

An Introduction to Graphs

Let's begin by watching the video "An Introduction to Graphs" (3:08).



Below are a list of essential definitions for understanding graphs as we go through this module.

Vertex

A vertex (also called a “node”) is a fundamental part of a graph. It can have a name, which we will call the “key.” A vertex may also have additional information. We will call this additional information the “payload.”

Edge

An edge (also called an “arc”) is another fundamental part of a graph. An edge connects two vertices to show that there is a relationship between them. Edges may be one-way or two-way. If the edges in a graph are all one-way, we say that the graph is a *directed graph*, or a *digraph*. The class prerequisites graph shown above is clearly a digraph since you must take some classes before others.

Weight

Edges may be weighted to show that there is a cost to go from one vertex to another. For example in a graph of roads that connect one city to another, the weight on the edge might represent the distance between the two cities.

Path

A path in a graph is a sequence of vertices that are connected by edges. Formally we would define a path as w_1, w_2, \dots, w_n such that $(w_i, w_{i+1}) \in E$ for all $1 \leq i \leq n - 1$. The unweighted path length is the number of edges in the path, specifically $n - 1$. The weighted path length is the sum of the weights of all the edges in the path.

Cycle

A cycle in a directed graph is a path that starts and ends at the same vertex. A graph with no cycles is called an *acyclic graph*. A directed graph with no cycles is called a *directed acyclic graph* or a *DAG*. We will see that we can solve several important problems if the problem can be represented as a DAG.

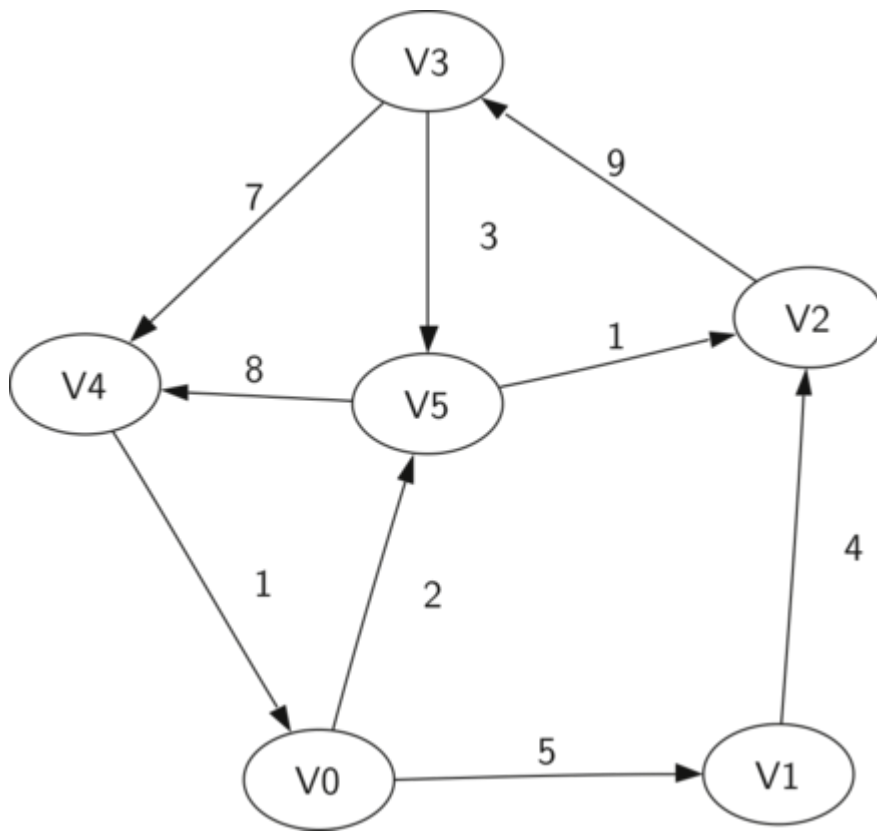
What is a Graph?

With those definitions in hand we can formally define a graph. A graph can be represented by G where $G = (V, E)$. For the graph G , V is a set of vertices and E is a set of edges. Each edge is a tuple (v, w) where $w, v \in V$. We can add a third component to the edge tuple to represent a weight. A subgraph s is a set of edges e and vertices v such that $e \subset E$ and $v \subset V$.

The figure below shows another example of a simple weighted digraph. Formally we can represent this graph as the set of six vertices: $V = \{V0, V1, V2, V3, V4, V5\}$

and the set of nine edges:

$$E = \{(v0, v1, 5), (v1, v2, 4), (v2, v3, 9), (v3, v4, 7), (v4, v0, 1), (v0, v5, 2), (v5, v4, 8), (v3, v5, 1), (v5, v2, 1)\}$$



Source: [Problem Solving and Algorithms in Python](http://interactivepython.org/runestone/static/pythonds/index.html#) [_\(http://interactivepython.org/runestone/static/pythonds/index.html#\)](http://interactivepython.org/runestone/static/pythonds/index.html#) from Bradley Miller on [www.interactivepython.org](http://interactivepython.org) [_\(http://interactivepython.org/runestone/static/pythonds/index.html#\)](http://interactivepython.org/runestone/static/pythonds/index.html#).