Pre Quiz

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(a), and (b) if "is friends" a reflexive relation.

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Cannot conclude

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$$(\exists f \in F, f(CS4269)) \wedge (TY \in F)$$

a) $TY(CS4269)$ cannot conclude
b) $\forall f \in F, f(CS4269)$ cannot conclude

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$$\forall n \in N, \exists r \in R, r = \sqrt{n}$$

$5.5 \notin N$, cannot conclude $\exists r \in R, r = \sqrt{5.5}$
$5 \in N \implies \exists r \in R, r = \sqrt{5}$

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A set is a collection of unique elements. like $\mathbb{N}$. The cardinality of a set describes its size. like $|\mathbb{N}| = \aleph_0$

$$\exists \text{ injection } f : A \mapsto B, |A| \leq |B|$$

$$\exists \text{ surjection } f : A \mapsto B, |A| \geq |B|$$

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A finite set is a set that contains, well finite amount of element. For example $\emptyset$
A countable set is either finite or has the same cardinality as $\mathbb{N}$. for example $\mathbb{Z}^+$
An uncountable set is a set that has the same cardinality as $\mathbb{R}$. for example $\mathcal{P}(\mathbb{N})$

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A function is a mapping from a set to another set. $f : D_f \mapsto R_f$, where $R_f \subseteq CoDomain_f$. Codomain is the set where the elements a function can possibly maps to, while range is the set of elements which the function actually maps to.

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The power set of S is a set of all subsets of S, which has cardinality $2^{|S|}$ For example $f : \emptyset \mapsto \{\emptyset\}$

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Base case:

$$\sum_{k=0}^{0} k^2 = \frac{0(0+1)(2 \times 0+1)}{6} = 0$$

Inductive case: Suppose

$$\sum_{k=0}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\begin{aligned}
\sum_{k=0}^{n+1} k^2 &= \frac{n(n+1)(2n+1)}{6} + (n+1)^2 \\
&= \frac{1}{6}(2n^3 + 3n^2 + n + 6n^2 + 12n + 6) \\
&= \frac{1}{6}(2n^3 + 9n^2 + 13n + 6) \\
&= \frac{1}{6}((n+1)(2n^2 + 7n + 6)) \\
&= \frac{1}{6}(n+1)(n+2)(2n+3)
\end{aligned}$$

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Base case: $h = 1, |V| = 2^1 - 1 = 1$ Inductive steps: strong induction:
Suppose a CBST of height h has $2^h - 1$ nodes, and a CBST with height $h-1$ has $2^{h-1} - 1$ nodes, The number of leaves $= 2^h - 1 - (2^{h-1} - 1) = 2^{h-1}$, to add a new level each leaf grows 2 new leave, total $2 \cdot 2^{h-1} = 2^h$
Thus a CBST of height $h+1$ has $2^h - 1 + 2^h = 2^{h+1} - 1$

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A string w over an alphabet Σ is a finitely long sequence of characters from Σ. A langauge is all strings formed from an alphabet based on certain rules. Σ^* is a regular language that consists of all finitely long sequence from Σ.

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A regular language: the empty language $\emptyset$. proof: trivial, a DFA that accepts nothing.
A context free language: valid parentheses, rules:

$$\begin{aligned}
S &\to \epsilon \\
S &\to SS \\
S &\to (S)
\end{aligned}$$

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A turing machine is a state machine defined as such: $\{Q, \Gamma, \Sigma, \delta, b, s, f\}$ each represents: the set of states, alphabet symbols, tape sympols, transition rules $(Q - F \times \Gamma) \mapsto (Q \times \Gamma \times \{L, R\})$, blank symbol, starting and final states. Languages can be either accepted or rejected by a Turing machine is called recursive. Languages that may be accepted but not guarenteed to terminate on a Turing machine is recursively enumerable.

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Decideable means always terminate of a Turing machine, either accepted or rejected in finite amount of time. For example $\emptyset$, as regular languages are decidable.

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Post's correspondence problem is undecidable.

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One turing machine terminate on a string in time polynomial wrt input size. Meaning the language is in complexity class P

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NP complexity class consists of languages that can be accepted or rejected a non-deterministic turing machine in polynomial time wrt input size.