Exam 2 Practice Solutions

1] $f(x) = x^3 - 2x^2 - 5x + 6$

1 a root of $f(x) \iff x-1$ a factor of $f(x)$

$$\begin{array}{c|cccc} 1 & 1 & -2 & -5 & 6 \\ & & 1 & -1 & -6 \\ \hline & 1 & -1 & -6 & \big| \; 0 \end{array}$$

so $f(x) = (x-1)(x^2 - x - 6)$

$x^2 - x - 6 = (x-6)(x+1)$

so $\boxed{f(x) = (x-1)(x+1)(x-6)}$

2]

$$\begin{array}{r} 3x^3 - 6x^2 + 13x - 25 \\ x^2+2x+1 \enclose{longdiv}{3x^5 + 0x^4 + 4x^3 - 5x^2 + 3x + 2} \\ -3x^5 - 6x^4 - 3x^3 \qquad\qquad\qquad\quad \\ \hline -6x^4 + x^3 - 5x^2 + 3x + 2 \\ 6x^4 + 12x^3 + 6x^2 \qquad\qquad \\ \hline 13x^3 + x^2 + 3x + 2 \\ -13x^3 - 26x^2 - 13x \qquad \\ \hline -25x^2 - 10x + 2 \\ 25x^2 + 50x + 25 \\ \hline 40x + 27 \end{array}$$

So, $\boxed{\begin{array}{l} Q(x) = 3x^3 - 6x^2 + 13x - 25 \\ R(x) = 40x + 27 \end{array}}$

3] $\log(x) - 7\log(y) + 5\log(z) = \boxed{\log\left(\frac{x z^5}{y^7}\right)}$

$\ln(x+2) + 2\ln(x) = \boxed{\ln\left(x^2(x+2)\right)}$

$\log_3(1) + 3\log_3(y) = \boxed{\log_3(y^3)}$

4] $\log(x^2 - 5x + 6) = \log((x-3)(x-2)) = \boxed{\log(x-3) + \log(x-2)}$

$\log_2(4x^3) = \log_2(4) + 3\log_2(x) = \boxed{2 + 3\log_2(x)}$

$\ln(xe^{2x}) = \ln(x) + \ln(e^{2x}) = \boxed{\ln(x) + 2x}$

5] Let $u = e^x$ then this equation becomes $2u^2 + 9u - 5 = 0$

use quadratic formula: $u = \frac{-9 \pm \sqrt{81 - 4\cdot 2\cdot(-5)}}{4}$

$= \frac{-9 \pm \sqrt{81 + 40}}{4} = \frac{-9 \pm \sqrt{121}}{4}$

$= \frac{-9 \pm 11}{4} = \frac{1}{2}$ or -5

$e^x = \frac{1}{2}$ or $\cancel{e^x = -5}$ ← negative #s not in range of e^x

so $\boxed{x = \ln\left(\frac{1}{2}\right)}$

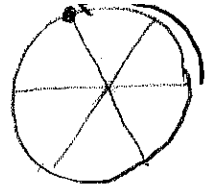
6] domain of $\log(x)$ is $(0, \infty)$ so $3x - 2 > 0$

$\Rightarrow 3x > 2 \Rightarrow x > \frac{2}{3}$ $\boxed{\text{domain} = \left(\frac{2}{3}, \infty\right)}$

7] $A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$ $P = 1200$, $r = 0.012$, $n = 12$, $t = 2$

$\boxed{1200\left(1 + \frac{0.012}{12}\right)^{12\cdot 2}}$

8] $\frac{2\pi}{3} = \frac{2}{3} \cdot \pi \rightsquigarrow$



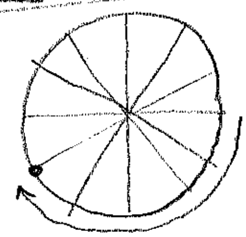
reference angle is $\frac{\pi}{3}$

$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

reflecting across y-axis does not change y-coordinate so

$$\boxed{\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}}$$

$-\frac{5\pi}{6} = -\frac{5}{6} \cdot \pi \rightsquigarrow$



reference angle is $\frac{\pi}{6}$

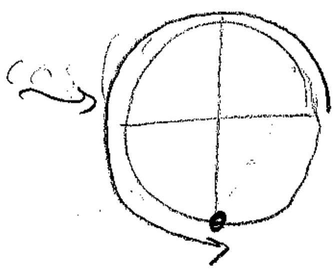
$\sin\frac{\pi}{6} = \frac{1}{2}$ $\quad \tan\frac{\pi}{6} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

Quadrant III means x and y coords are negative, but this does not change sign of tan, so

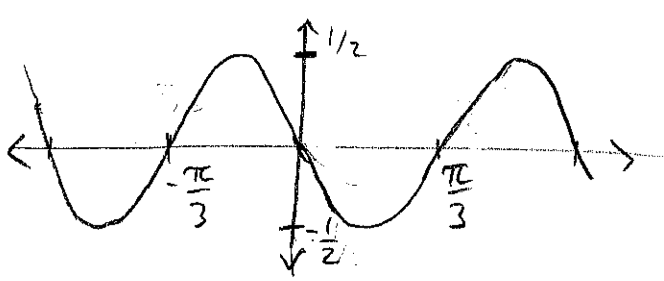
$$\boxed{\tan\left(-\frac{5\pi}{6}\right) = \frac{1}{\sqrt{3}}}$$

$\frac{7\pi}{2} = \frac{4}{2}\pi + \frac{3}{2}\pi = 2\pi + \frac{3}{2}\pi$ so $\frac{7\pi}{2}$ is coterminal to $\frac{3\pi}{2}$



$\sin\left(\frac{3\pi}{2}\right) = -1 \Rightarrow \boxed{\csc\left(\frac{3\pi}{2}\right) = \frac{1}{-1} = -1}$

9] amplitude $= \frac{1}{2}$, period $= \frac{2\pi}{3}$, horiz. shift to right by π sends $\sin\theta$ to $-\sin\theta$



10] (c)