$$
\begin{aligned}
n \log n &= \mathcal{O}(\log(n!)) \\
&= \mathcal{O}\left(\frac{n}{2} \log n - \frac{n}{2} \log 2\right) \\
&= \mathcal{O}\left(\frac{n}{2} \log \frac{n}{2}\right) \\
&= \mathcal{O}\left(\sum_{i=0}^{\frac{n}{2}} \log\left(\frac{n}{2} + i\right)\right) \\
&= \mathcal{O}\left(\sum_{i=\frac{n}{2}}^{n} \log i\right) \\
&= \mathcal{O}\left(\sum_{i=1}^{n} \log i - \sum_{i=1}^{\frac{n}{2}-1} \log i\right) \\
&= \mathcal{O}\left(\log(n!) - \log\left(\left(\frac{n}{2} - 1\right)!\right)\right) \\
&= \mathcal{O}\left(\log\left(\frac{n!}{(\frac{n}{2} - 1)!}\right)\right) \\
&= \mathcal{O}\left(\log \prod_{i=\frac{n}{2}}^{n} i\right) \\
&\text{(sloppily...)} \\
&= \mathcal{O}(\log(n!))
\end{aligned}
$$