Introduction to Computational Physics – Exercise 9

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The Lorenz Attractor

Task: Solve numerically, using `rk4`, the above coupled set of equations for the values $r \in \{0.5, 1.17, 1.3456, 25.0, 29.0\}$. Choose the initial conditions near one of the fixed points: $C_\pm$ for $r > 1$ and $\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$ for $r < 1$. Explain the behavior, as much as possible, with the stability properties of the fixed points.

Task: Determine the sequence z_k for $r = 26.5$, where z_k is a local maximum in z on the solution curve after k periods. Plot z_{k+1} as a function of z_k. When sufficient points are there, connect the points. The resulting function $z_{k+1} = f(z_k)$ has an intersection with the diagonal $z_{k+1} = z_k$. It is a fixed point of the function $f(z_k)$. Is the slope m of this function > 1, < -1 or between -1 and $+1$? Notice: The theory of discrete maps says that there is no periodic solution if $|m| > 1$. So, in such a case we can deduce that this solution of the Lorenz system is not periodic.