Group 1
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1. The grammar for the language of balanced parentheses.
   $S \rightarrow (S) \mid SS \mid \varepsilon$

   Add new start state:
   $S_0 \rightarrow S$
   $S \rightarrow (S) \mid SS \mid \varepsilon$

   Split long rules:
   $S_0 \rightarrow S$
   $S \rightarrow (Z_1 \mid SS \mid \varepsilon$
   $Z_1 \rightarrow S)$

   Eliminate ε rules:
   $S_0 \rightarrow S \mid \varepsilon$
   $S \rightarrow (Z_1 \mid SS$
   $Z_1 \rightarrow S) \mid)$

   Eliminate unit rules:
   $S_0 \rightarrow (Z_1 \mid SS \mid \varepsilon$
   $S \rightarrow (Z_1 \mid SS$
   $Z_1 \rightarrow S) \mid)$

   Remove terminals from compound rules:
   $\mathbf{S \rightarrow X_(Z_1 \mid SS \mid \varepsilon}$
   $\mathbf{S \rightarrow X_(Z_1 \mid SS}$
   $\mathbf{Z_1 \rightarrow SX_) \mid)}$
   $\mathbf{X_(\rightarrow (}$
   $\mathbf{X_) \rightarrow)}$

2. The grammar for the language $(a^i b^j c^k \mid i = j \text{ or } j = k\}$.
   $S \rightarrow A \mid B$
   $A \rightarrow Ac \mid C \mid \varepsilon$
   $C \rightarrow aCb \mid \varepsilon$

$B \rightarrow aB \mid D \mid \varepsilon$
$D \rightarrow bDc \mid \varepsilon$

Split long rules:
$S \rightarrow A \mid B$
$A \rightarrow Ac \mid C \mid \varepsilon$
$C \rightarrow aZ_1 \mid \varepsilon$
$B \rightarrow aB \mid D \mid \varepsilon$
$D \rightarrow bZ_2 \mid \varepsilon$
$Z_1 \rightarrow Cb$
$Z_2 \rightarrow Dc$

Eliminate ε rules:
$S \rightarrow A \mid B \mid \varepsilon$
$A \rightarrow Ac \mid C \mid c$
$C \rightarrow aZ_1$
$B \rightarrow aB \mid D \mid a$
$D \rightarrow bZ_2$
$Z_1 \rightarrow Cb \mid b$
$Z_2 \rightarrow Dc \mid c$

Eliminate unit rules:
$S \rightarrow Ac \mid aZ_1 \mid c \mid aB \mid bZ_2 \mid a \mid \varepsilon$
$A \rightarrow Ac \mid aZ1 \mid c$
$C \rightarrow aZ_1$
$B \rightarrow aB \mid bZ_2 \mid a$
$D \rightarrow bZ_2$
$Z_1 \rightarrow Cb \mid b$
$Z_2 \rightarrow Dc \mid c$

Remove terminals from compound rules:
$\mathbf{S \rightarrow AX_c \mid X_aZ_1 \mid c \mid X_aB \mid X_bZ_2 \mid a \mid \varepsilon}$
$\mathbf{A \rightarrow AX_c \mid X_aZ_1 \mid c}$
$\mathbf{C \rightarrow X_aZ_1}$
$\mathbf{B \rightarrow X_aB \mid X_bZ_2 \mid a}$
$\mathbf{D \rightarrow X_bZ_2}$
$\mathbf{Z_1 \rightarrow CX_b \mid b}$
$\mathbf{Z_2 \rightarrow DX_c \mid c}$
$\mathbf{X_a \rightarrow a}$
$\mathbf{X_b \rightarrow b}$

$\mathbf{X_c \rightarrow c}$

3. The grammar for the language $\{x_1\#x_2\#...\#x_k \mid k >= 1$, each $x_i \in \{a, b\}^*$, and for some i and j $xi = x_j^R\}$.
   $S \rightarrow LCR$
   $M \rightarrow MM \mid a \mid b \mid \#$
   $L \rightarrow M\# \mid \varepsilon$
   $R \rightarrow \#M \mid \varepsilon$
   $C \rightarrow aCa \mid bCb \mid \#L$

   Split long rules:
   $S \rightarrow LZ_1$
   $M \rightarrow MM \mid a \mid b \mid \#$
   $L \rightarrow M\# \mid \varepsilon$
   $R \rightarrow \#M \mid \varepsilon$
   $C \rightarrow aZ_2 \mid bZ_3 \mid \#L$
   $Z1 \rightarrow CR$
   $Z2 \rightarrow Ca$
   $Z3 \rightarrow Cb$

   Eliminate ε rules:
   $S \rightarrow LZ_1 \mid Z_1$
   $M \rightarrow MM \mid a \mid b \mid \#$
   $L \rightarrow M\#$
   $R \rightarrow \#M$
   $C \rightarrow aZ_2 \mid bZ_3 \mid \#L \mid \#$
   $Z_1 \rightarrow CR \mid C$
   $Z_2 \rightarrow Ca$
   $Z_3 \rightarrow Cb$

   Eliminate unit rules:
   $S \rightarrow LZ_1 \mid CR \mid aZ_2 \mid bZ_3 \mid \#L \mid \#$
   $M \rightarrow MM \mid a \mid b \mid \#$
   $L \rightarrow M\#$
   $R \rightarrow \#M$
   $C \rightarrow aZ_2 \mid bZ_3 \mid \#L \mid \#$
   $Z_1 \rightarrow CR \mid aZ_2 \mid bZ_3 \mid \#L \mid \#$
   $Z_2 \rightarrow Ca$
   $Z_3 \rightarrow Cb$

Remove terminals from compound rules:
$\mathbf{S \rightarrow LZ_1 \mid CR \mid X_aZ_2 \mid X_bZ_3 \mid X_\#L \mid \#}$
$\mathbf{M \rightarrow MM \mid a \mid b \mid \#}$
$\mathbf{L \rightarrow M\#}$
$\mathbf{R \rightarrow \#M}$
$\mathbf{C \rightarrow X_aZ_2 \mid X_bZ_3 \mid X_\#L \mid \#}$
$\mathbf{Z_1 \rightarrow CR \mid X_aZ_2 \mid X_bZ_3 \mid X_\#L \mid \#}$
$\mathbf{Z_2 \rightarrow Ca}$
$\mathbf{Z_3 \rightarrow Cb}$
$\mathbf{X_a \rightarrow a}$
$\mathbf{X_b \rightarrow b}$
$\mathbf{X_\# \rightarrow \#}$

4. The grammar given for the language in Sipser 2.13.
   $S \rightarrow TT \mid U$
   $T \rightarrow 0T \mid T0 \mid \#$
   $U \rightarrow 0U00 \mid \#$

   Split long rules:
   $S \rightarrow TT \mid U$
   $T \rightarrow 0T \mid T0 \mid \#$
   $U \rightarrow Z_1Z_2 \mid \#$
   $Z_1 \rightarrow 0U$
   $Z_2 \rightarrow 00$

   Eliminate unit rules:
   $S \rightarrow TT \mid Z_1Z_2 \mid \#$
   $T \rightarrow 0T \mid T0 \mid \#$
   $U \rightarrow Z_1Z_2 \mid \#$
   $Z_1 \rightarrow 0U$
   $Z_2 \rightarrow 00$

   Remove terminals from compound rules:
   $\mathbf{S \rightarrow TT \mid Z_1Z_2 \mid \#}$
   $\mathbf{T \rightarrow X_0T \mid TX_0 \mid \#}$
   $\mathbf{U \rightarrow Z_1Z_2 \mid \#}$
   $\mathbf{Z_1 \rightarrow X_0U}$
   $\mathbf{Z_2 \rightarrow X_0X_0}$
   $\mathbf{X_0 \rightarrow 0}$