Consistency and Admissibility Proof for Total Manhattan Distance Heuristic for Cube problem

Wang Xiyu

Simple Manhattan distance heuristic where onlt horizontal and vertical moves are allowed is trivial;

each move costs 1, and each horizontal move will change the horizontal index by 1, so do vertical moves
1 2 3 4 4 1 2 3 2 3 4 1

the largest possible manhatten diatance between a pair is $\lceil\frac{r}{2}\rceil + \lceil\frac{c}{2}\rceil$, therefore optimistically the max cost of one swap will be $\lceil\frac{r}{2}\rceil + \lceil\frac{c}{2}\rceil$. A generous upper bound to the total manhatten distance from the goal state will be $\frac{r \times c}{2}(\lceil\frac{r}{2}\rceil + \lceil\frac{c}{2}\rceil)$