Introduction to Computational Physics SS2020

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Exercise 7 from June 10, 2020
Return before noon of June 19, 2020

1 Many Species Population Dynamics

In this exercise we study the evolution of 6 populations according to the following equations for population dynamics: 3 predator- (P_i) and 3 prey-species (N_i), all parameters positive, always $i, j = 1, \ldots, 3$:

$$\begin{aligned}
\frac{\mathrm{d}N_i}{\mathrm{d}t} &= N_i \left(a_i - N_i - \sum_j b_{ij} P_j \right) \\
\frac{\mathrm{d}P_i}{\mathrm{d}t} &= P_i \left(\sum_j c_{ij} N_j - d_i \right)
\end{aligned}$$

The parameters chosen are $a_1 = 56$, $a_2 = 12$, $a_3 = 35$; $d_1 = 85$, $d_2 = 9$, $d_3 = 35$; the parameters bij and c_{ij} are given in matrix form here:

$$b_{ij} = \begin{pmatrix} 20 & 30 & 5 \\ 1 & 3 & 7 \\ 4 & 10 & 20 \end{pmatrix}$$

$$c_{ij} = \begin{pmatrix} 20 & 30 & 35 \\ 3 & 3 & 3 \\ 7 & 8 & 20 \end{pmatrix}$$

Notice: the unusual feature here in the equations is that the prey populations N_i have a Verhulst style growth limiting factor in their equations, which limits their growth even if there is no predator (model for limited resources even in absence of predators).

Another Notice: please do not try to make the equations dimensionless, just use the numbers given here.

2 Stability Analysis (Homework)

1. (5 points) What are the fixed points for the system of equations given above? Hint 1: No complicated computations are necessary, the idea is that you should guess the fixed points very easily. Compare our previous examples.

Hint 2: This time there are three fixed points! In addition to our 'usual' ones, there is a third one related to the Verhulst growth limiting factor in the first three equations.

2. (5 points) What is the Jacobi matrix $\mathbf{A}$ at the non-trivial fixed point? (non-trivial FP means here that ALL elements are unequal to zero. There is only one FP with this property)

3. (10 points) Determine the eigenvalues and eigenvectors λ_i and $\mathbf{v}_i$, $i = 1, \ldots 6$ of $\mathbf{A}$ for this fixed point.

Choose an initial state $\mathbf{n} = \sum_{i=1}^{6} c_i \mathbf{v}_i$, with $c_1 = c_2 = 3$, $c_3 = c_4 = 1$, $c_5 = -5$, $c_6 = 0.1$.

Plot and discuss the time dependent evolution of the six populations (given by this linear model); in particular what about oscillations, growth or decay?

Notice: In this mathematical example the solutions can become negative in some cases, which is unphysical for population dynamics. Nevertheless let us please use this case to show the mathematical features.