Introduction to Computational Physics SS2020

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Exercise 2 from April 29, 2020
Return before noon of May 8, 2020

1 Numerical Simulation of the 2-Body Problem

- Write a computer program that computes the relative motion of two point-like bodies under their mutual gravitational influence. Use a step-by-step Euler integration procedure. Set $G = 1$, $M_1 = M_2 = 1$.
- For which velocity v_0 can the two bodies rotate around each other in a circular fashion with a separation of 1?
- Now perform the numerical integration. Use, for the moment, a constant time step of $\Delta t = 0.01$. What happens with the numerical model if you choose $v_0/\sqrt{2}$ as the initial velocity? Make a plot of the orbits.
- Compute the eccentricity from the Runge-Lenz-Laplace vector.
- What happens if you choose an initial velocity larger than $\sqrt{2}v_0$?
- Simulate another case, like e.g. the $v_0/3$ initial velocity case. Experiment with decreasing the time step (and simultaneously increasing the number of time steps) and see how the results change.

2 Error Analysis of Euler Scheme (HOMEWORK)

(a) Choose 3 different eccentricities by varying the initial velocity, and study the set-up for a wide range of different time steps. Make sure that Δt spans orders of magnitude. Integrate the 2-body problem for two orbits. Plot, in a double-logarithmic fashion, the error in the energy at the end of this orbit as a function of the time step Δt. Discuss the result, is it consistent with what one should expect? (10 points)

(b) Do the same as above, but now employ the leapfrog integrator scheme. How does the result change? (10 points)

NOTE again, for this and all following homework exercises, which require writing a computer program, it is necessary to submit all parts of the program written by you, together with the results. It must be comprehensible for your tutor how your program generates your results.