

Basic Properties of Numbers [Triangle Inequality] For all numbers a and b , we have

We know that for all $x \geq 0$, $x = \sqrt{x^2}$ and for all $x \geq 0$, $x \leq \sqrt{x^2}$, hence $x \leq \sqrt{x^2}$. And given a real number x , $x = \sqrt{x^2}$. Then

Since for all $x^2 \leq y^2$, $x \leq y$, as long as both x and y are nonnegative.

Numbers of Various Sorts [The Principle of Mathematical Induction] Let $n, N \in \mathbb{Z}$, and for all $n \geq N$, let $P(n)$ be a statement. If $P(N)$ is true, and for all $k \geq N$, $P(k) \Rightarrow P(k+1)$ is true then for all $n \geq N$, $P(n)$ is true. [The Well-ordering Principle] Every nonempty set of natural numbers has a least element. Let

If $F \neq \emptyset$, then by the Well-Ordering Principle, F contains a least element, say l . Since $P(N)$ is true, we know $l \geq N$, and

Graphs The number $a-b$ is the distance between a and b . Then the set of numbers x which satisfy $x - a < \varepsilon$ may be pictured as an interval. There exist $\varepsilon > 0$ such that for all $\delta > 0$ there exist x which satisfies $0 < x - a < \delta$ but not $f(x) - l < \varepsilon$. A function can be continuous at a if and only if

We simply choose $\delta = \min(\delta_1, \delta_2)$.

Hence the number l which f approaches near a is denoted by $\lim_{x \rightarrow a} f(x)$ is possible. Continuous Functions Three Hard Theorems