Wyatt Blair

CS-556 HW4

33) $A \sim \text{normal}(46.8\text{km/hr},\ 1.75\text{km/hr})$

CDF of standard normal dist   standard normal dist

a) $\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-u^2/2}\, du = \mathbb{P}(Z \le x)$

$\Rightarrow \mathbb{P}(A \le x) = \Phi\left(\frac{x-\mu}{\sigma}\right) = \Phi\left(\frac{50\text{km/hr} - 46.8\text{km/hr}}{1.75\text{km/hr}}\right) = \Phi(1.829) \approx 96.6\%$

b) $\mathbb{P}(Z \ge x) = 1 - \Phi(x)$

$\Rightarrow \mathbb{P}(A \ge 48\text{km/hr}) = 1 - \Phi\left(\frac{48\text{km/hr} - 46.8\text{km/hr}}{1.75\text{km/hr}}\right) = 1 - \Phi(0.686) \approx 24.6\%$

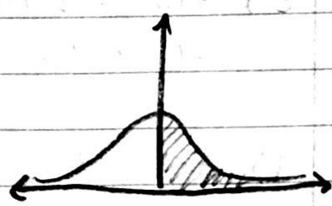
c) $\mathbb{P}(((46.8) - 1.5(1.75) \le X \le ((46.8) + 1.5(1.75))) = \mathbb{P}(44.175 \le X \le 49.425)$

$= \mathbb{P}(49.425 \ge X) - \mathbb{P}(44.175 \le X) = \Phi\left(\frac{49.425 - 46.8}{1.75}\right) - \Phi\left(\frac{44.175 - 46.8}{1.75}\right)$

$= 0.9332 - 0.0668 = 0.8664 = 86.6\%$

35) $A \sim \text{normal}(8.8, 2.8)$

a) $\mathbb{P}(A \ge x) = \Phi\left(\frac{x-\mu}{\sigma}\right)$ $\mathbb{P}(A \le x) = 1 - \Phi\left(\frac{x-\mu}{\sigma}\right)$



$\Rightarrow \mathbb{P}(A \le 10) = \Phi\left(\frac{10 - 8.8}{2.8}\right) = \Phi(0.429) \approx 66.6\%$

$\Rightarrow \mathbb{P}(A \ge 10) = 1 - \Phi(0.429) \approx 33.4\%$

Both questions are asking the same thing. So the probabilities for "will be at least 10" and "will exceed 10" are the same: 33.4%

b) $\mathbb{P}(A \ge 20) = 1 - \Phi\left(\frac{20 - 8.8}{2.8}\right) = 3.167 \times 10^{-5} = 0.003167\%$

c) $\mathbb{P}(5 \le A \le 10) = \Phi\left(\frac{10 - 8.8}{2.8}\right) - \Phi\left(\frac{5 - 8.8}{2.8}\right) = 0.666 - 0.08737 = 57.863\%$

d) Z-score associated w/ 98% $\rightarrow$ 2.1

$\Rightarrow Z_{upper} = 2.1 = \frac{8.8 + c - 8.8}{2.8} \Rightarrow c = 2.8(2.1) = 5.88$

e) $\mathbb{P}(A \ge 10) = 33.4\%$

$\Rightarrow \mathbb{P}(A \le 10) = 1 - \mathbb{P}(A \ge 10) = 66.6\%$

$\mathbb{P}(\text{no trees have } A \le 10) = (66.6\%)^4 = 19.67\%$

$\mathbb{P}(\text{at least one tree has } A \ge 10) = 1 - \mathbb{P}(\text{no trees have } A \le 10) = 0.8033 = 80.33\%$

Scanned with CamScanner

49) $A \sim norm(3432, 482)$

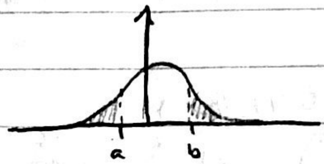
a) $\mathbb{P}(A \geq x) = 1 - \Phi\left(\frac{x-\mu}{\sigma}\right)$

$\Rightarrow \mathbb{P}(A \geq 4000) = 1 - \Phi\left(\frac{4000-3432}{482}\right) = 1 - \Phi(1.178) = 1 - 0.8807 = 0.1193 = 11.9\%$

$\mathbb{P}(a \leq A \leq b) = \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right)$

$\Rightarrow \mathbb{P}(3000 \leq A \leq 4000) = \Phi(1.178) - \Phi(-0.8962) = 0.8807 - 0.1851 = 69.6\%$

b)



$\Rightarrow \mathbb{P}(A \leq a \cap A \geq b) = \Phi\left(\frac{a-\mu}{\sigma}\right) + \left(1 - \Phi\left(\frac{b-\mu}{\sigma}\right)\right)$

$\Rightarrow \mathbb{P}(A \leq 2000 \cap A \geq 5000) = \Phi\left(\frac{2000-3432}{482}\right) + \left(1 - \Phi\left(\frac{5000-3432}{482}\right)\right)$

$= 0.001484 + (1 - 0.9994)$

$= 0.002084 = 0.2084\%$

c) 7 lbs = 3175.15 g

$\mathbb{P}(A \geq 3175.15) = -\Phi\left(\frac{3175.15-3432}{482}\right) + 1 = 1 - 0.2971 = 70.29\%$

d) z-score corresponding to 0.1% $\rightsquigarrow$ -3.09

$z_{lower} = -3.09 = \frac{x-3432}{482} \Rightarrow x = 1942.62$

$z_{upper} = 3.09 = \frac{y-3432}{482} \Rightarrow y = 4921.38$

If the baby's weight $\notin [1942.62, 4921.38]$ then the baby is in the 0.1% th percentile

e) $\Rightarrow \frac{1 \text{ lbs}}{453.592 \text{ g}} = a \qquad B = aA$

$\Rightarrow \mathbb{P}(B \geq 7) = 1 - \Phi\left(\frac{7 - a(3432)}{a(482)}\right) = 1 - 0.2971 = 70.29\%$

It's the same as part c)

Scanned with CamScanner