Introduction to Computational Physics – Exercise 11

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Importance Sampling

Task: Compute the integral

$$I = \frac{1}{\pi} \int_{-\infty}^{+\infty} \exp(-y_1^2 - y_2^2)\,\mathrm{d}y_1 \mathrm{d}y_2 \tag{1}$$

numerically with Monte Carlo methods; compare equally distributed random numbers (RN's) again with an appropriate importance sampling. *Hints:*

- First use equally distributed RN's in $[-5, +5]$.
- Second use importance sampling with a suitable function. *Hint:* Use the function provided in the Box Muller algorithm as given in the lecture:

$$y_1 = \sqrt{-2\ln(x_1)}\,\cos(2\pi x_2) \qquad y_2 = \sqrt{-2\ln(x_1)}\,\sin(2\pi x_2) \qquad \text{with } x_1,\, x_2 \in [0,\, 1] \tag{2}$$

Random Walk

Task: Construct a stochastic process, which has the probability density function

$$g(y_1,\, y_2) = \frac{1}{2\pi} \exp\left\{-\frac{y_1^2}{2} - \frac{y_2^2}{2}\right\} \tag{3}$$

as an equilibrium distribution (Metropolis method). After an initial phase of i random walk steps store the sequence of k pairs of numbers y_1, y_2; the sequence y_1, y_2 is a stochastic representation of the underlying equilibrium distribution. For each sequence a "measurement" of the value of the integral is obtained, and the expectation value is the average of many such measurements.
Create $N \in \{100,\, 1000,\, 10000\}$ of such sequences y_1, y_2 and determine the integral as average of all measurements, and plot the error as function of N. What is a good choice for the number of initial steps i? Just check for convergence of the result.