

# Introduction to Computational Physics – Exercise 11

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## Importance Sampling

**Task:** Compute the integral

$$I = \frac{1}{\pi} \int_{-\infty}^{+\infty} \exp(-y_1^2 - y_2^2) dy_1 dy_2 \quad (1)$$

numerically with Monte Carlo methods; compare equally distributed random numbers (RN's) again with an appropriate importance sampling. *Hints:*

- First use equally distributed RN's in  $[-5, +5]$ .
- Second use importance sampling with a suitable function. *Hint:* Use the function provided in the Box Muller algorithm as given in the lecture:

$$y_1 = \sqrt{-2 \ln(x_1)} \cos(2\pi x_2) \quad y_2 = \sqrt{-2 \ln(x_1)} \sin(2\pi x_2) \quad \text{with } x_1, x_2 \in [0, 1] \quad (2)$$

## Random Walk

**Task:** Construct a stochastic process, which has the probability density function

$$g(y_1, y_2) = \frac{1}{2\pi} \exp\left\{-\frac{y_1^2}{2} - \frac{y_2^2}{2}\right\} \quad (3)$$

as an equilibrium distribution (Metropolis method). After an initial phase of  $i$  random walk steps store the sequence of  $k$  pairs of numbers  $y_1, y_2$ ; the sequence  $y_1, y_2$  is a stochastic representation of the underlying equilibrium distribution. For each sequence a “measurement” of the value of the integral is obtained, and the expectation value is the average of many such measurements.

Create  $N \in \{100, 1000, 10000\}$  of such sequences  $y_1, y_2$  and determine the integral as average of all measurements, and plot the error as function of  $N$ . What is a good choice for the number of initial steps  $i$ ? Just check for convergence of the result.