# Introduction to Computational Physics – Exercise 7

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June 12, 2020

## Population dynamics

In this exercise we study the following equation for population dynamics:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = rN(1 - N/K) - \frac{BN^2}{A^2 + N^2} \tag{1}$$

where all parameters $r$, $K$, $A$ and $B$ are positive. It is a more complex example, in which the growth behaviour depends on whether $N$ is smaller or larger than a critical populations size $A$.

### Dimensional analysis

Determine the dimension of the parameters and rewrite the equation in dimensionless form. Note that there are different possibilities. Please formulate a dimensionless time $\tau$ that is not defined on the basis of $r$. Use $n = N/A$ as the dimensionless version of $N$.

### Stationary points

Determine the stationary points $n^*$ for $K/A = 7.3$. Note that for $n^* \neq 0$ these values are solutions of a cubic equation; it depends on $n$ and the remaining free parameter. The cubic equation should be derived by yourself analytically; its zero points you can obtain numerically / graphically by using e.g. Mathematica. When do one or three real solutions exist as a function of the remaining free parameter? (Hint: we do not ask for some analytical formula here! It is enough to vary the free parameter and check using Mathematica which three solutions for the stationary points you get; as stationary points only real solutions are valid. Only one digit after the comma is enough, in other words you vary the free parameter by about 0.05.). Which of the stationary points is stable and unstable?