homework02

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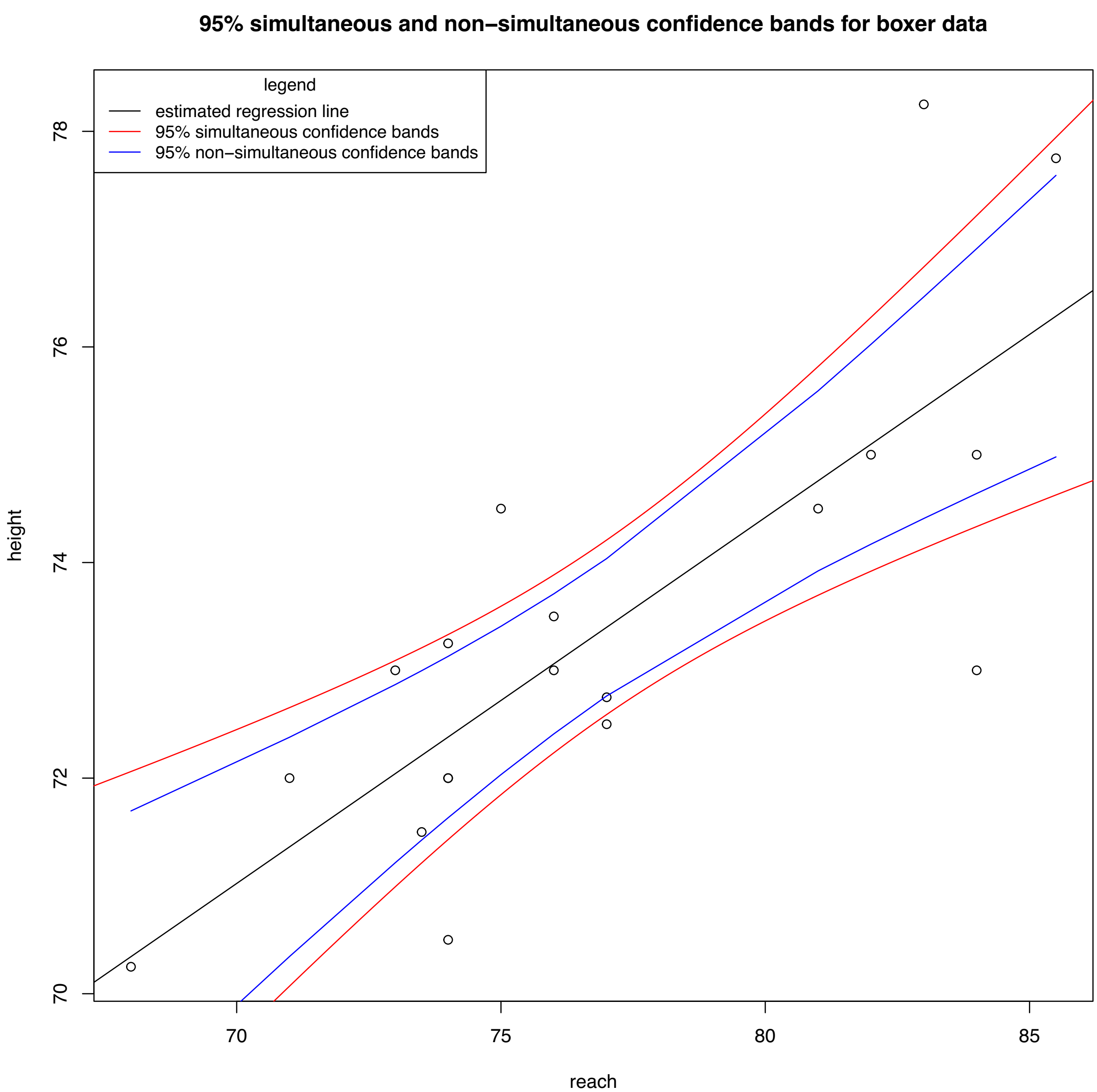
Question1:

```
rm(list = ls())

n = 1000
data <- read.csv("boxers.csv", header = TRUE, sep = ",")

poi <- seq(65, 90, length.out = 1000)

x = data$reach
y = data$height

model = lm(y ~ x)
predictions <- predict(model, interval = "confidence", level = 0.95)

row1 <- rep(1, 19)

X1 <- rbind(row1, data$reach)
X1 = t(X1)

L <- solve(t(X1) %*% X1)

y_hat = fitted(model)
e = y - y_hat
s = sqrt(sum(e^2) / (19 - 2))
s_y = rep(0,1000)
for (i in 1:1000) {
  xx = matrix(c(1,poi[i]),nrow = 2)
  value <- qf(0.95, df1=2, df2=17, lower.tail=TRUE)
  s_y[i] = s * sqrt(2 * t(xx) %*% L %*% xx * value)
}
newy = rep(0,1000)
for (i in 1:1000) {
  newy[i] = model$coefficients[2]*poi[i] + model$coefficients[1]
}

y_lower = newy - s_y
y_upper = newy + s_y

plot(x, y, main = '95% simultaneous and non-simultaneous confidence bands for boxer data',
     xlab = 'reach',
     ylab = 'height')
```

```
abline(a = model$coefficients[1], b = model$coefficients[2], col='black')
lines(poi, y_lower, col='red', lty = 1)
lines(poi, y_upper, col='red', lty = 1)
lines(sort(x), sort(predictions[, 2]), col='blue', lty = 1)
lines(sort(x), sort(predictions[, 3]), col='blue', lty = 1)
legend("topleft",
       cex = 0.9,
       title = "legend",
       legend=c("estimated regression line", "95% simultaneous confidence bands", "95% non-simultaneous
       col=c("black", "red", "blue"),
       lty = c(1, 1, 1))
```



Question 2:

$y = X\beta + \varepsilon = X_1\beta_1 + X_2\beta_2 + \varepsilon$, H_0: $C\beta = (0, 1)\begin{pmatrix}\beta_1 \\ \beta_2\end{pmatrix} = \beta_2 = 0$

under H_0, LSE is $\hat{\beta}_r$, under H_1, LSE is $\hat{\beta}_1$

$SSE_0 - SSE_1 = SSR_1 - SSR_0 = (\hat{\beta} - \hat{\beta}_r)' X'y$

$SSH = (C\hat{\beta})' [C(X'X)^{-1}C']^{-1} C\hat{\beta} = (C\hat{\beta})' [C(X'X)^{-1}C'] C(X'X)^{-1} X'y$

So prove $SSE_0 - SSE_1 = SSH$ is equal to prove $\hat{\beta}_r = \hat{\beta} - (X^TX)^{-1}C^T[C(X^TX)^{-1}C']^{-1}C\hat{\beta}$

using Lagrange multipliers to minimize,

$f(\beta, \lambda) = (y - X\beta)^T(y - X\beta) + 2\lambda^T(C\beta - 0)$

$$\begin{cases} \partial f / \partial \beta = 2X'X\beta - 2X^Ty + 2C^T\lambda = 0 \\ \partial f / \partial \lambda = C\beta = 0 \end{cases} \Rightarrow \begin{cases} X'X\beta + C'\lambda = X'y \\ C\beta = 0 \end{cases}$$

We know that $\hat{\beta} = (X'X)^{-1}X'y$ $\Rightarrow$ $\begin{cases} \beta = \hat{\beta} - (X'X)^{-1}C'\lambda \\ C\hat{\beta} - C(X'X)^{-1}C'\lambda = 0 \end{cases}$

$$\Rightarrow \begin{cases} \lambda = [C(X'X)^{-1}C^T]^{-1}C\hat{\beta} \\ \hat{\beta}_r = \hat{\beta} - (X'X)^{-1}C'[C(X^TX^{-1})C^T]^{-1}C\hat{\beta} \end{cases}$$

$\square$