

Defining a Language from a DFA

To define a language from a given DFA (Deterministic Finite Automaton), you describe the set of strings that the DFA accepts. Here's a step-by-step guide:

1. Describe the DFA

A DFA is typically defined by a 5-tuple $M = (Q, \Sigma, \delta, q_0, F)$, where:

- Q : A finite set of states.
- Σ : The input alphabet (a finite set of symbols).
- δ : The transition function $\delta : Q \times \Sigma \rightarrow Q$, which determines the state transitions based on the current state and input symbol.
- q_0 : The start state, where $q_0 \in Q$.
- F : The set of accept (final) states, where $F \subseteq Q$.

2. Define the Language $L(M)$

The language recognized (accepted) by the DFA M , denoted as $L(M)$, is the set of all strings over the alphabet Σ that the DFA accepts.

Formally, $L(M)$ is defined as:

$$L(M) = \{w \in \Sigma^* \mid \hat{\delta}(q_0, w) \in F\}$$

where:

- w is a string over the alphabet Σ .
- Σ^* is the set of all strings (including the empty string) over the alphabet Σ .
- $\hat{\delta}$ is the extended transition function, which is recursively defined as:

$$\begin{aligned}\hat{\delta}(q, \epsilon) &= q && \text{(for the empty string } \epsilon) \\ \hat{\delta}(q, aw) &= \delta(\hat{\delta}(q, a), w) && \text{(for } a \in \Sigma \text{ and } w \in \Sigma^*)\end{aligned}$$

3. Interpretation

- The language $L(M)$ consists of all strings that, when processed by the DFA starting from the start state q_0 , lead to one of the accept states in F .
- If a string w is in $L(M)$, the DFA can process w and end in an accept state.
- If a string w is not in $L(M)$, the DFA either ends in a non-accepting state or cannot process the string according to its transition rules.

Example

Consider a DFA $M = (Q, \Sigma, \delta, q_0, F)$ with:

- $Q = \{q_0, q_1, q_2\}$
- $\Sigma = \{0, 1\}$
- δ defined as:

$$\delta(q_0, 0) = q_0$$

$$\delta(q_0, 1) = q_1$$

$$\delta(q_1, 0) = q_2$$

$$\delta(q_1, 1) = q_0$$

$$\delta(q_2, 0) = q_1$$

$$\delta(q_2, 1) = q_2$$

- q_0 is the start state.
- $F = \{q_2\}$.

Language $L(M)$:

- The DFA accepts a string if it ends in state q_2 .
- $L(M)$ is the set of strings over $\{0, 1\}$ that result in the DFA being in state q_2 after reading the entire string.

Conclusion

To define a language from a DFA, you describe the set of all strings that the DFA accepts. This set is the language recognized by the DFA, and it is formally defined using the extended transition function $\hat{\delta}$ that describes how the DFA processes each string.

Finite Automata: Language Accepted by DFA

Contents

Here we are going to formally define what is meant by a DFA (Deterministic Finite Automaton) accepting a string or a language.

Acceptance of a String by a DFA

A string w is accepted by a DFA $M = \langle Q, \Sigma, q_0, \delta, A \rangle$ if and only if $\hat{\delta}(q_0, w) \in A$. That is, a string is accepted by a DFA if and only if the DFA starting at the initial state ends in an accepting state after reading the string.

Acceptance of a Language by a DFA

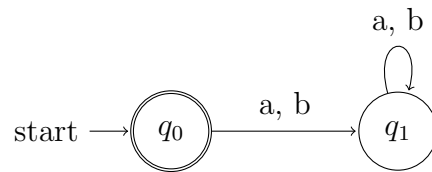
A language L is accepted by a DFA $M = \langle Q, \Sigma, q_0, \delta, A \rangle$ if and only if

$$L = \{w \mid \hat{\delta}(q_0, w) \in A\}$$

That is, the language accepted by a DFA is the set of strings accepted by the DFA.

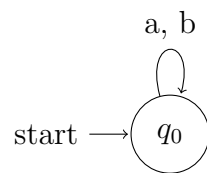
Examples

Example 1



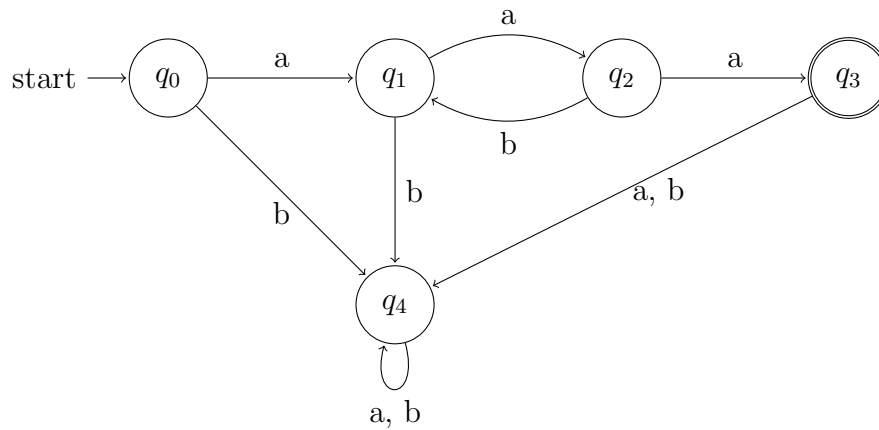
This DFA accepts $\{\epsilon\}$ because it can go from the initial state to the accepting state (also the initial state) without reading any symbol of the alphabet, i.e., by reading an empty string ϵ . It accepts nothing else because any non-empty symbol would take it to state 1, which is not an accepting state, and it stays there.

Example 2



This DFA does not accept any string because it has no accepting state. Thus, the language it accepts is the empty set \emptyset .

Example 3: DFA with One Cycle



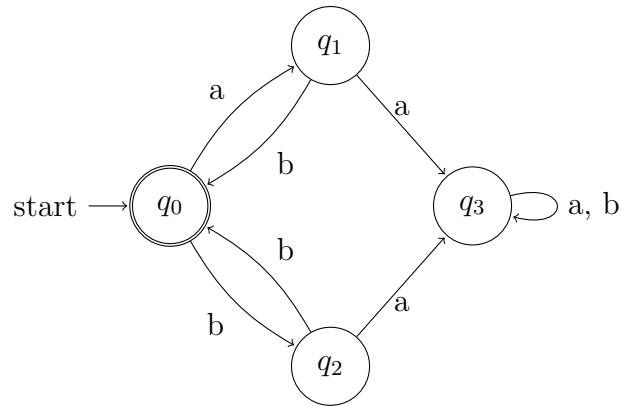
This DFA has a cycle: 1 - 2 - 1, and it can go through this cycle any number of times by reading the substring "ab" repeatedly.

To find the language it accepts:

- First, from the initial state go to state 1 by reading one "a".
- Then from state 1, go through the cycle 1 - 2 - 1 any number of times by reading the substring "ab" any number of times to come back to state 1. This is represented by $(ab)^*$.
- Finally, from state 1 go to state 2 and then to state 3 by reading "aa".

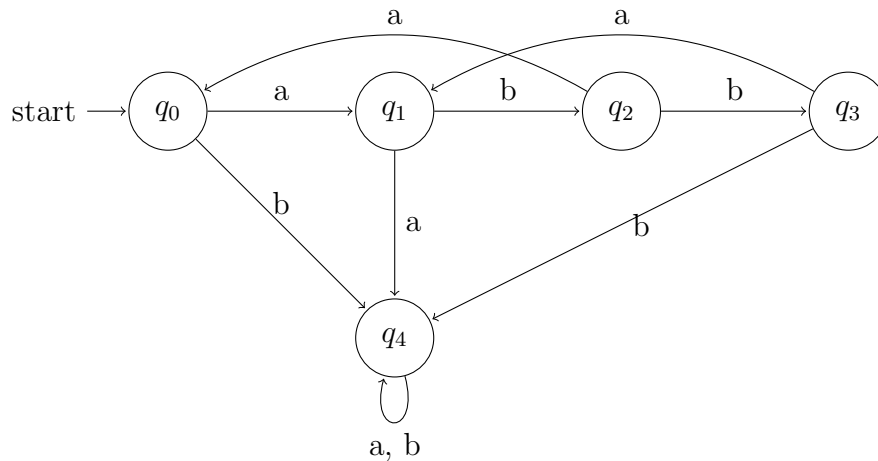
Thus, a string that is accepted by this DFA can be represented by $a(ab)^*aa$.

Example 4: DFA with Two Independent Cycles



This DFA has two independent cycles: $0 - 1 - 0$ and $0 - 2 - 0$, and it can move through these cycles any number of times in any order to reach the accepting state from the initial state, such as $0 - 1 - 0 - 2 - 0 - 2 - 0$. Thus, a string that is accepted by this DFA can be represented by $(ab + bb)^*$.

Example 5: DFA with Two Interleaved Cycles



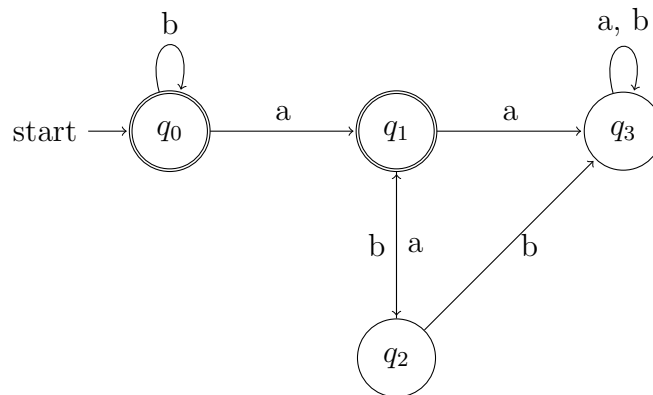
This DFA has two cycles: 1 - 2 - 0 - 1 and 1 - 2 - 3 - 1.

To find the language accepted by this DFA:

- First, from state 0 go to state 1 by reading "a". (Any other state which is common to these cycles such as state 2 can also be used instead of state 1).
- Then from state 1, go through the two cycles 1 - 2 - 0 - 1 and 1 - 2 - 3 - 1 any number of times in any order by reading substrings "baa" and "bba", respectively. At this point, a substring $a(baa + bba)^*$ will have been read.
- Then go from state 1 to state 2 and then to state 3 by reading "bb".

Thus, altogether $a(baa + bba)^*bb$ will have been read when state 3 is reached from state 0.

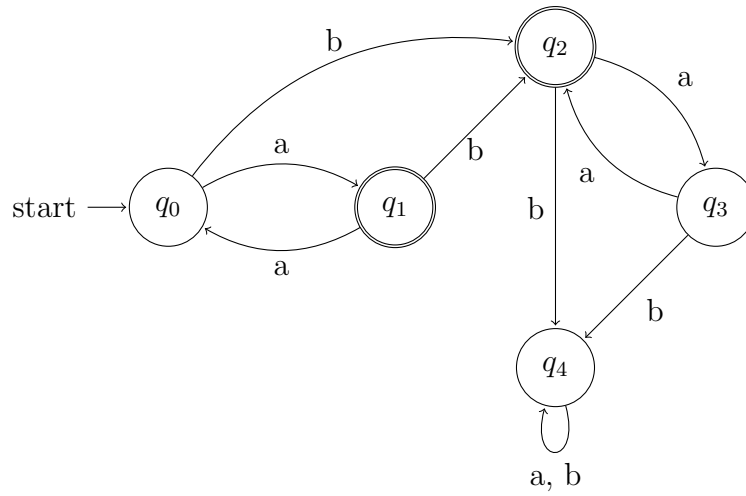
Example 6



This DFA has two accepting states: 0 and 1. Thus, the language that is accepted by this DFA is the union of the language accepted at state 0 and the one accepted at state 1.

- The language accepted at state 0 is b^* .
- To find the language accepted at state 1, first at state 0 read any number of "b"s. Then go to state 1 by reading one "a". At this point (b^*a) will have been read. At state 1, go through the cycle 1 - 2 - 1 any number of times by reading the substring "ba" repeatedly. Thus, the language accepted at state 1 is $b^*a(ba)^*$.

More examples



This DFA contains 2 accepting states, suggesting the union of the language accepted at state 1 and state 2.

This DFA also contains 2 separated cycles,