

## 1 Overview

- Week 1-3: Classical AI, search algorithms
  1. Uninformed search
  2. Local search: hill climbing
  3. Informaed search: A\*
  4. Adversarial search Minimax
- Week 4-7: Classical ML
  1. Decision trees
  2. Linear/Logistic regression
  3. Kernels and support vector machines
  4. "Classical" unsuperivese learning
- Week 10-12: Modern ML
  1. Neural networks
  2. Deep learning
  3. Sequential data
- Week 13: Misc.

## 2 AI: Computers Trying to Behave Like Humans

- **PEAS Framework:**
  - **Performance measure:** define “goodness” of a solution
  - **Environment:** define what the agent can and cannot do
  - **Actuators:** outputs
  - **Sensors:** inputs
- Agent function is sufficient.
- Common agent structures (to define an AI agent):
  - Reflex
  - Goal-based
  - Utility-based
  - Learning
  - (Others possible; can mix and match!)
- Exploration vs exploitation

### 3 Problem Statement

fully observable  $\wedge$  deterministic  $\wedge$  static  $\wedge$  discrete  $\implies$  only need to observe once

To solve a prob using search:

- A goal or a set of goals
- a model of the enironment
- a search algorithm

goal formulation -> problem formulation -> search -> execute

1. goal formulation

2. problem formulation, eg. path finding

- states: nodes representation invariant:: abstract states should correspond to concrete states
- initial state: starting node
- goal states/test: dest node  
Goal test: define the goal using a function *is\_goal*
- actions: move along an edge ::  $|actions(state)| \leq branching\_factor$
- transition model:  $f(curr\_state, action) \implies next\_state$
- action cost function: see edges

3. Important facts:

- Representation Invariant: ensure that the abstract states correspond to concrete states
- Goal Test: Goal defined via a function *is\_goal*
- Action: a set of  $action(state)$ ,  $|actions(state)| \leq branching\_factor$
- Transition model:  $f(curr\_state, action) \implies next\_state$

## Search

### Uninformed search

No information that could guide the seaech: no clue how good a state is

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```
create frontier
// create visited // with vsited memory
insert Node(initial_state) to frontier
while frontier is not empty:
    node = frontier.pop()
    if node.state is goal:
        return solution
// if node.state in visited: // with vsited memory
//     continue
// visited.add(state)
for action in actions(node.state):
    next_state = transition(node.state, action)
    frontier.add(Node(next_state))
return failure
```

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Different subvariant of tree search uses differen DS for the frontier.

Search Type	Data Structure for Frontier
BFS	Queue
DFS	Stack
UCS (Uniform-cost Search)	Priority Queue

## Depth limited search

limit the search to depth  $l$

backtrack when the limit is hit.

time complexity: exponential to search depth

space complexity: size of the frontier

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```
create frontier
tier = 0
insert Node(initial_state) to frontier
while (!empty(frontier)) && (tier <= limit):
    node = frontier.pop()
    tier++
    if node.state is goal:
        return solution
    for action in actions(node.state):
        next_state = transition(node.state, action)
        frontier.add(Node(next_state))
return failure
```

---

## Iterative deepening search

search with depth from 0 to  $\infty$

return soln when found. Both complete

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```
create frontier
tier = 0
insert Node(initial_state) to frontier
while (!empty(frontier)) && (tier <= limit):
    node = frontier.pop()
    tier++
    if node.state is goal:
        return solution
    for action in actions(node.state):
        next_state = transition(node.state, action)
        frontier.add(Node(next_state))
return failure
```

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## Summary

Name	Time Complexity*	Space Complexity*	Complete?	Optimal?
Breadth-first Search	Exponential	Exponential	Yes	Yes
Uniform-cost Search	Exponential	Exponential	Yes	Yes
Depth-first Search	Exponential	Polynomial	No#	No
Depth-limited Search	Exponential	Polynomial**	No**	No**
Iterative Deepening Search	Exponential	Exponential**	Yes	Yes

# Not complete if not tracking visited nodes, search may stuck in loop before visiting all nodes.

\* In terms of some notion of depth/tier

\*\* If used with DFS

## 4 Local Search

Systematic search: typically complete and optimal under certain constraints. However intractable sometimes  
Local search: typically incomplete and suboptimal, but has anytime property, ie. longer time -> better solution. Able to provide good enough solution under reasonable amount of time.

### 4.1

1. Start at random position in the state space
2. iteratively move from a state to another neighbouring state via perturbation or construction
3. solution is the final state

State space: all possible configuration (1)

Search space: a subset of state space that will be explored (2)

#### 4.1.1 Perturbation search

- Search space: complete candidate solutions
- search step: modification of one or more solution

For example: swap a path with another path

#### 4.1.2 Constructive search

- partial candidate solution
- extension of one or more solution

For example: path finding

### 4.2

goal formulation -> problem formulation -> search -> execute

1. goal formulation
2. problem formulation, eg. path finding
  - states: nodes representation invariant:: abstract states MAYNOT directly correspond to concrete states
  - initial state: starting node, a candidate solution
  - goal states/test: dest node [optional]  
Goal test: define the goal using a function  $is\_goal\ f(curr\_state, action) \implies next\_state$
  - Successor function: a function that generates neighbouring states by applying modifications from the current state. This defines the local search space

### 4.3 Evaluation function

A math function that assess the quality or desirability of the solution.

Some solutions may be unacceptable but there are some less bad than the others.

## 4.4 Hill Climbing/ Greedy Local Search

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```
curr_state = init_state
while 1:
    best_succ = best(successor(curr_state))
    if (eval(best) <= eval(curr_state))
        return curr_state
    curr_state = best_succ
```

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## 4.5 State space landscape

- Global max:
- Local max:
- shoulder:

# 5 Adversarial search

## 5.1 Classical adversarial games

- Fully observable
- Deterministic
- discrete
- No infinite run
- 2-player zero-sum
- turn taking

### terms

- Player: agent
- Turn:
- Move
- End state
- winning conditon
- 

## 5.2 Problem formulation in adversarial search

- states: nodes representation invariant:: abstract states MAYNOT directly correspond to concrete states
- initial state: starting node, a candidate solution
- Terminal State: state the outcome of the game when it terminates
- Utility function: output the value of a state from the perspetive of our agent

## 5.3 Minimax

In the view of A, A try to maximize the outcome of the game, B will try to minimize A's outcome, as the gam is zero sum

- $\text{expand}(\text{state}) \Rightarrow [a]$

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```
max_value(state):
    if is_terminal(state): return utility(state)
    v = -∞
    for next_state in expand(state):
        v = max(v, min value for player A in next_state)
    return v

min_value(state):
    if is_terminal(state): return utility(state)
    v = ∞
    for next_state in expand(state):
        v = min(v, max value for player A in next_state)
    return v

minimax(state):
    v = max_value(state)
    return action in expand(state) with value v
```

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## 5.4 Alpha-beta prunning

From the viewpoint of the MAX player:

- $\alpha$ : the value of the best choice for the MAX player so far
- $\beta$ : the value of the best choice for the MIN player so far

An optimized version of the Minimax algorithm using pruning:

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```
max_value(state,  $\alpha$ ,  $\beta$ ):
    if is_terminal(state): return utility(state)
    v = -∞
    for next_state in expand(state):
        v = max(v, min( $\alpha$ ,  $\beta$ ))
    return v

min_value(state,  $\alpha$ ,  $\beta$ ):
    if is_terminal(state): return utility(state)
    v = ∞
    for next_state in expand(state):
        v = min(v, max( $\alpha$ ,  $\beta$ ))
    return v

alpha-beta search(state):
    v = max_value(state, -∞, ∞) // initialized  $\alpha$  to be -∞,  $\beta$  to ∞
    return action in expand(state) with value v
```

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## 6 Learning agent

For problems that the function are difficult to specify, solutions re intractable to compute in general. Typically episodic,

$$DL \subset ML \subset AL$$

### 6.1 supervised

Learn the mapping input, feedback given  $\rightarrow$  output, given a dataset, it minimizes the difference between the prediction and the provided correct ans using a learnign algorithm e.g. image ientification

1. Train phase: try minimizing the diff between pred and correct ans given using the training set, resulting in a trainign agen function, known as the model/hypothesis
2. Testing/evaluation phase: using a test set to measure the performance of the model. The performance on unseen data measures the generalization of the model

#### Task

- Classification: to predict discrete labels or catagories on the input features
- Regression: predict continuous numetical value based on input features

#### Dataset

$$D = \bigcup_{[1,n]} \{(x^{(i)}, y^{(i)})\}$$

#### True data generation function

$$y = f^*(x) + \epsilon$$

where  $f^*(x)$  is true but unknow, which generates the label from the input features;  $\epsilon$  is some noise or error term, which account for the randomness or imperfection in the date generating process.  
the goal is to find a function that best approximately  $f^*(x)$

#### Hypothesis class

THE set of models or functions that maps from inputs to outputs  $h : X \implies Y$  that can be learned by a learing algorithm. Each element of the hypo clas  $h \in \mathbb{H}$

#### LEarnign algorithm

$$A(D_{train}, H_{hypo}) = h(x), h \in \mathbb{H} \approx f^*(x)$$

#### PERformance measure

$$h(x) \approx f^*(x)$$

$$PM(D_{test}, h \in H_{hypo}) \mapsto$$

Try the hypothesis h on a new set of examples (test data)

## Regression: error

$$\text{Absolute error} = |\hat{y} - y| \quad (3)$$

$$\text{Squared error} = (\hat{y} - y)^2 \quad (4)$$

Mean absolute error

$$MAE = \dots$$

where

$$\hat{y}^{(i)} = h(x^{(i)})$$

Accuracy:

$$A = \frac{1}{N} \sum_{i=1}^n \mathbb{1}_{\hat{y}^{(i)}=y^{(i)}}$$

## confusion matrix

True positive, false positive, false negative, true negative (2, 1, 3, 4 quadrant)

$$\text{Accuracy} = \frac{TP + TN}{TP + FN + FP + TN}$$

$$\text{Precision} = \frac{TP}{(TP + FP)}$$

maximize if FP is costly

$$\text{Recall} = \frac{TP}{(TP + FN)}$$

maximize recall if FN is dangerous

$$F1 = \frac{2}{\frac{1}{P} + \frac{1}{R}}$$

## 6.2 Decision Tree

Greedy, top-down, recursive, use

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```
DTL(examples, attributes, default):
    if (examples == ∅): return default
    if (∀e ∈ examples, e has the same classification c): return c
    if (attributes == ∅): return mode(examples)

    best = choose_attribute(attributes, examples)
    tree = new decision tree with root test best
    for vi of best do:
        examplesi = {e | e ∈ examples, e.best = vi}
        subtree = DTL(examplesi, attributes \ best, mode(examplesi))
        tree.add(vi: subtree)
```

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$$f : [\text{attribute vector}] \mapsto \text{Boolean}$$

Basically nested if-else

## Expressiveness

Decision trees can express any function of the input attributes. Trivially, Consistent training set  $\rightarrow$  Consistent decision tree, but unlikely to generalize to new examples

Each row in the truth table is represented as a path in the decision tree



### Size of the hypothesis class

For  $n$  boolean attributes, there are  $n$  boolean func,  $n$  distinct truth tables each with  $2^n$  rows  $\rightarrow 2^{2^n}$  decision trees

### Informativeness

Ideally we want to select an attribute that splits the examples into all positive or all negative.

Entropy: The higher the more random, thus less informative

$$I(P(v_1) \dots P(v_k)) = - \sum_{i=1}^k P(v_i) \log_2 P(v_i)$$

For data set contains boolean outputs,

$$I(P(+), P(-)) = - \frac{p}{p+n} \log_2 \frac{p}{p+n} - \frac{n}{p+n} \log_2 \frac{n}{p+n}$$

### Information gain

Information gain = entropy of this node - entropy of children nodes

$$IG(A) = I\left(\frac{p}{p+n}, \frac{n}{p+n}\right) - remainder(A)$$

$$remainder(A) = \sum_{i=1}^v \frac{p_i + n_i}{p+n} I\left(\frac{p_i}{p_i + n_i}, \frac{n_i}{p_i + n_i}\right)$$

### Decision tree pruning

- By sample size
- by max search depth

## 6.3 unsupervised

Find pattern. No feedback given. e.g, group images by char

## 6.4 Reinforcement

Trial and error, reward given based on observation and action. eg. chess