

MSE 2034 - Elements of Materials Engineering  
Big Gift 2 – Formula Sheet

Brinell to tensile strength  
↓  
 $\frac{T}{S} = 3.45 \cdot HB$

$\tau_R = \sigma \cos(\phi) \cos(\lambda)$

$\sigma_y = \sigma_0 + k_y d^{-1/2}$

$\%CW = \left(\frac{A_o - A_d}{A_o}\right) \times 100$   
 $r_o = \frac{r_d}{\sqrt{1 - \frac{CW}{100}}}$

$d^n - d_0^n = Kt$   
 $\sigma = \frac{16 FL}{\pi d_o^3}$

$\sigma_m = 2\sigma_0 \left(\frac{a}{\rho_t}\right)^{1/2}$   
 $\frac{\sigma}{N} = \frac{16 FL}{\pi d_o^3}$

$\sigma_c = \left(\frac{2E\gamma_s}{\pi a}\right)^{1/2}$   
 $K_{IC} = Y\sigma_c \sqrt{\pi a}$   
*make it be in terms of MPa<sup>1/2</sup> (If no data, Y = 1)*

$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$

$X = X_0 \exp\left(-\frac{Q}{RT}\right)$

$R = 8.314 \text{ J/(mol K)}$   
 $k = 1.38 \times 10^{-23} \text{ J/(atom K)}$   
 $N_a = 6.023 \times 10^{23} \text{ atom/mol}$

$\sigma = \frac{F}{A}$

$\sigma_{allow} = \frac{\sigma_{actual}}{FoS}$

$\sigma_r = \sigma_{max} - \sigma_{min}$

$\sigma_a = \frac{\sigma_r}{2} = \frac{\sigma_{max} - \sigma_{min}}{2}$

$R = \frac{\sigma_{min}}{\sigma_{max}}$

$LM = T(20 + \log(t_r))$

$\dot{\epsilon}_s = K_1 \sigma^n$

$\dot{\epsilon}_s = K_2 \sigma^n \exp\left(-\frac{Q_c}{RT}\right)$

$y = 1 - \exp(-kt^n)$

Consider a single crystal of silver oriented such that a tensile stress is applied along a [001] direction. If slip occurs on a {111} plane and in a  $\bar{1}\bar{1}0$  direction, and is initiated at an applied tensile stress of 1.1 MPa (160 psi), compute the critical resolved shear stress.

$\lambda = \cos^{-1} \left[ \frac{(\phi \chi + \psi)(\phi + \chi)}{\sqrt{(\phi^2 + \psi^2 + \chi^2)(\phi^2 + \psi^2 + \chi^2)}} \right]$   
 $\phi = \cos^{-1} \left[ \frac{(\phi \chi + \psi)(\phi + \chi)}{\sqrt{(\phi^2 + \psi^2 + \chi^2)(\phi^2 + \psi^2 + \chi^2)}} \right]$   
 $1.1 (\cos \lambda) (\cos \psi) = 0.449 = \tau_{crs}$

Chapter 10

Homogeneous Nucleation

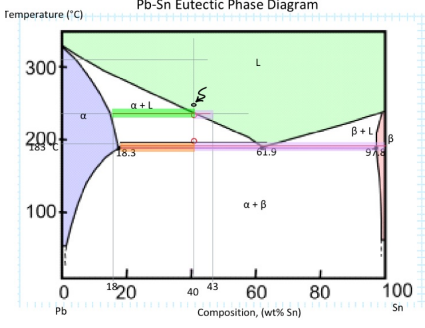
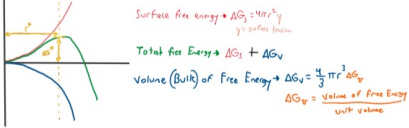
$r^* = \frac{-2\gamma_m}{\Delta H_f \Delta T}$   
 $G^* = \frac{16\pi\gamma_m^2}{3\Delta H_f \Delta T}$   
 $r^* \downarrow$  w/  $\Delta T \uparrow$   
 $T_m$  = melting temp  
 $\Delta T \geq T_m$  = supercooling

Heterogeneous Nucleation

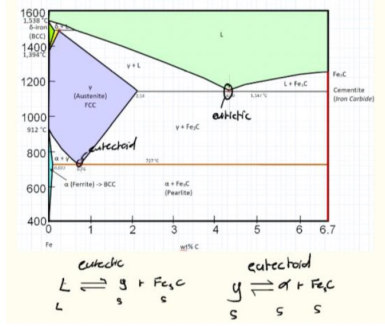
$r^* = \frac{-2\gamma_s}{\Delta G_v}$   
 $\Delta G_v = \left(\frac{16\pi\gamma_s^2}{3\Delta G_v}\right) \cdot f(\theta)$   
 $\gamma_{SL} = \gamma_{SV} + \gamma_{LV} \cos \theta$

Growth  $G = \exp\left(-\frac{Q}{RT}\right)$

Rate of Phase Transformation  
Avrami Equation  $\rightarrow y = 1 - \exp(-kt^n)$



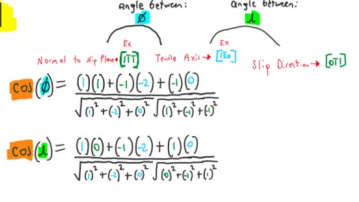
- Label the regions on the Eutectic Phase Diagram.
- Then answer the 4 questions for each of the following:
- For a 40 wt% Sn alloy:
- 300 °C  
a.  $\alpha + L$   
b. Compositions  
i.  $\alpha \rightarrow 18.3 \text{ wt\% Sn}$   
ii.  $L \rightarrow 43 \text{ wt\% Sn}$   
c. Amounts  
i.  $\alpha \rightarrow (43 - 40)/(43 - 18) = 0.12 \rightarrow 12\%$   
ii.  $L \rightarrow (40 - 18)/(43 - 18) = 0.88 \rightarrow 88\%$
  - 220 °C  
a.  $\alpha + L$   
b. Compositions  
i.  $\alpha \rightarrow 18.3 \text{ wt\% Sn}$   
ii.  $L \rightarrow 97.8 \text{ wt\% Sn}$   
c. Amounts  
i.  $\alpha \rightarrow (97.8 - 40)/(97.8 - 18.3) = 0.50 \rightarrow 50\%$   
ii.  $L \rightarrow (40 - 18.3)/(97.8 - 18.3) = 0.4977 \rightarrow 50\%$
  - 184 °C  
a.  $\alpha + L$   
b. Compositions  
i.  $\alpha \rightarrow 18.3 \text{ wt\% Sn}$   
ii.  $\beta \rightarrow 97.8 \text{ wt\% Sn}$   
c. Amounts  
i.  $\alpha \rightarrow (97.8 - 40)/(97.8 - 18.3) = 0.727 \rightarrow 73\%$   
ii.  $\beta \rightarrow (40 - 18.3)/(97.8 - 18.3) = 0.273 \rightarrow 27\%$
  - 182 °C  
a.  $\alpha + L$   
b. Compositions  
i.  $\alpha \rightarrow 18.3 \text{ wt\% Sn}$   
ii.  $\beta \rightarrow 97.8 \text{ wt\% Sn}$   
c. Amounts  
i.  $\alpha \rightarrow (97.8 - 40)/(97.8 - 18.3) = 0.727 \rightarrow 73\%$   
ii.  $\beta \rightarrow (40 - 18.3)/(97.8 - 18.3) = 0.273 \rightarrow 27\%$



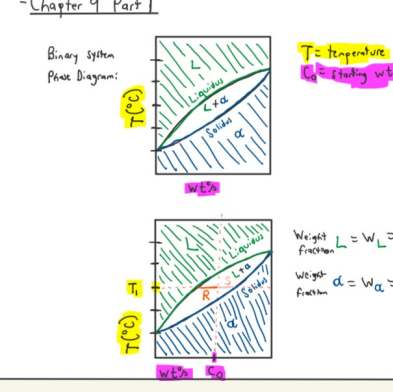
Color Key Per Chapter

Equations:

- Chapter 7 Part 1, 2
  - Critical resolved shear stress  $\rightarrow \tau_{CRSS}$
  - maximum elastic yield stress  $\rightarrow \tau_y = \frac{\tau_{CRSS}}{\cos \phi}$
  - Hall Petch Equation  $\sigma_y = \sigma_0 + K_y d^{-1/2}$
- Chapter 7 Part 3
  - Cold work %  $\rightarrow \frac{\Delta A_0}{A_0} \times 100$
  - Dislocation Density  $\rho_d$  = total dislocation / Unit volume



- Chapter 8
  - Stress at crack tip  $\rightarrow \sigma_a = 2 \sqrt{\frac{E \gamma_s}{\pi a}}$
  - Stress concentration factor  $K_t = \frac{\sigma_{max}}{\sigma_{nom}}$
  - Fracture Toughness  $K_{Ic} = Y \sqrt{\frac{E \gamma_s}{\pi a}}$
  - Mean stress  $\rightarrow \sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$
  - Range of stress  $\sigma_r = \sigma_{max} - \sigma_{min}$
  - Stress amplitude  $\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$
  - Stress Ratio  $\rightarrow R = \frac{\sigma_{min}}{\sigma_{max}}$
  - Steady state creep  $\rightarrow \dot{\epsilon}_s = K \sigma^n \exp(-\frac{Q_a}{RT})$
  - Prediction of creep rupture Lifetime  $\rightarrow T(C + \log t_r) = m$
- Chapter 9 Part 1



- Chapter 8: Failure
  - Types of Failure
    - Fracture
      - Simple fracture: Segregation of body into 2+ pieces in response to static stress
      - 2 general types of fracture
        - Brittle fracture: Slow crack growth, no plastic deformation, fails with warning, fails under tension
        - Ductile fracture: Significant plastic deformation, fails with warning, fails under tension
    - Fatigue fracture: Occurs due to cyclic loading, under long term period of repeated stress/strain, increasing damage, reducing magnitude of mean stress, Surface treatments, Compressed spots where crack propagation is prevented, Design changes to remove stress concentrations, Fatigue crack growth rate, Time depends deformation caused by constant stress, Elongation is important for metals that have high O/C absorbing temp, Many Equations!!!

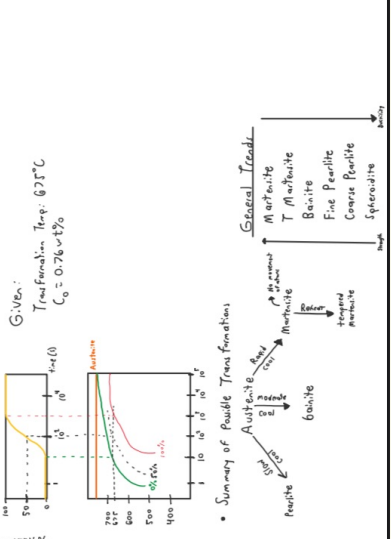
- Chapter 9 Phase Diagrams
  - Eutectic = L  $\rightarrow$  A + B, isotherm
  - Eutectoid =  $\gamma \rightarrow \alpha + \beta$ , isotherm
  - Peritectic =  $L + \alpha \rightarrow \beta$ , isotherm
  - Hypo = left of tie line
  - Hyper = right of tie line
- Chapter 10
  - Kinetics of phase transformation
    - Nucleation
      - Rate of nucleation is not at same time (all rate)
      - Growth proceeds until equilibrium
      - RT, time, nucleation
      - Supercooling (enthalpy, enthalpy)
      - Free energy (low activation rate, low energy, low energy)
      - Low energy nucleation sites, any surface, low energy
    - Solidification: Nucleation types
      - Heterogeneous nucleation: Form on the surface, small liquidus, more gradual
      - Homogeneous nucleation: Nuclei form in bulk of liquid metal, large supercooling
    - Diffusion and energy effects
      - Homogeneous nucleation: Called nucleation or nucleation energy
      - For nucleation to occur,  $\Delta G < 0$
      - $\Delta G$ : free energy difference between solid and liquid phases  $\rightarrow$  Volume free energy
      - $\Delta G_s$ : free energy difference between solid and liquid phases  $\rightarrow$  Surface free energy

Chapter 7 Part 1 + 2 : Dislocations

- Dislocations: Linear Defects (one-dimensional) that cause misalignment of nearby atoms
- Types: edge, screw, mixed
- Plastic Deformation by Dislocation Motion
  - called slip
  - Slip Systems
    - Combination of slip plane and slip direction
    - Slip Plane: Crystallographic Plane where slip occurs
    - Slip Direction: Crystallographic direction in which slip occurs
  - For FCC crystal structure
    - Dislocation motion on {111} family of planes
    - Dislocation motion in  $\langle 110 \rangle$  family of directions
    - 12 independent slip systems for FCC

Chapter 7 Part 3: Strategies for Strengthening

- 4 Strategies for Strengthening Materials (make dislocation motion difficult)
  1. Reduce Grain Size
    - increases # of slip orientations, which increases barrier strength
  2. Form Solid Solutions
    - Impurity atoms distort the lattice and generate lattice strains (Fill in holes where material can fracture)
  3. Precipitation Strengthening
    - hard precipitates are difficult to shear
  4. Cold Work (Strain Hardening)
    - $\frac{\Delta \sigma_y}{\sigma_y} = \frac{\Delta A_0}{A_0} \times 100$
    - Dislocations entangle with one another
    - Dislocation Density ( $\rho_d$ ) = total dislocation length / Unit volume
- Impact of Cold Work
  - As  $\rho_d \uparrow$ ...
    - yield strength ( $\sigma_y$ )  $\uparrow$
    - tensile strength ( $\sigma_t$ )  $\uparrow$
    - Ductility (%EL)  $\downarrow$
  - Effect of Heat Treating after Cold Work
    - Cold work nullified
    - TS  $\downarrow$ , %EL  $\uparrow$
- 3 Stages during Heat Treatment
  1. Recovery
    - reduction of dislocation density by annihilation
  2. Recrystallization
    - New grains formed w/ low dislocation density, small in size
    - grains regrow, small grains
  3. Grain Growth
    - Small grains disappear and big grains grow
    - $d^n - d_0^n = kt$
- Grain Size Influences Properties
  - Metals w/ small grains  $\rightarrow$  strong and tough at low temperatures
  - metals w/ large grains  $\rightarrow$  good creep resistance at high temperatures



Chapter 8: Failure

Chapter 9 Phase Diagrams

Chapter 10 Kinetics of phase transformation

Summary of Phase Transformation Diagrams