

Basic Properties of Numbers [Triangle Inequality] For all numbers a and b , we have

We know that for all $x \geq 0$, $x = x$ and for all $x < 0$, $x < x$, hence $x \leq x$. And given a real number x , $x = \sqrt{x^2}$. Then

Since for all $x^2 \leq y^2$, $x \leq y$, as long as both x and y are nonnegative.

Numbers of Various Sorts [The Principle of Mathematical Induction] Let $n, N \in \mathbb{Z}$, and for all $n \geq N$, let $P(n)$ be a statement. If $P(N)$ is true, and for all $k \geq N$, $P(k) \Rightarrow P(k+1)$ is true, then for all $n \geq N$, $P(n)$ is true. [The Well-ordering Principle] Every nonempty set of positive integers has a least element.

If $F \neq \emptyset$, then by the Well-ordering principle, F contains a least element, say l . Since $P(N)$ is true, we know $l > N$, and

Graphs The number $a-b$ is the distance between a and b . Then the set of numbers x which satisfy $x-a < \varepsilon$ may be pic
there exist $\varepsilon > 0$ such that for all $\delta > 0$ there exist x which satisfies $0 < x-a < \delta$ but not $f(x)-l < \varepsilon$. A function can

We simply choose $\delta = \min(\delta_1, \delta_2)$.

Hence the number l which f approaches near a is denoted by $\lim_{x \rightarrow a} f(x)$ is possible. Continuous Functions Three Hard Theorems