Lab 4 - Wyatt Madden & Dan Crowley

February 16, 2020

1 3.1

In [1]:
```
import scipy.io as scipy
import seaborn as sns
import matplotlib.pyplot as plt
import pandas as pd
import numpy as np
import numpy.linalg as lg
from eval_basis import *
from func_gauss import *
from func_hat import *
from gauss_basis import *
from hat_basis import *
```

In [2]:
```
# Least-Squared Error FIT
#  Find the linear combination of basis functions which best model the data.
#
#  Inputs:
#
#  x - Vector with observation locations in 1D. (indep. variable)
#  t - Vector with observations in 1D. (dep. variable)
#  params - Parameters for the basis functions to be used in func, e.g. as
#    produced by gauss_basis.
#  func - Function handle which evaluates a basis function with parameters
#    given by the columns of params and at the specified locations. e.g.
#    @gauss_basis, or @hat_basis.
#    For example, the first basis function at x = 2 is func(2, params(:,1)).
#  mu - Scalar representing the standard deviation of the prior Gaussian on
#    the model parameters.
#
#  Outputs:
#  w - Coefficients used to generate a linear combination of the basis
#    functions which is the maximum likelihood learned model.

def lsefit(x, t, params, func, mu):
    design_matrix = better_eval_basis(params = params,
                                      func = func,
```

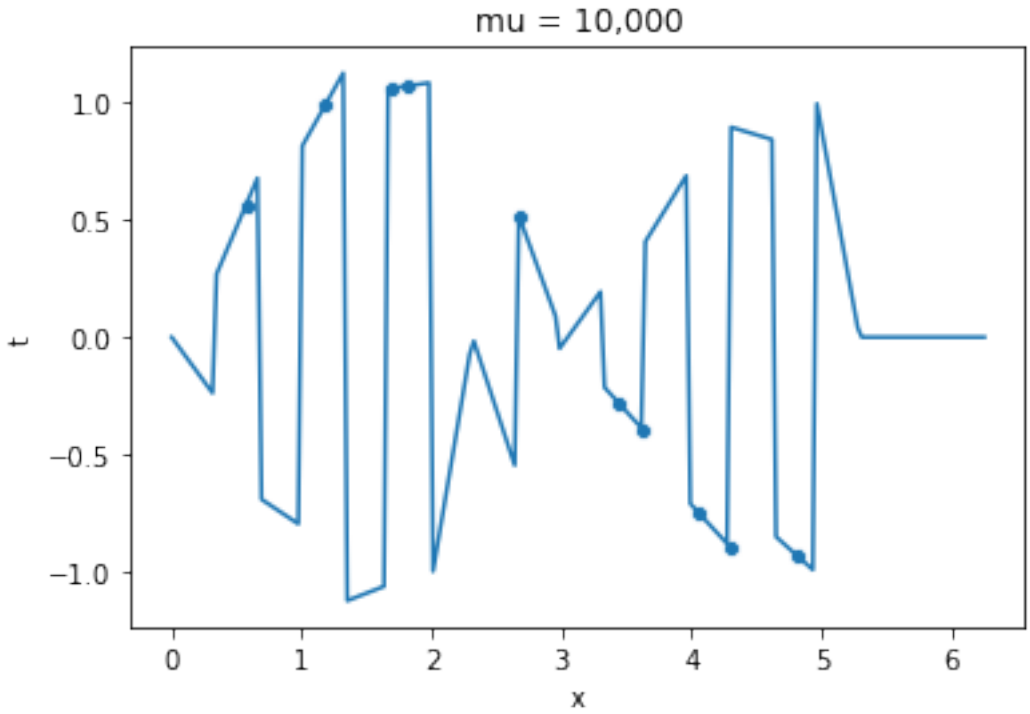
```
                                        xeval = x)

    w_hat = lg.inv(np.dot(np.transpose(design_matrix), design_matrix) +
                   np.identity(design_matrix.shape[1])*(1/mu**2))
    w = np.dot(w_hat, np.dot(np.transpose(design_matrix), t))
    return w
```

```
In [3]: lab_4_dat = scipy.loadmat('/Users/wyattmadden/Documents/school/' +
                                  'MSU/2020/spring/m508/lab_info/lab_4/simple.mat',
                                  squeeze_me = True)

    x = lab_4_dat['x']
    t = lab_4_dat['t']

    data_orig = {'x': x,
                 't': t}

    data_orig = pd.DataFrame(data_orig)
```

2 3.2

```
In [4]: #function to automate fitting process
    def df_of_preds(x, t, basis, func, mu, M, at):
        fits = lsefit(x = x,
                      t = t,
                      params = basis(0, 2*np.pi, M),
                      func = func,
                      mu = mu)
        preds_df = {'fits': np.dot(fits,
                                   np.transpose(better_eval_basis(basis(0,
                                                                  2*np.pi,
                                                                  M),
                                                                  func,
                                                                  at))),
                    'x': at}
        preds_df = pd.DataFrame(preds_df)
        return preds_df

    #function to automate plotting
    def plot_preds_and_data(preds, data, title):
        sns.lineplot(x = "x", y = "fits", data = preds)
        sns.scatterplot(x = "x", y = "t", data = data).set(title = title)
```

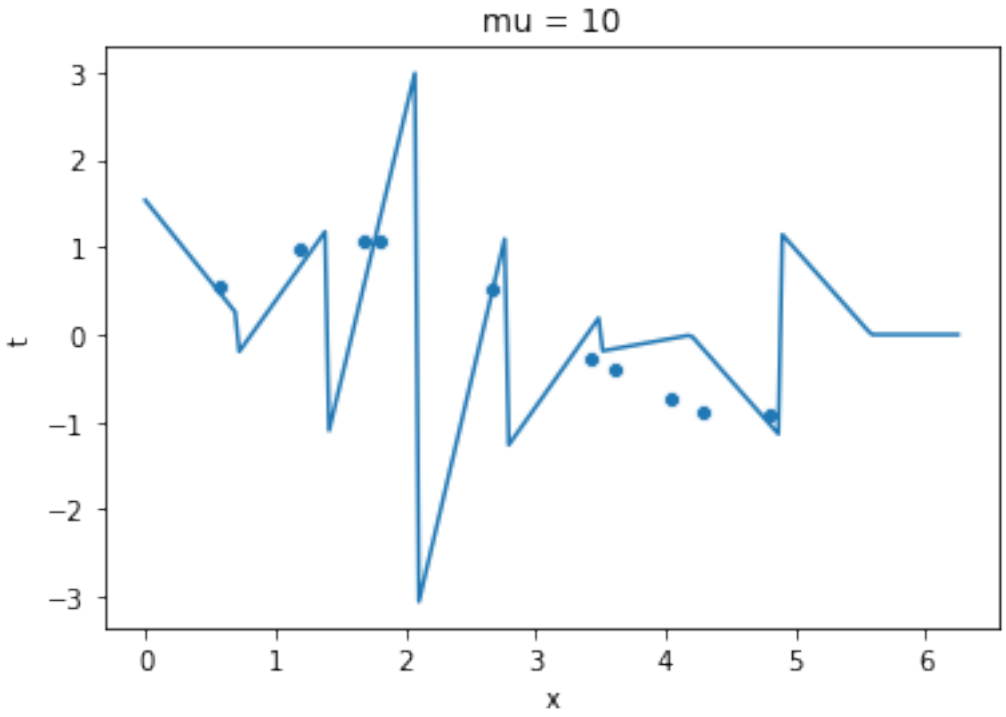
```
In [48]: zero_to_two_pi = np.arange(0, 2*np.pi, np.pi/100)

         fits = df_of_preds(x = x, t = t, basis = hat_basis,
                            func = func_hat, mu = 10**5, M = len(x),
                            at = zero_to_two_pi)

         plot_preds_and_data(fits, data_orig, "mu = 10,000")
```



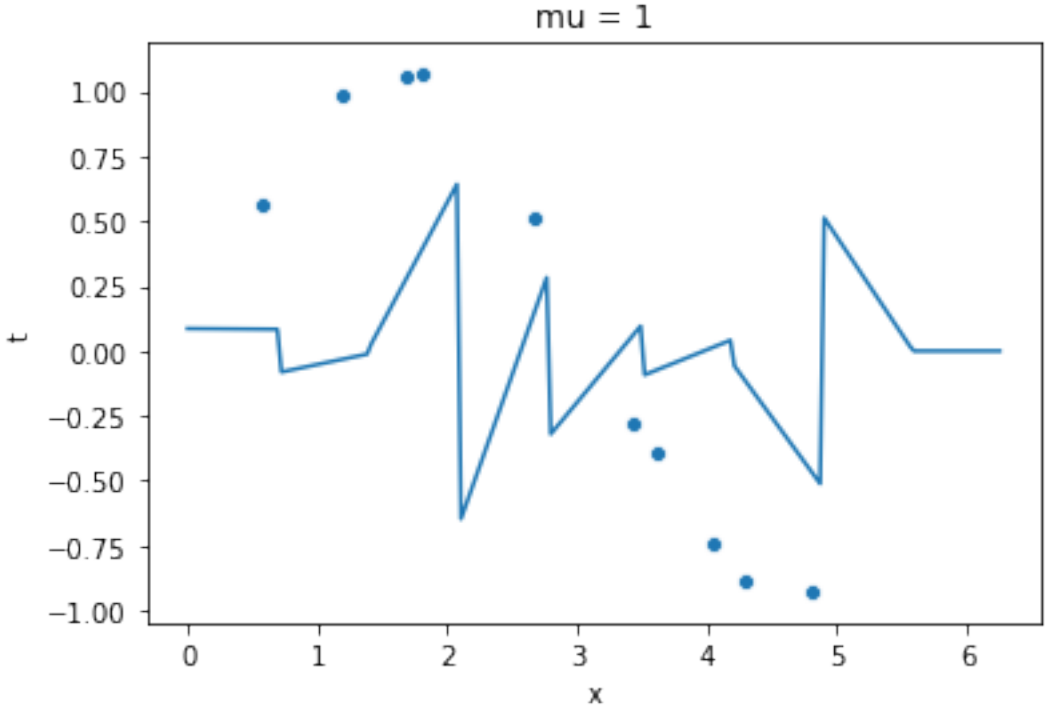
The fit of the hat basis function with mu of 10,000 is not a good approximation of the data. It is too responsive to subsequent data points, especially on the lower end of the x space.

3 3.3

```
In [50]: fits = df_of_preds(x, t, hat_basis, func_hat, 10, len(x),
                            at = zero_to_two_pi)
         plot_preds_and_data(fits, data_orig, "mu = 10")
```



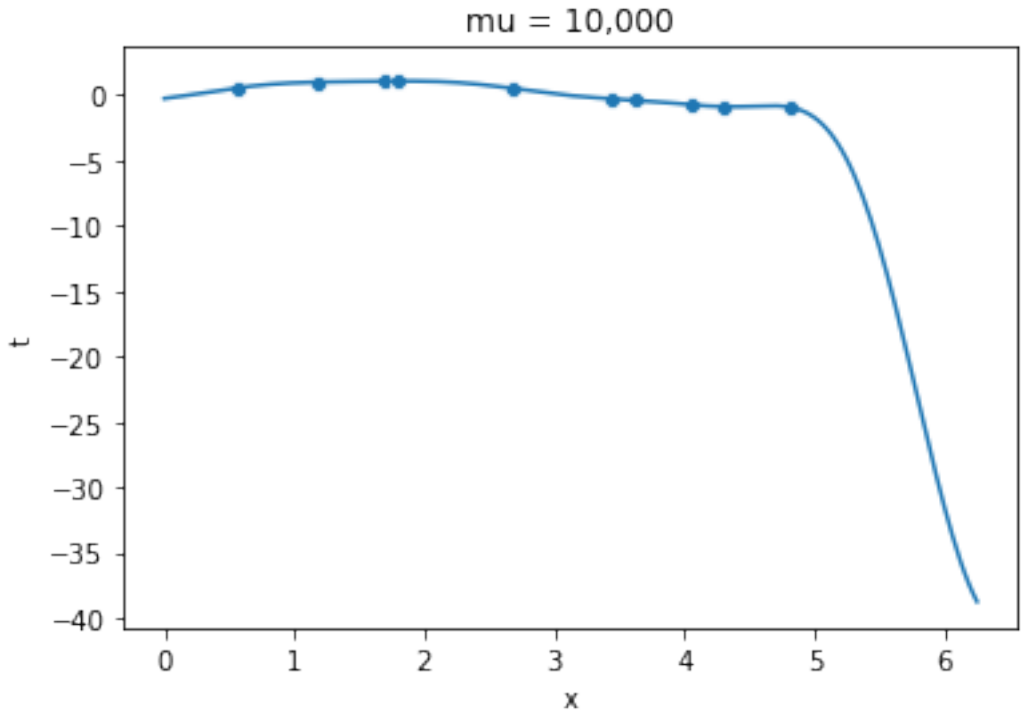

```
In [51]: fits = df_of_preds(x, t, hat_basis, func_hat, 1, len(x),
                            at = zero_to_two_pi)
         plot_preds_and_data(fits, data_orig, "mu = 1")
```



4 3.4

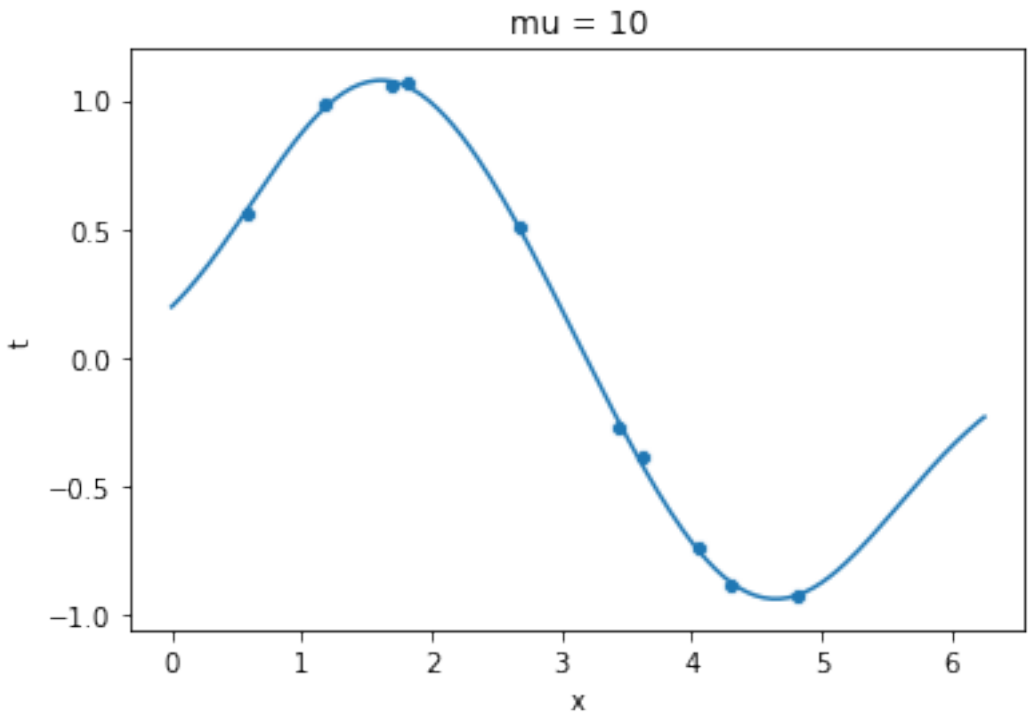
5 3.5

```
In [52]: fits = df_of_preds(x, t, gauss_basis, func_gauss, 10**5, len(x),
                            at = zero_to_two_pi)
         plot_preds_and_data(fits, data_orig, "mu = 10,000")
```

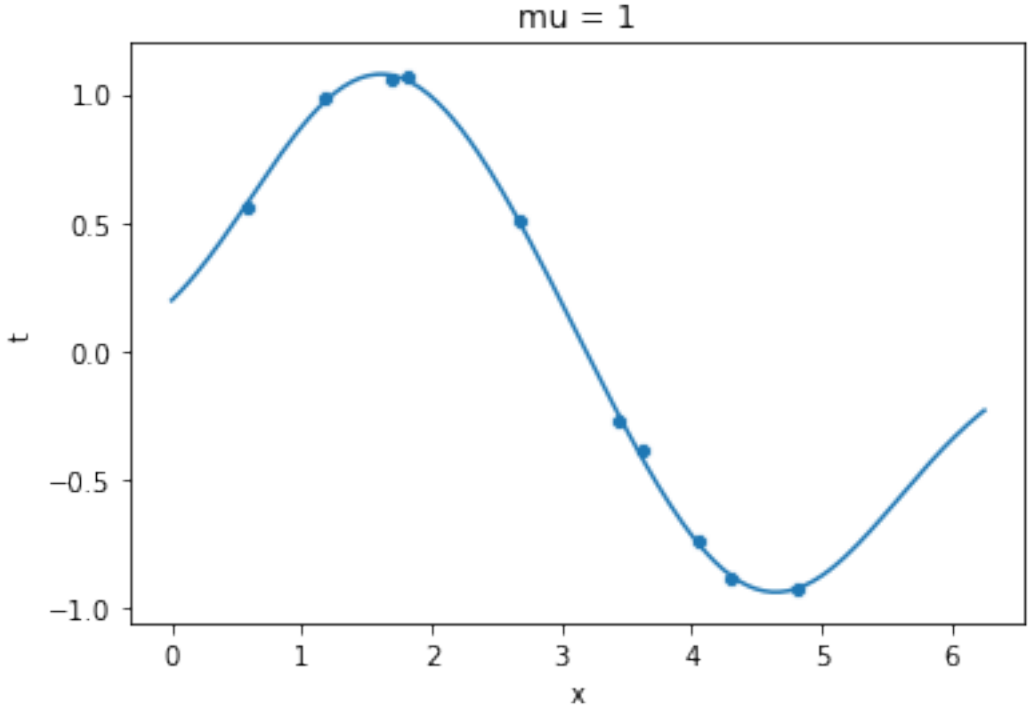


6 3.6

```
In [53]: fits = df_of_preds(x, t, gauss_basis, func_gauss, 10, len(x),
                            at = zero_to_two_pi)
         plot_preds_and_data(fits, data_orig, "mu = 10")
```




```
In [54]: fits = df_of_preds(x, t, gauss_basis, func_gauss, 10, len(x),
                            at = zero_to_two_pi)
         plot_preds_and_data(fits, data_orig, "mu = 1")
```

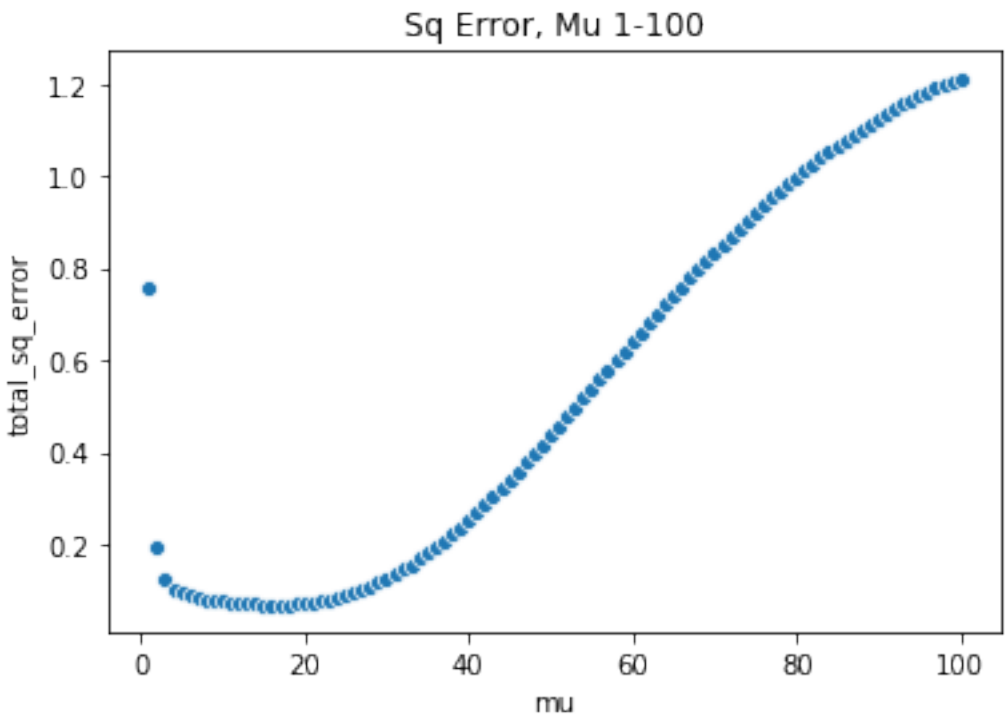


7 3.7

8 3.8

In [12]:
```
lab_4_test = scipy.loadmat('/Users/wyattmadden/Documents/school/' +
                           'MSU/2020/spring/m508/lab_info/lab_4/test.mat',
                           squeeze_me = True)

x_test = lab_4_test['test_x']
t_test = lab_4_test['test_t']

data_test = {'x': x_test,
             't': t_test}

data_test = pd.DataFrame(data_test)
sq_errors = np.empty(100)

for i in range(1, 101):
    fits = df_of_preds(x, t, gauss_basis, func_gauss, i, 10,
                       at = x_test)
    sq_errors[i-1] = np.sum((fits['fits'] - t_test)**2)


sq_errors_and_mus = {'mu': range(1, 101),
                     'total_sq_error': sq_errors}
sq_errors_and_mus = pd.DataFrame(sq_errors_and_mus)

sns.scatterplot(x = "mu",
                y = "total_sq_error",
                data = sq_errors_and_mus).set(title = "Sq Error, Mu 1-100")
```

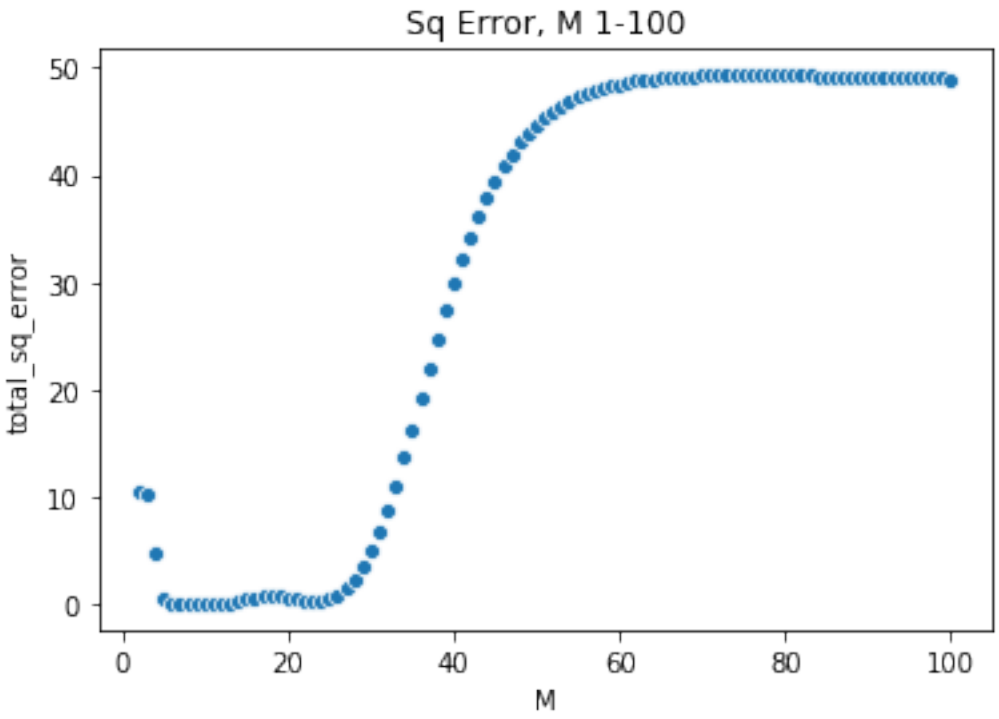
Out[12]:
```
[Text(0.5,1,'Sq Error, Mu 1-100')]
```



9 3.9

10 3.10

```
In [19]: m_sq_errors = np.empty(99)

         for i in range(2, 101):
             fits = df_of_preds(x, t, gauss_basis, func_gauss, 13, i,
                                at = x_test)
             m_sq_errors[i-2] = np.sum((fits['fits'] - t_test)**2)


         m_sq_errors_and_mus = {'M': range(2, 101),
                                'total_sq_error': m_sq_errors}
         m_sq_errors_and_mus = pd.DataFrame(m_sq_errors_and_mus)

         sns.scatterplot(x = "M",
                         y = "total_sq_error",
                         data = m_sq_errors_and_mus).set(title = "Sq Error, M 1-100")

Out[19]: [Text(0.5,1,'Sq Error, M 1-100')]
```


Sq Error, M 1-100
total_sq_error
M
0
10
20
30
40
50
0
20
40
60
80
100