STAT 610: Discussion 0

1 Some useful formula

$$n! \sim \sqrt{2\pi n}(\frac{n}{e})^n, \quad (1+\frac{x}{n})^n \longrightarrow e^x, \quad e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\frac{d}{d\theta}\int_{a(\theta)}^{b(\theta)} f(x,\theta)dx =$$

2 Review of Probability

- $(\Omega, \mathscr{F}, \mu)$ (omitted)
- cdf, pdf, pmf (joint and marginal)
  Given pdf $f(x,y)$,
  - $\mathbb{E}$[g(X,Y)]=
  - $\mathbb{E}$[h(X)]=
- Expectation, Variance, Moments, (mgf, chf, pgf)
  - $\mathbb{E}X =$
  - Tail sum formula:
  - $\mathrm{Var}X =$
  - $\mathbb{E}X^k =$
- Independence and Correlation
  - $\mathrm{Cov}(X,Y) =$
  - $\mathrm{Var}(\sum_{i=1}^n X_i) =$
  - uncorrelated but dependent example:
- Conditional dist, prob, expectation (omitted)
- Transformation of RVs: Jacobian and range (see Q3 as an example)
- Multivariate Normal and $\chi^2,\ t,\ F$ dist (omitted)
- LLN (Law of Large Numbers): For iid sequences $X_1, \cdots, X_n$ with finite mean (i,e, $\mathbb{E}|X_1| < \infty$), then

$$\overline{X}_n \xrightarrow{d/a.s.} \mu$$

- CLT (Central Limit Thm): For iid sequences $X_1, \cdots, X_n$, if $\sigma^2 \in (0,\infty)$, we have

$$\sqrt{n}\frac{\overline{X}_n - \mu}{\sigma} \xrightarrow{d} \mathcal{N}(0,1).$$

3 Order Statistics

Observe sequence $X_1, X_2, \cdots, X_n$, rearrange them as $X_{(1)} \le X_{(2)} \le \cdots \le X_{(n)}$.

- Marginal cdf:

$$F_{X_{(j)}}(x) = \mathbf{P}(X_{(j)} \le x) = \mathbf{P}(\text{at least } j \text{ out of } n \text{ values} \le x)$$
$$= \sum_{k=j}^{n} \binom{n}{k} [F(x)]^k [1 - F(x)]^{n-k}, \qquad j = 1, 2, \cdots, n.$$

- Marginal pdf: $f_{X_{(j)}}(x) = \frac{n!}{(j-1)!(n-j)!} f(x)[F(x)]^{j-1}[1 - F(x)]^{n-j}$.

- Note: $\begin{cases} \sum_{i=1}^n X_i = \sum_{i=1}^n X_{(i)}, \quad \sum_{i=1}^n X_i^2 = \sum_{i=1}^n X_{(i)}^2, \quad \cdots \\ \prod_{i=1}^n X_i = \prod_{i=1}^n X_{(i)}, \quad \prod_{i=1}^n X_i^2 = \prod_{i=1}^n X_{(i)}^2, \quad \cdots \end{cases}$

  Q: $f_{X_1}(x_1) \cdots f_{X_n}(x_n) \stackrel{?}{=} f_{X_{(1)}}(x_{(1)}) \cdots f_{X_{(n)}}(x_{(n)})$?

4 Questions

1. $\mathbf{P}(X_{(n)} > x) =$

2. (joint pdf) For $1 \le i1 < i2 < \cdots < ik \le n$, assume that there is no tie (which is natural for continuous dist), then
   $f(x_{(i1)}, \cdots, x_{(ik)}) =$

3. (Range) Let $X_1, \cdots, X_n \stackrel{iid}{\sim} U[0, a]$, let $R = X_{(n)} - X_{(1)}$, then $f_R(r) =$?

4. $X_1, \cdots, X_n \stackrel{iid}{\sim} U[0, 1]$, $\frac{X_1}{X_{(1)}}$ cdf?