareProfile--CRM) | CRM | | XOM | [**+ 0.89**](http://www.macroaxis.com/invest/market/XOM--compareProfile--F) | F | | DIS | [**+ 0.86**](http://www.macroaxis.com/invest/market/DIS--compareProfile--MSFT) | MSFT | | XOM | [**+ 0.86**](http://www.macroaxis.com/invest/market/XOM--compareProfile--T) | T | | DIS | [**+ 0.85**](http://www.macroaxis.com/invest/market/DIS--compareProfile--F) | F | | T | [**+ 0.84**](http://www.macroaxis.com/invest/market/T--compareProfile--F) | F | | XOM | [**+ 0.83**](http://www.macroaxis.com/invest/market/XOM--compareProfile--MSFT) | MSFT | | CRM | [**+ 0.81**](http://www.macroaxis.com/invest/market/CRM--compareProfile--MSFT) | MSFT | | T | [**+ 0.77**](http://www.macroaxis.com/invest/market/T--compareProfile--MSFT) | MSFT | | DIS | [**+ 0.77**](http://www.macroaxis.com/invest/market/DIS--compareProfile--XOM) | XOM | | F | [**+ 0.75**](http://www.macroaxis.com/invest/market/F--compareProfile--MSFT) | MSFT | | YHOO | [**+ 0.74**](http://www.macroaxis.com/invest/market/YHOO--compareProfile--GOOG) | GOOG | | CRM | [**+ 0.7**](http://www.macroaxis.com/invest/market/CRM--compareProfile--F) | F | | DIS | [**+ 0.65**](http://www.macroaxis.com/invest/market/DIS--compareProfile--T) | T | |  | Recommended Pairs http://cdn.macroaxis.netdna-cdn.com/images/whats_this.gif   |  |  |  | | --- | --- | --- | | MRK | [**- 0.0**](http://www.macroaxis.com/invest/market/MRK--compareProfile--YHOO) | YHOO | | XOM | [**- 0.12**](http://www.macroaxis.com/invest/market/XOM--compareProfile--JPM) | JPM | | DIS | [**- 0.18**](http://www.macroaxis.com/invest/market/DIS--compareProfile--JPM) | JPM | | MRK | [**- 0.2**](http://www.macroaxis.com/invest/market/MRK--compareProfile--GOOG) | GOOG | | JPM | [**- 0.25**](http://www.macroaxis.com/invest/market/JPM--compareProfile--MSFT) | MSFT | | XOM | [**- 0.28**](http://www.macroaxis.com/invest/market/XOM--compareProfile--MRK) | MRK | | MRK | [**- 0.29**](http://www.macroaxis.com/invest/market/MRK--compareProfile--F) | F |  |  |  |  | | --- | --- | --- | | MRK | [**+ 0.0**](http://www.macroaxis.com/invest/market/MRK--compareProfile--YHOO) | YHOO | | MRK | [**+ 0.03**](http://www.macroaxis.com/invest/market/MRK--compareProfile--JPM) | JPM | | JPM | [**+ 0.08**](http://www.macroaxis.com/invest/market/JPM--compareProfile--F) | F | | T | [**+ 0.14**](http://www.macroaxis.com/invest/market/T--compareProfile--YHOO) | YHOO | | M | [**+ 0.22**](http://www.macroaxis.com/invest/market/M--compareProfile--CRM) | CRM | | JPM | [**+ 0.25**](http://www.macroaxis.com/invest/market/JPM--compareProfile--T) | T | | T | [**+ 0.25**](http://www.macroaxis.com/invest/market/T--compareProfile--GOOG) | GOOG | |  | High negative correlations   |  |  |  | | --- | --- | --- | | JPM | [**- 0.74**](http://www.macroaxis.com/invest/market/JPM--compareProfile--YHOO) | YHOO | | JPM | [**- 0.68**](http://www.macroaxis.com/invest/market/JPM--compareProfile--GOOG) | GOOG | | MRK | [**- 0.58**](http://www.macroaxis.com/invest/market/MRK--compareProfile--MSFT) | MSFT | | MRK | [**- 0.46**](http://www.macroaxis.com/invest/market/MRK--compareProfile--CRM) | CRM | | DIS | [**- 0.4**](http://www.macroaxis.com/invest/market/DIS--compareProfile--MRK) | MRK | | M | [**- 0.36**](http://www.macroaxis.com/invest/market/M--compareProfile--GOOG) | GOOG | | JPM | [**- 0.35**](http://www.macroaxis.com/invest/market/JPM--compareProfile--CRM) | CRM | | MRK | [**- 0.33**](http://www.macroaxis.com/invest/market/MRK--compareProfile--T) | T | | M | [**- 0.33**](http://www.macroaxis.com/invest/market/M--compareProfile--YHOO) | YHOO | | M | [**- 0.31**](http://www.macroaxis.com/invest/market/M--compareProfile--MRK) | MRK | | MRK | [**- 0.29**](http://www.macroaxis.com/invest/market/MRK--compareProfile--F) | F | | XOM | [**- 0.28**](http://www.macroaxis.com/invest/market/XOM--compareProfile--MRK) | MRK | | JPM | [**- 0.25**](http://www.macroaxis.com/invest/market/JPM--compareProfile--MSFT) | MSFT | | MRK | [**- 0.2**](http://www.macroaxis.com/invest/market/MRK--compareProfile--GOOG) | GOOG | |

Source: Investorama.com

In order to increase growth and get more allocations, a portfolio that is similar to the recommended pairs is considered better because of their correlation structures. These are only for example purpose.

Note that as we increase growth/profit, we also increase the risk of the portfolio. Low levels of uncertainty (low risk) are associated with low potential returns. High levels of uncertainty (high risk) are associated with high potential returns. The risk/return tradeoff is the balance between the desire for the lowest possible risk and the highest possible return. This is demonstrated graphically in the chart below. A higher standard deviation means a higher risk and higher possible return.

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| --- |
| http://i.investopedia.com/inv/tutorials/site/concepts1_riskreturn.gif |

Source: *Investorama*

On the lower end of the scale, the risk-free rate of return is represented in our case by the return on U.S. Government Bond (AGG) because their chance of default is next to nothing.

This portfolio optimization based on ETFs can generate some returns that are related to the risk.

The relative risk helps determine a better portfolio. However, it is not a validation process. Validation will prove how good the portfolio optimization is. It is a long process and thus can be carried in further research.

10. **References**

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