Lab 4 - Wyatt Madden & Dan Crowley

February 16, 2020

1 3.1

In [1]:
```
import scipy.io as scipy
import seaborn as sns
import matplotlib.pyplot as plt
import pandas as pd
import numpy as np
import numpy.linalg as lg
from eval_basis import *
from func_gauss import *
from func_hat import *
from gauss_basis import *
from hat_basis import *
```

In [2]:
```
# Least-Squared Error FIT
#  Find the linear combination of basis functions which best model the data.
#
#  Inputs:
#
#  x - Vector with observation locations in 1D. (indep. variable)
#  t - Vector with observations in 1D. (dep. variable)
#  params - Parameters for the basis functions to be used in func, e.g. as
#    produced by gauss_basis.
#  func - Function handle which evaluates a basis function with parameters
#    given by the columns of params and at the specified locations. e.g.
#    @gauss_basis, or @hat_basis.
#    For example, the first basis function at x = 2 is func(2, params(:,1)).
#  mu - Scalar representing the standard deviation of the prior Gaussian on
#    the model parameters.
#
#  Outputs:
#  w - Coefficients used to generate a linear combination of the basis
#    functions which is the maximum likelihood learned model.

def lsefit(x, t, params, func, mu):
    design_matrix = better_eval_basis(params = params,
                                      func = func,
```

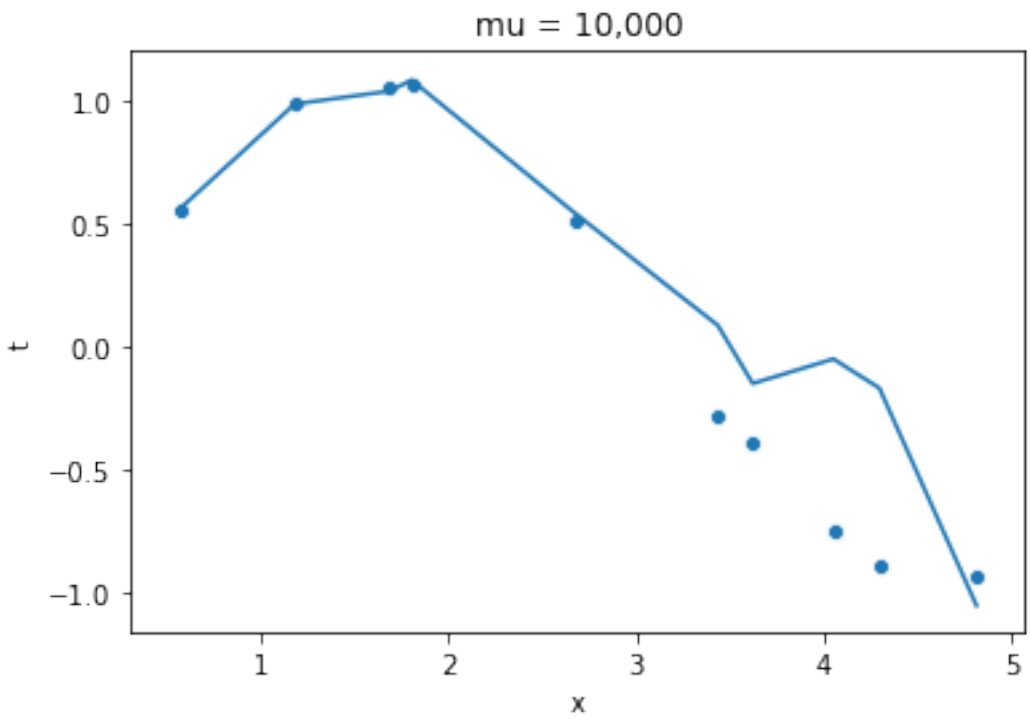
```
                                        xeval = x)

    w_hat = lg.inv(np.dot(np.transpose(design_matrix), design_matrix) +
                   np.identity(design_matrix.shape[1])*(1/mu**2))
    w = np.dot(w_hat, np.dot(np.transpose(design_matrix), t))
    return w
```

```
In [3]: lab_4_dat = scipy.loadmat('/Users/wyattmadden/Documents/school/' +
                                  'MSU/2020/spring/m508/lab_info/lab_4/simple.mat',
                                  squeeze_me = True)

    x = lab_4_dat['x']
    t = lab_4_dat['t']
```

2 3.2

```
In [43]: hat_ten_10000 = lsefit(x = x,
                                t = t,
                                params = hat_basis(0, 2*np.pi, len(x)),
                                func = func_hat,
                                mu = 10**5)

    hat_df = {'hat_ten_10000': np.dot(np.transpose(hat_ten_10000),
                                      np.transpose(better_eval_basis(hat_basis(0, 2*np.pi,
                                                                               func_hat,
                                                                               x))),
              'x': x,
              't': t}

    hat_df = pd.DataFrame(hat_df)

    sns.lineplot(x = "x", y = "hat_ten_10000", data = hat_df)
    sns.scatterplot(x = "x", y = "t", data = hat_df).set(title = "mu = 10,000")
```

```
Out[43]: [Text(0.5,1,'mu = 10,000')]
```



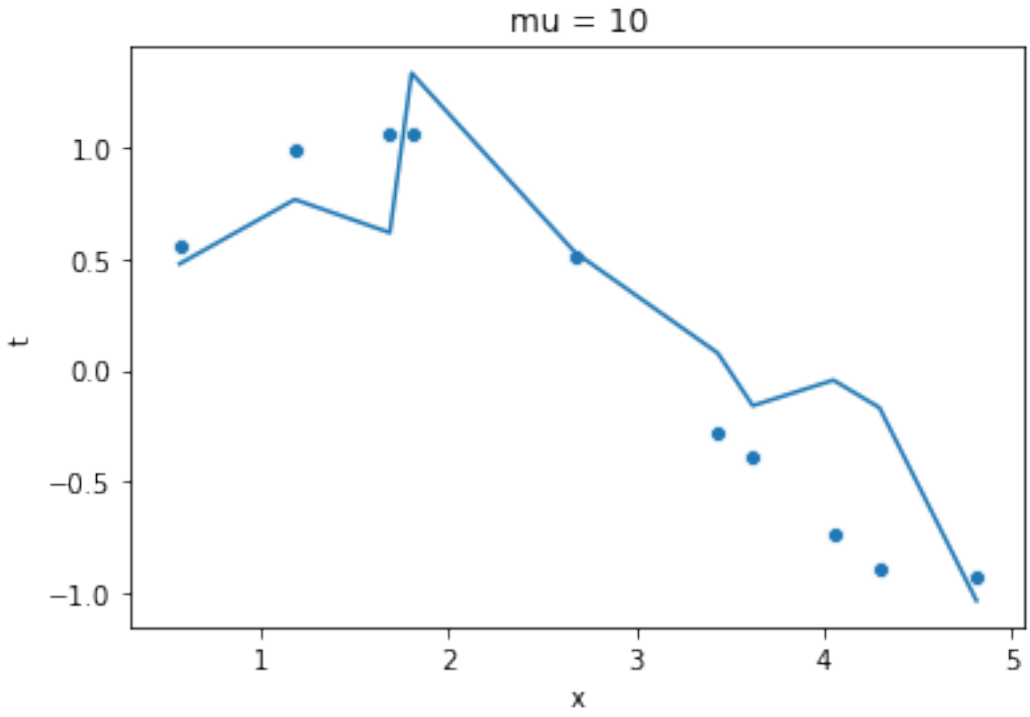
The fit of the hat basis function with mu of 10,000 is not a good approximation of the data. It is too responsive to subsequent data points, especially on the lower end of the x space.

3 3.3

```
In [44]: hat_ten_10 = lsefit(x = x,
                             t = t,
                             params = hat_basis(0, 2*np.pi, len(x)),
                             func = func_hat,
                             mu = 10)
         hat_ten_1 = lsefit(x = x,
                             t = t,
                             params = hat_basis(0, 2*np.pi, len(x)),
                             func = func_hat,
                             mu = 1)

         hat_df = {'hat_ten_10': np.dot(np.transpose(hat_ten_10),
                                        np.transpose(better_eval_basis(hat_basis(0, 2*np.pi,
                                                                                 func_hat,
                                                                                 x))),
                   'hat_ten_1': np.dot(np.transpose(hat_ten_1),
                                        np.transpose(better_eval_basis(hat_basis(0, 2*np.pi,
                                                                                 func_hat,
                                                                                 x))),
```

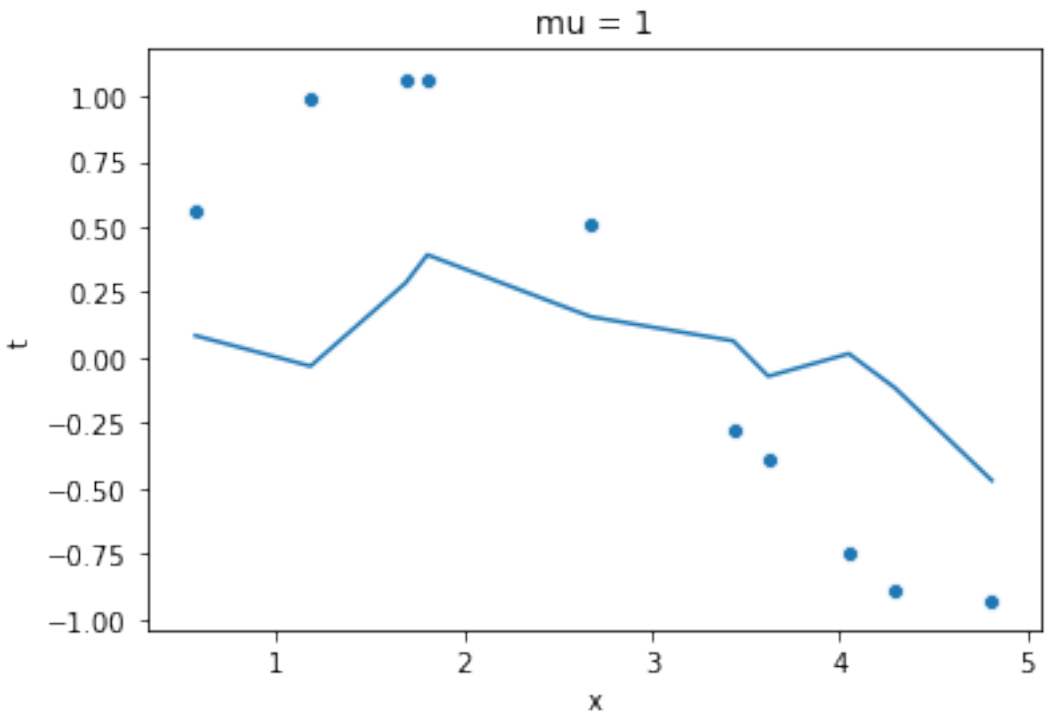
```
                'x': x,
                't': t}

        hat_df = pd.DataFrame(hat_df)

        sns.lineplot(x = "x", y = "hat_ten_10", data = hat_df)
        sns.scatterplot(x = "x", y = "t", data = hat_df).set(title = "mu = 10")
```

Out[44]: [Text(0.5,1,'mu = 10')]




```
In [45]: sns.lineplot(x = "x", y = "hat_ten_1", data = hat_df)
         sns.scatterplot(x = "x", y = "t", data = hat_df).set(title = "mu = 1")
```

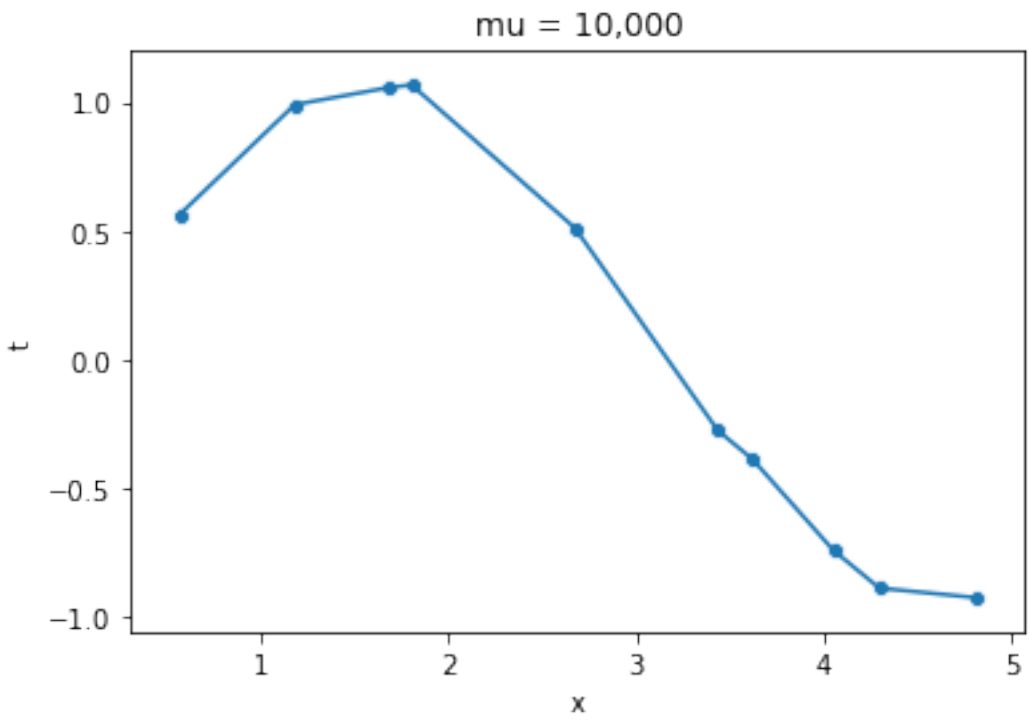
Out[45]: [Text(0.5,1,'mu = 1')]



4 3.4

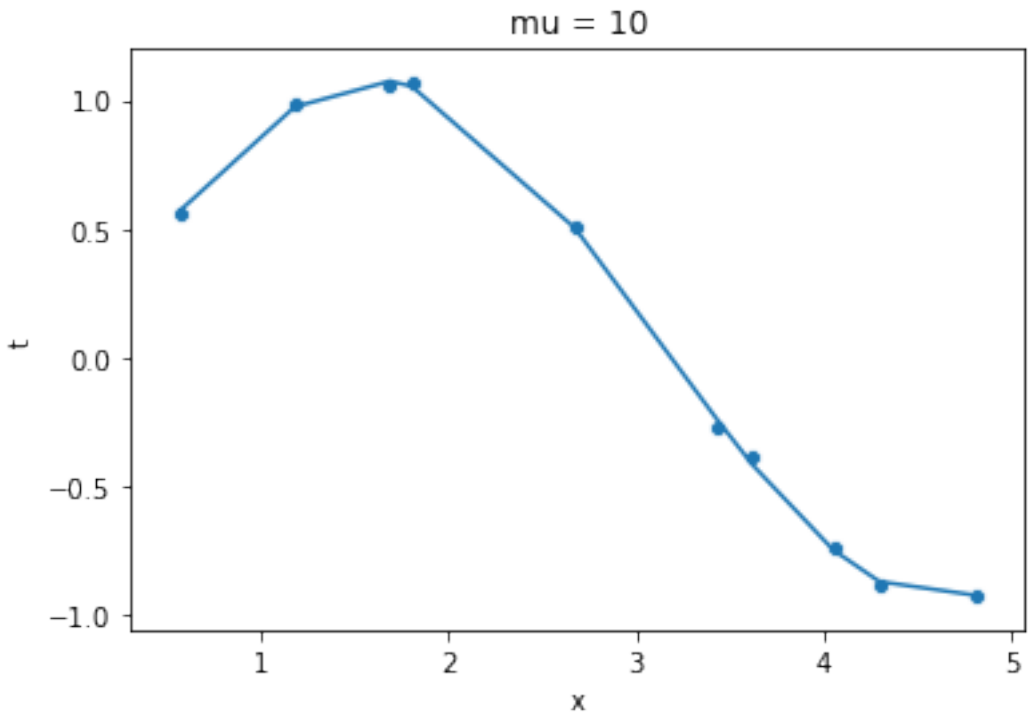
5 3.5

```
In [48]: gauss_ten_10000 = lsefit(x = x,
                                  t = t,
                                  params = gauss_basis(0, 2*np.pi, len(x)),
                                  func = func_gauss,
                                  mu = 10**5)

         gauss_df = {'gauss_ten_10000': np.dot(np.transpose(gauss_ten_10000),
                                               np.transpose(better_eval_basis(gauss_basis(0, 2*np.pi
                                                                              func_gauss,
                                                                              x))),
                     'x': x,
                     't': t}

         gauss_df = pd.DataFrame(gauss_df)

         sns.lineplot(x = "x", y = "gauss_ten_10000", data = gauss_df)
         sns.scatterplot(x = "x", y = "t", data = hat_df).set(title = "mu = 10,000")

Out[48]: [Text(0.5,1,'mu = 10,000')]
```

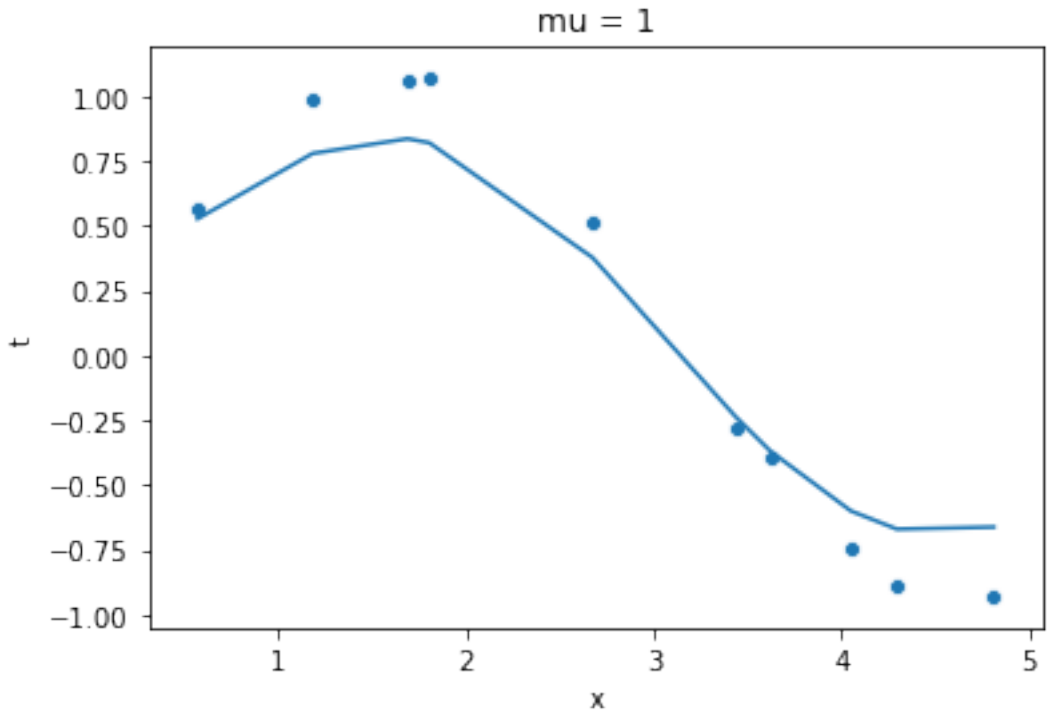


6 3.6

```
In [49]: gauss_ten_10 = lsefit(x = x,
                               t = t,
                               params = gauss_basis(0, 2*np.pi, len(x)),
                               func = func_gauss,
                               mu = 10)

         gauss_df = {'gauss_ten_10': np.dot(np.transpose(gauss_ten_10),
                                            np.transpose(better_eval_basis(gauss_basis(0, 2*np.pi
                                                                           func_gauss,
                                                                           x))),
                     'x': x,
                     't': t}

         gauss_df = pd.DataFrame(gauss_df)

         sns.lineplot(x = "x", y = "gauss_ten_10", data = gauss_df)
         sns.scatterplot(x = "x", y = "t", data = hat_df).set(title = "mu = 10")

Out[49]: [Text(0.5,1,'mu = 10')]
```




```
In [50]: gauss_ten_1 = lsefit(x = x,
                              t = t,
                              params = gauss_basis(0, 2*np.pi, len(x)),
                              func = func_gauss,
                              mu = 1)

         gauss_df = {'gauss_ten_1': np.dot(np.transpose(gauss_ten_1),
                                           np.transpose(better_eval_basis(gauss_basis(0, 2*np.pi
                                                                          func_gauss,
                                                                          x))),
                     'x': x,
                     't': t}

         gauss_df = pd.DataFrame(gauss_df)

         sns.lineplot(x = "x", y = "gauss_ten_1", data = gauss_df)
         sns.scatterplot(x = "x", y = "t", data = hat_df).set(title = "mu = 1")

Out[50]: [Text(0.5,1,'mu = 1')]
```



7 3.7

8 3.8

9 3.9

10 3.10

```
In [51]: gauss_ten_1 = lsefit(x = x,
                              t = t,
                              params = gauss_basis(0, 2*np.pi, 100),
                              func = func_gauss,
                              mu = 1)

         gauss_df = {'gauss_ten_1': np.dot(np.transpose(gauss_ten_1),
                                           np.transpose(better_eval_basis(gauss_basis(0, 2*np.pi
                                                                          func_gauss, x))),
                     'x': x,
                     't': t}

         gauss_df = pd.DataFrame(gauss_df)
```

```
sns.lineplot(x = "x", y = "gauss_ten_1", data = gauss_df)
sns.scatterplot(x = "x", y = "t", data = hat_df).set(title = "mu = 1")
```

Out[51]: [Text(0.5,1,'mu = 1')]