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2/12/23

CS-556 HW1

1)

① $2x + 3y + z = 8$
② $4x + 7y + 5z = 20$
③ $0x - 2y + 2z = 0$

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③ $\Rightarrow -2y + 2z = 0 \Rightarrow y = z$
① $\Rightarrow 2x + 4y = 8 \Rightarrow x = 4 - 2y$
② $\Rightarrow 4(4-2y) + 12y = 20 \Rightarrow 16 + 4y = 20 \Rightarrow y = 1$

$x = 4 - 2y = 4 - 2 = 2$
$y = 1$
$z = y = 1$

2)

$$\begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 3 & 1 & 1 & 3 \end{bmatrix}$$

columns: $\underset{A}{\begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}}, \underset{B}{\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}}, \underset{C}{\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}}, \underset{D}{\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}}$

rows: $\underset{E}{\begin{pmatrix} 0 \\ 1 \\ 2 \\ 1 \end{pmatrix}}, \underset{F}{\begin{pmatrix} 1 \\ 2 \\ 3 \\ 2 \end{pmatrix}}, \underset{G}{\begin{pmatrix} 3 \\ 1 \\ 1 \\ 3 \end{pmatrix}}$

$B + D = \begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix}$

$B + D - A = \begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} = C$

$\Rightarrow C = B + D - A$

$\Rightarrow$ C is linearly dependent on the other 3 columns

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$ $\Rightarrow$ all linearly independent $\Rightarrow$ rank: 3

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3) construct A s.t. column space contains $\begin{pmatrix}3\\6\\2\end{pmatrix}, \begin{pmatrix}4\\0\\1\end{pmatrix}$ & null space has $\begin{pmatrix}2\\2\\1\end{pmatrix}$

$$R = \alpha\begin{pmatrix}3\\6\\2\end{pmatrix} + \beta\begin{pmatrix}4\\0\\1\end{pmatrix} \qquad N = \gamma\begin{pmatrix}2\\2\\1\end{pmatrix}$$

$$\Rightarrow A = \begin{bmatrix}3 & 4 & a_{13}\\ 6 & 0 & a_{23}\\ 2 & 1 & a_{33}\end{bmatrix} \Rightarrow A\begin{pmatrix}2\\2\\1\end{pmatrix} = \vec{0} \Rightarrow \begin{bmatrix}3 & 4 & a_{13}\\ 6 & 0 & a_{23}\\ 2 & 1 & a_{33}\end{bmatrix}\begin{bmatrix}2\\2\\1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}14\\12\\6\end{bmatrix} + \begin{bmatrix}a_{13}\\a_{23}\\a_{33}\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix} \Rightarrow \begin{bmatrix}a_{13}\\a_{23}\\a_{33}\end{bmatrix} = \begin{bmatrix}-14\\-12\\-6\end{bmatrix} \Rightarrow A = \begin{bmatrix}3 & 4 & -14\\ 6 & 0 & -12\\ 2 & 1 & -6\end{bmatrix}$$

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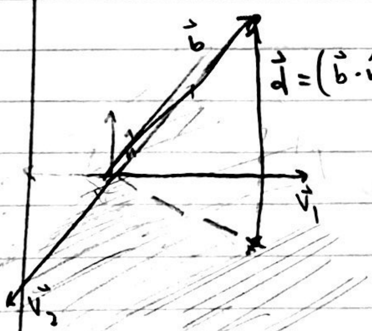
4) $\vec{b} = \begin{pmatrix}4\\1\\2\end{pmatrix} \quad \vec{v}_1 = \begin{pmatrix}-1\\1\\1\end{pmatrix} \quad \vec{v}_2 = \begin{pmatrix}1\\-2\\1\end{pmatrix}$

$$\vec{v}_1 \times \vec{v}_2 = \begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ -1 & 1 & 1\\ 1 & -2 & 1\end{vmatrix} = \hat{x}(1+2) - \hat{y}(-1-1) + \hat{z}(2-1) = \begin{pmatrix}3\\2\\1\end{pmatrix} = \vec{n}$$

$$\|\vec{n}\| = \sqrt{3^2+2^2+1^2} = \sqrt{14} \Rightarrow \hat{n} = \frac{\vec{n}}{\|\vec{n}\|} = \frac{1}{\sqrt{14}}\begin{pmatrix}3\\2\\1\end{pmatrix}$$



$$\vec{d} = (\vec{b}\cdot\hat{n})\hat{n} \qquad \|\vec{d}\| = \vec{b}\cdot\hat{n} = \frac{1}{\sqrt{14}}\begin{pmatrix}4\\1\\2\end{pmatrix}\cdot\begin{pmatrix}3\\2\\1\end{pmatrix} = \frac{1}{\sqrt{14}}(12+2+2) = \frac{14}{\sqrt{14}}$$

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5)

a)

$$\begin{bmatrix} 2 & 1 & 3 \\ 7 & 1 & 0 \\ 3 & 5 & 9 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} = 2 \begin{bmatrix} 2 \\ 7 \\ 3 \end{bmatrix} + 4 \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix} + 1 \begin{bmatrix} 3 \\ 0 \\ 9 \end{bmatrix} = \begin{bmatrix} 11 \\ 18 \\ 35 \end{bmatrix}$$

b)

$$\begin{bmatrix} 2 & 1 & 3 \\ 7 & 1 & 0 \\ 3 & 5 & 9 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} (2\cdot2)+(1\cdot4)+(3\cdot1) \\ (7\cdot2)+(1\cdot4)+(0\cdot1) \\ (3\cdot2)+(5\cdot4)+(9\cdot1) \end{bmatrix} = \begin{bmatrix} 11 \\ 18 \\ 35 \end{bmatrix}$$

6)

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 3 & 1 \\ 4 & 8 & k \end{bmatrix}$$

a)

$$a \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} + b \begin{bmatrix} 3 \\ 3 \\ 8 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ k \end{bmatrix} \rightsquigarrow \text{let } a=-1,\ b=1$$

$$\Rightarrow -\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} + \begin{bmatrix} 3 \\ 3 \\ 8 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ k \end{bmatrix} \Rightarrow k = 4$$

b)

$$\forall a,b: \quad a \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} + b \begin{bmatrix} 3 \\ 3 \\ 8 \end{bmatrix} \neq \begin{bmatrix} 2 \\ 1 \\ k \end{bmatrix}$$

$$\begin{bmatrix} a + 3b - 2 \\ 2a + 3b - 1 \\ 4a + 8b - k \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightsquigarrow$$

k can be any value other than 4

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7)

$$G(\vec{u},\vec{v}) = \frac{\vec{u}\cdot\vec{v}}{\vec{u}\cdot\vec{u}}\vec{u} \qquad \vec{a} = \begin{bmatrix}1\\0\\3\end{bmatrix} \quad \vec{b} = \begin{bmatrix}0\\1\\2\end{bmatrix} \quad \vec{c} = \begin{bmatrix}1\\1\\3\end{bmatrix}$$

$$\vec{u}_1 = \vec{a}$$

$$\vec{u}_2 = \vec{b} - G(\vec{u}_1,\vec{b}) = \vec{b} - \frac{\vec{u}_1\cdot\vec{b}}{\vec{u}_1\cdot\vec{u}_1}\vec{u}_1 = \vec{b} - \frac{\vec{a}\cdot\vec{b}}{\vec{a}\cdot\vec{a}}\vec{a} = \begin{bmatrix}0\\1\\2\end{bmatrix} - \frac{6}{10}\begin{bmatrix}1\\0\\3\end{bmatrix}$$

$$= \begin{bmatrix}0 - 6/10\\1 - 0\\2 - 18/10\end{bmatrix} = \begin{bmatrix}2/5\\1\\1/5\end{bmatrix} = \vec{u}_2$$

$$\vec{u}_3 = \vec{c} - G(\vec{u}_1,\vec{c}) - G(\vec{u}_2,\vec{c}) = \begin{bmatrix}1\\1\\3\end{bmatrix} - \frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{a}}\vec{a} - \frac{\vec{u}_2\cdot\vec{c}}{\vec{u}_2\cdot\vec{u}_2}\vec{u}_2$$

$$= \begin{bmatrix}1\\1\\3\end{bmatrix} - \frac{\begin{pmatrix}1\\0\\3\end{pmatrix}\cdot\begin{pmatrix}1\\1\\3\end{pmatrix}}{\begin{pmatrix}1\\0\\3\end{pmatrix}\cdot\begin{pmatrix}1\\0\\3\end{pmatrix}}\begin{pmatrix}1\\0\\3\end{pmatrix} - \frac{\begin{pmatrix}2/5\\1\\1/5\end{pmatrix}\cdot\begin{pmatrix}1\\1\\3\end{pmatrix}}{\begin{pmatrix}2/5\\1\\1/5\end{pmatrix}\cdot\begin{pmatrix}2/5\\1\\1/5\end{pmatrix}}\begin{pmatrix}2/5\\1\\1/5\end{pmatrix}$$

$$= \begin{bmatrix}1\\1\\3\end{bmatrix} - \frac{10}{\cancel{10}}\begin{bmatrix}1\\0\\3\end{bmatrix} - \frac{\cancel{10}^{5}}{\cancel{6}_{3}}\begin{bmatrix}2/5\\1\\1/5\end{bmatrix}.$$

$$= \begin{bmatrix}1\\1\\3\end{bmatrix} - \begin{bmatrix}1\\0\\3\end{bmatrix} - \begin{bmatrix}2/3\\5/3\\1/3\end{bmatrix} = \begin{bmatrix}-2/3\\-2/3\\-1/3\end{bmatrix} = \vec{u}_3$$

$$\beta : \left\{ \begin{pmatrix}1\\0\\3\end{pmatrix}, \begin{pmatrix}2/5\\1\\1/5\end{pmatrix}, \begin{pmatrix}-2/3\\-2/3\\-1/3\end{pmatrix} \right\}$$