Poor pigs

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Problem Statement

Given integer n, t_1, t_2, number of buckets, among which exactly one bucket contains poison; time taken for poison to kill a pig, and total time allowed. Find the minimum number of pigs needed to find the bucket containing poison within total time allowed.

Problem Constraints

Concept Proof

Theorem 0.1. *p pigs and q attempts can find the poisonous bucket among* $(q + 1)^p$ *buckets.*

Proof. Proof by Induction:

Induction over attempts

Base case: One attempt allowed $\implies 2^p$ buckets to be searched. Encode each bucket in binary, where at digit i a 0 represents it not being consumed by pig i, 1 otherwise. It is clear that for every possible poisonous bucket, there exists a unique pattern of dead pigs, thus sufficient to identify the bucket.
Inductive step: Suppose with q attempts, $(q + 1)^p$ buckets can be searched. $\qquad\square$

Corollary 0.2. *To find the poisonous bucket among n buckets, in x attempts,* $\lceil log_x(n) \rceil$ *pigs are needed.*

Solution

Easy