CS-541: HW-1

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1 Question 1

1.1 A

1.1.1 Question

Suppose you repeatedly roll a fair six-sided die until you roll a 1 (and then you stop). Every time you roll a 2, you lose A points, and every time you roll a 6, you win B points. You do not win or lose any points if you roll a 3, 4, or a 5. What is the expected number of points (as a function of A and B) you will have when you stop?

1.1.2 Answer

There are four options for what can happen to one's score after a turn: they can lose A points, gain B points, remain at the same score, or end the game. The first three of these options are each weighted with a probability of $\frac{1}{6}$ while the last option is weighted with a probability of $\frac{1}{2}$. This means that the expected value of a player's score can be calculated with the following equation:

$$\mathbb{E} = (\mathbb{E} - A)(\frac{1}{6}) + (\mathbb{E} + B)(\frac{1}{6}) + (0)(\frac{1}{6}) + (\mathbb{E})(\frac{1}{2}) \tag{1}$$

Upon rearranging and isolating $\mathbb{E}$, I obtain the answer:

$$\mathbb{E} = B - A \tag{2}$$

1.2 B

1.2.1 Question

Suppose that we have a collection of customer reviews for two restaurants, i.e., Chipotle and Five Guys. Chipotle gets 200 reviews, where 120 reviews are positive while the other 80 reviews are negative. Five Guys gets 40 positive reviews and 60 negative reviews with a total of 100 reviews. Suppose we first randomly choose a restaurant (thus each restaurant would have an equal probability of being selected), then select one of its customer reviews at random. Given the result that we finally get a positive review, what is the probability that this review is about Chipotle? Solve the problem using Bayes rule.

1.2.2 Answer

The first thing I like to do for problems such as these is draw a chart:

	Chipotle	Five Guys
+	120	40
-	80	60

Now according to Bayes Rule, we need to solve the following equation using the chart above:

$$\mathbb{P}(\text{Chipotle}|+) = \frac{\mathbb{P}(+|\text{Chipotle})\mathbb{P}(\text{Chipotle})}{\mathbb{P}(+)} \tag{3}$$

So we need to calculate every term on the right hand side of the equation and plug them in to determine the left hand side.

$$\mathbb{P}(+|\text{Chipotle}) = \frac{120}{120+80} = 0.6 \tag{4}$$

$$\mathbb{P}(\text{Chipotle}) = \frac{1}{2} = 0.5 \tag{5}$$

$$\mathbb{P}(+) = \frac{120+40}{200+100} = \frac{160}{300} = 0.533 \tag{6}$$

Therefore,

$$\mathbb{P}(\text{Chipotle}|+) = \frac{0.6 * 0.5}{0.533} = 0.563 = 56.3\% \tag{7}$$

1.3 C

1.3.1 Question

Suppose the probability of a coin turning up heads is $0 < p < 1$, and that we flip it 7 times and get H, H, T, H, T, T, H. We know the probability of obtaining this sequence is:
$L(p) = pp(1-p)p(1-p)(1-p)p = p^4(1-p)^3$. What is the value of p that maximizes L(p)? What is an intuitive interpretation of this value p?
Hint: Consider taking the derivative of log L(p). You can also directly take the derivative of L(p), but its cleaner and more natural to differentiate log L(p).(No need to prove this in the solution)

1.3.2 Answer

$$L(p) = p^4(1-p)^3 \tag{8}$$

$$\log(L(p)) = 4\log(p) + 3\log(1-p) \tag{9}$$

$$\frac{\partial}{\partial p}\log(L(p)) = \frac{4}{p} - \frac{3}{1-p} \tag{10}$$

Set derivative to zero to find extreme point:

$$0 = \frac{\partial}{\partial p}\log(L(p)) \implies \frac{4}{p} = \frac{3}{1-p} \implies 4 - 4p = 3p \implies \frac{4}{7} = p \tag{11}$$

Because of the associative property, it does not matter what order one obtains the sequence HHTHTTH in, it has the same probability of showing up as any other sequence with four heads and three tails. The optimized probability $p = \frac{4}{7}$ is a reflection of this in that we want the probability of seeing a head when we flip a coin to be 4:7, which implies tails has a probability of 3:7. Even though every other state with four heads and three tails has an identical probability of appearing, they are all tied for most likely state to appear when flipping an unfair coin ($p(\text{heads}) = \frac{4}{7}$) seven times.