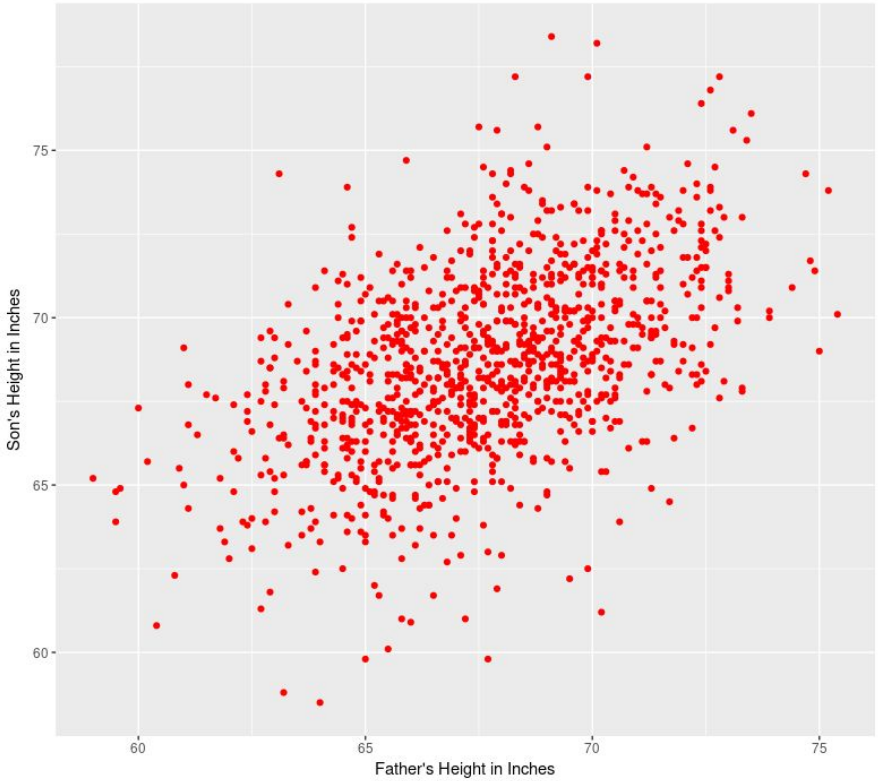
https://cs.brown.edu/courses/cs100/lectures/lecture15c.pdf

A Brief History of Regression

Heights of Fathers and their Sons

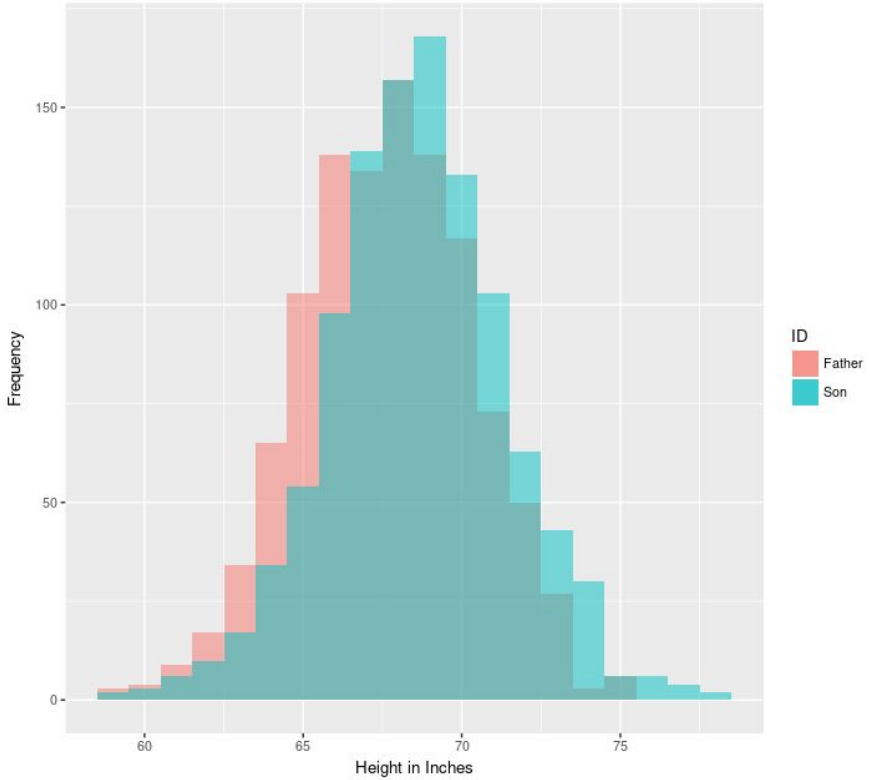
- The scatter plot to the right depicts data collected by Pearson and his colleagues in the early 1900's
- It consists of 1078 pairs of heights of father and their sons
- The plot is shaped like an American football, with a dense center and fewer points around the perimeter



Histograms of their Heights

- The histograms of the fathers' and sons' heights are both bell-shaped.
- The histograms mostly overlap.
- But sons are about an inch taller than their fathers, on average.

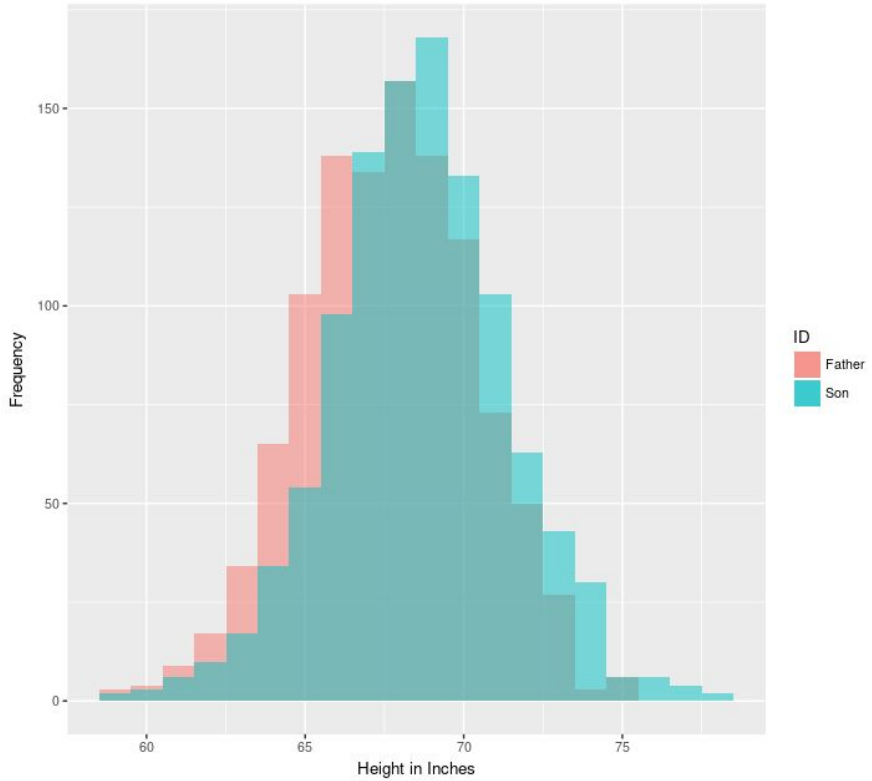
```
> summary(heights)
     Father           Son
 Min.   :59.00   Min.   :58.50
 1st Qu.:65.80   1st Qu.:66.90
 Median :67.80   Median :68.60
 Mean   :67.69   Mean   :68.68
 3rd Qu.:69.60   3rd Qu.:70.50
 Max.   :75.40   Max.   :78.40
```



Correlation in their Heights

The correlation in their heights is exactly what leads to the American football (i.e., ellipsoidal) shape

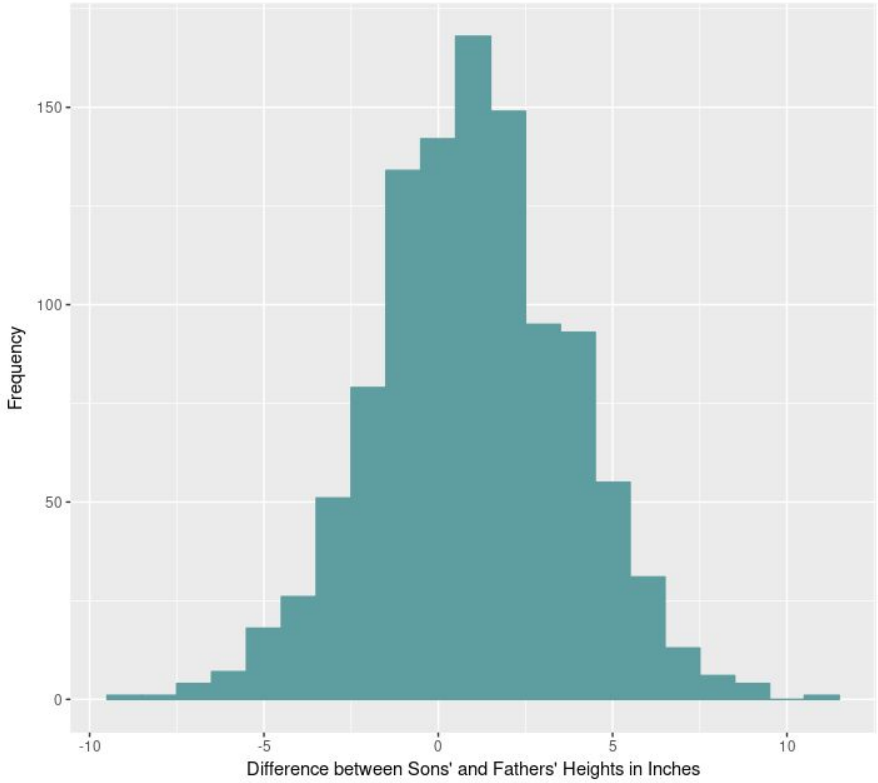
```
> pearson <- read.csv("pearson.csv")
> cor(pearson$Son, pearson$Father)
[1] 0.5011627
```



Histogram of the Differences

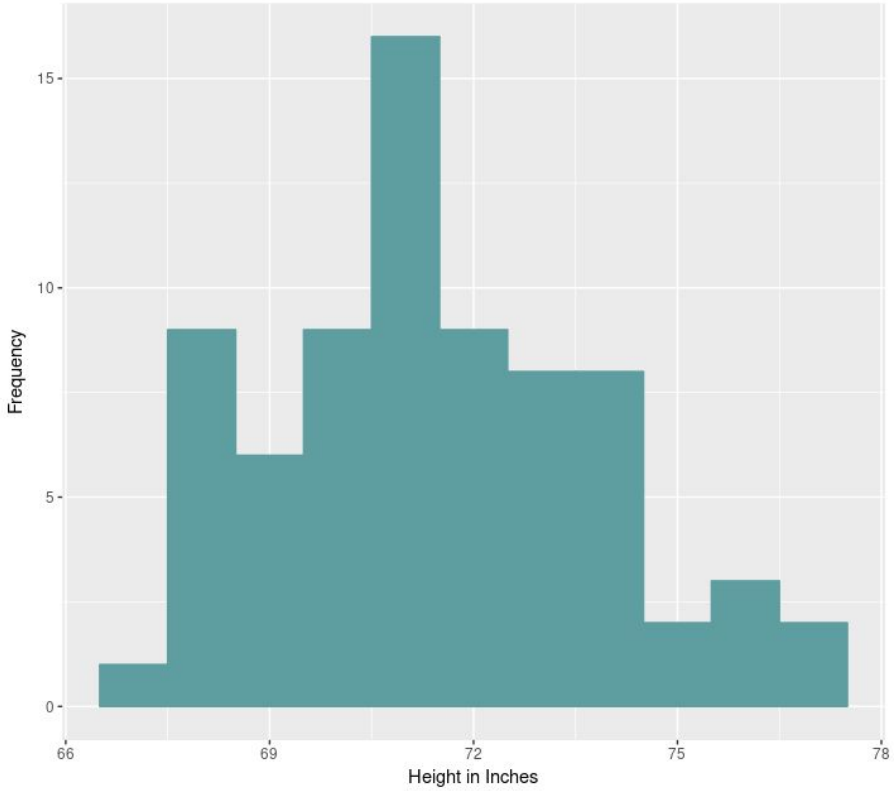
The bulk (95%) of the data lie between -4.4 and 6.4 inches.

```
> summary(heights)
    Father          Son            Diff        
Min.   :59.00   Min.   :58.50   Min.   :-9.0000  
1st Qu.:65.80   1st Qu.:66.90   1st Qu.:-0.8000  
Median :67.80   Median :68.60   Median : 1.0000  
Mean   :67.69   Mean   :68.68   Mean   : 0.9974  
3rd Qu.:69.60   3rd Qu.:70.50   3rd Qu.: 2.7750  
Max.   :75.40   Max.   :78.40   Max.   :11.2000  
```



The Regression Effect

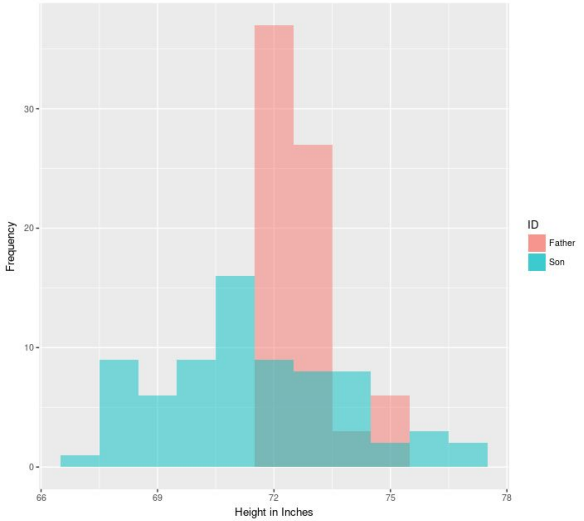
- We might expect the sons of tall fathers to be tall as well.
- This histogram shows the heights of sons of 72 inch fathers.
- Most (68%) of these sons are less than 72 inches tall!



The Regression Effect (cont'd)

- This is surprising!
  - Sons are an inch taller than their fathers, on average.
  - But sons of tall fathers are an inch shorter than their fathers!

```
> tall_fathers <- heights %>% filter(Father >= 72)
> mean_tall_fathers <- tall %>% summarize(father =
mean(Father), son = mean(Son), diff = mean(Diff))
> mean_tall_fathers
   father     son     diff
1 72.8178 71.4575 -1.36027
```



History of the Regression Effect

- The regression effect was first documented by the statistician Francis Galton, who had thought (hoped, even) that tall fathers would have tall sons.
- These data show that tall fathers' sons were not quite as tall.
- Galton, who is sometimes called the father of eugenics, called this effect "regression to mediocrity".
- Galton also noticed that short fathers had sons who were somewhat taller than their generation on average. Today, this is called the regression effect.
- Individuals who are below average after a first measurement tend to move towards the mean after a second, and vice versa. Why?

The Regression Effect, Explained

- Imagine pre-test and a post-test measurements for a set of individuals who receive a null treatment (i.e., a placebo).
- Some individuals will test below the mean, and others will test above.
- Assuming perfect measurements (no measurement error), those who test below (or above) in the pre-test will do so for one of two reasons. Either:
  - Their measurements are truly below (or above) the mean, or
  - Random fluctuations
- In the post-test, if they are truly below (or above) the mean, they will likely measure that way again. But if their pre-test measurements were due to random fluctuations, they will move in the direction of the mean!
- So, conditioned on measuring below (or above) the mean in the pre-test, measurements will be closer to the mean in the post-test!

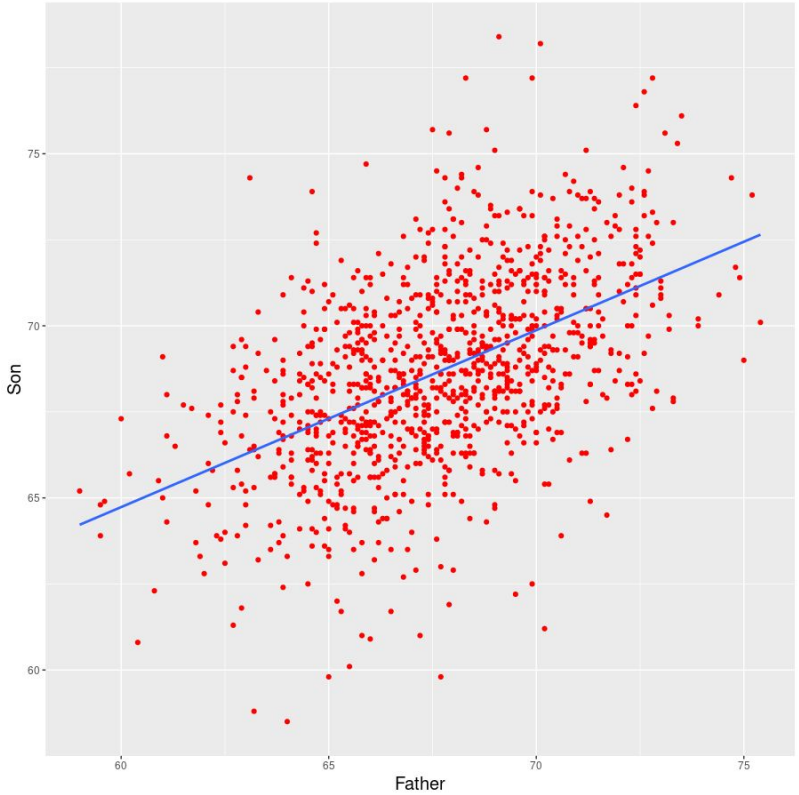
Extras

Fitting a Regression Line in R

The blue line follows the angle of the cloud of points, and is called the regression line.

```
> attach(pearson)
> plot(Father, Son, col = "red")
> fit <- lm(Son ~ Father)
> abline(fit, col = "blue")
> detach(pearson)
```

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 33.89280    1.83289   18.49   <2e-16 ***
Father       0.51401    0.02706   19.00   <2e-16 ***
```



The Regression Line, in Standard Units

- This scatter plot depicts the data in standard units.
- The black line has a slope of 1:
  - A one unit increase in father's height leads to corresponding one unit increase in son's.
- The slope of the regression line is less than 1. In fact, it is $r \approx 0.5$:
  - A one unit increase in father's height leads to corresponding one-half unit increase in son's.