

# Aircraft Performance

## 2 - Formula Overview

### Cruise Flight

#### Equations of motion

$$\frac{W}{g} \frac{dV}{dt} = T - D - W \sin \gamma \quad (1)$$

$$\frac{W}{g} V \frac{d\gamma}{dt} = L - W \cos \gamma \quad (2)$$

#### Kinematics equations

$$\frac{ds}{dt} = V \cos \gamma, \quad \frac{dH}{dt} = V \sin \gamma \quad (3)$$

#### Efficiency of jet aircrafts

$$\eta_{tot} = \frac{P_{out}}{P_{in}} = \frac{TV}{H \frac{F}{g}} = \frac{V}{c_T} \frac{g}{H} \quad (4)$$

#### Efficiency of propeller aircrafts

$$\eta_{tot} = \frac{P_{out}}{P_{in}} = \frac{TV}{H \frac{F}{g}} = \frac{\eta_p}{c_p} \frac{g}{H} \quad (5)$$

### Range

#### Range

$$R = \frac{V}{c_T} \frac{C_L}{C_D} \ln \left( \frac{W_0}{W_1} \right) = \frac{H}{g} \eta_{tot} \frac{C_L}{C_D} \ln \left( \frac{W_0}{W_1} \right) \quad (6)$$

#### Lift coefficient for maximum range

$$C_L = \sqrt{\frac{1}{3} C_{D_0} \pi A e} \quad (7)$$

#### Density for maximum range at speed limit

$$\rho_{opt} = \frac{W}{S} \frac{2}{V_{lim}^2} \frac{1}{C_L} \quad (8)$$

### Take-Off

#### Equations of motion

$$\frac{W}{g} \frac{dV}{dt} = T - D - D_g = T - D - \mu(W - L) \quad (9)$$

#### Ground drag

$$D_g = \mu N = \mu(W - L) \quad (10)$$

#### Mean acceleration ( $a$ at $\frac{1}{2}\sqrt{2}V_{lof}$ )

$$\bar{a} = \frac{g}{W} (\bar{T} - \bar{D} - \bar{D}_g) \quad (11)$$

#### Ground phase distance

$$s_{ground} = \frac{1}{2\bar{a}} V_{lof}^2 \quad (12)$$

#### Air phase distance

$$\gamma_{scr} s_{air} = \frac{V_{scr}^2 - V_{lof}^2}{2g} + H_{scr} \quad (13)$$

### Climbing Flight

#### Steady rate Of climb

$$RC_{st} = \frac{T - D}{W} V = \frac{Pa - Pr}{W} = P_s \quad (14)$$

#### Kinetic correction factor

$$\frac{RC_{st}}{RC} = 1 + \frac{1}{2g} \frac{d(V^2)}{dH} \quad (15)$$

#### Energy height

$$E_h = H + \frac{1}{2g} V^2 \quad (16)$$

#### Time derivative of energy height

$$\frac{dE_h}{dt} = \frac{T - D}{W} V = P_s \quad (17)$$

### Climb Time

#### Climb time to given height

$$t_{climb} = \int_0^{t_{climb}} dt = \int_0^H \frac{1}{RC} dH \quad (18)$$

#### Climb time to given energy height

$$t_{climb} = \int_0^{t_{climb}} dt = \int_0^{H_e} \frac{1}{P_s} dH_e \quad (19)$$

**Lift coefficient for minimal climb time**

$$C_L = \sqrt{3C_{D_0}\pi Ae} \quad (20)$$

**Flight speed for minimal climb time**

$$V = \sqrt{\frac{2}{\rho} \frac{1}{C_L} \frac{W}{S}}, \quad V_e = \sqrt{\frac{2}{\rho_0} \frac{1}{C_L} \frac{W}{S}} \quad (21)$$

## Landing

**Energy landing equation**

$$\frac{V_t^2}{2g} - \left( \frac{V_a^2}{2g} + H_{scr} \right) = -\frac{1}{2} s_{air} \left( \bar{\gamma}_a + \left( \frac{C_D}{C_L} \right)_t \right) \quad (22)$$

**Break distance**

$$s_{ground} = \int_{V_t}^0 \frac{V dV}{a} = -\frac{V_t^2}{2\bar{a}} \quad (23)$$

**Mean acceleration**

$$\bar{a} = \frac{g}{W} (\bar{D} + \bar{D}_g + \bar{T}_{rev}) \quad (24)$$

## Aircraft Systems

**Air path system**

$$E_a = [\mu] [\gamma] [\chi] E_e \quad (25)$$

**Moving earth system**

$$E_e = [\chi]^T [\gamma]^T [\mu]^T E_a \quad (26)$$

**General Equations of Motion**

$$m\dot{V} = T - D - W \sin \gamma \quad (27)$$

$$mV\dot{\chi} \cos \gamma = L \sin \mu \quad (28)$$

$$mV\dot{\gamma} = L \cos \mu - W \cos \gamma \quad (29)$$

## Effects of Wind

**Equations of motion**

$$T - D - W \sin \gamma = \frac{W}{g} \left( \dot{V} - \dot{V}_w \cos \gamma \right) \quad (30)$$

$$L - W \cos \gamma = \frac{W}{g} \left( V\dot{\gamma} + \dot{V}_w \sin \gamma \right) \quad (31)$$

**Kinematics equations**

$$\dot{H} = V \sin \gamma, \quad \dot{s} = V \cos \gamma - V_w \quad (32)$$

**Wind gradients**

$$RC \left( 1 + \frac{V}{g} \frac{dV}{dH} - \frac{V}{g} \frac{dV_w}{dH} \right) = RC_{st} \quad (33)$$

## Turning Flight

**Turn radius**

$$R = \frac{V^2}{g\sqrt{n^2 - 1}} \quad (34)$$

**Turn velocity**

$$V = \sqrt{\frac{2}{\rho} \frac{1}{C_L} \frac{nW}{S}} \quad (35)$$

**Turn power**

$$P_r = DV = \sqrt{\frac{2}{\rho} \frac{C_D^2}{C_L^3} \frac{(nW)^3}{S}} \quad (36)$$

## Helicopters

**Thrust**

$$T = 2mV_i = 2\rho\pi r^2 V_i \quad (37)$$

**Ideal hover power**

$$P_{id} = WV_i = W \sqrt{\frac{W}{2\rho\pi r^2}} \quad (38)$$

**Hover power**

$$P_{hov} = \frac{c_{d_p} \sigma}{8} \rho (\Omega R)^3 \pi R^2 \quad (39)$$

**Rotor solidity**

$$\sigma = \frac{nRc}{\pi R^2} \quad (40)$$

**Flight power**

$$P_p = P_{hov} (1 + \mu^2) \quad (41)$$

**Advance ratio**

$$\mu = \frac{V \cos \alpha_d}{\Omega R} \quad (42)$$