Proposal for Pound-Drever-Hall integrated laser stabilization system

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Project Description: Utilizing the Pound-Drever-Hall technique, we aim to invent our integrated device that can lock a laser to the resonance of a cavity whose quality factor(Q) is of 100 million at 976nm. Due to the high Q of the cavity, any laser wavelength drift larger than 0.00000976nm will cause a failure of cavity locking. Our goal is to lock the frequency at resonance for several minutes at a time, where Δt, the duration from the first drop of the transmission signal(t_i) to the first recovery back to the initial(t_f) is at the order of minutes, which exceeds the timescale for quantum experiments and demonstrates the stability of our integrated device.

1 Background and Impacts

With the great revolution in III-V semiconductor fabrication skills, various kinds of quantum systems become experimentally accessible to us. One type of system (this project aims to target) is an on-chip ring resonator, which can couple to an external electric field and generate a resonance inside. This is mathematically identical to a Fabry Perot resonator [1], characterized by the airy function:

$$F(\omega) = \frac{E_{\mathrm{ref}}}{E_{\mathrm{inc}}} = \frac{r\left(\exp\left(\frac{i\omega}{\Delta\nu_{\mathrm{fsr}}}\right) - 1\right)}{1 - r^2 \exp\left(\frac{i\omega}{\Delta\nu_{\mathrm{fsr}}}\right)}, \tag{1}$$

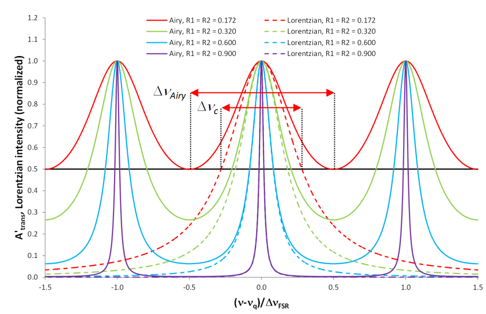

Figure 1: Different reflectance signal from the cavity, and a higher quality factor(Q) is represented by a more narrow Lorenzen shape

The narrower and more centered the Lorentzian shape is, the better the ability of the cavity to confine energy within it (characterized by the quality factor(Q) that is defined as $\frac{w_{peak}}{FWHM}$), which makes the generation of the on-chip nonlinear optical effect more power efficient. The ring resonator for this experiment is a device with $Q \approx 100M$ at a resonance of 976nm. *If one conducts a quantum information experiment on it (e.g. the generation of an entangled photon pair), a tiny fluctuation in laser wavelength of about 0.00000976nm would result in a 50% drop in the efficiency of generating an entangled photon pair.* Therefore, it is crucial to have control of the output wavelength to maintain it at the resonance of the cavity–this is known as $\boxed{\text{cavity locking}}$.

For any input-output system that requires a stabilized output, the key is to generate an error signal, which is an indicator of how far the output is away from the desired output. This error signal will be fed to a proportional-–integral–derivative(PID) controller, which adjusts the input to the system until an expected output is met.

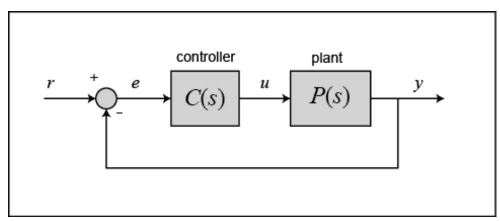

Figure 2: Flow Chart of a PID system. The plant will be the laser, and the controller is the integrated device we want to invent. The PID is an algorithm defined within the memory of the controller. The output of the system that loops back again is the voltage signal of the photo-detector (PD) of the ring cavity. The signal e is the error signal we need to experimentally generate through (PDH). The output of the controller is the voltage applied to the piezo of the laser, which can change the output wavelength of the laser.

There are various ways of generating the error signal, and the method we employed is the Pound-Drever-Hall technique (PDH). PDH applies a fast modulation through an electro-optic modulator on the input to the cavity and demodulates the output of the cavity with another sine wave to get the error signal. This signal will be generated in an experiment we will set up (see Project Design for details). Compared to other techniques, the essential advantage of PDH is that the generated error signal has a strong sensitivity that can respond quickly to the fluctuation of laser frequency when fed to the PID controller. For this reason, PDH locking is also the most widely used method in commercial laser lock boxes due to its strong sensitivity.

$$\epsilon = -2\sqrt{\frac{P_c}{P_s}} \Im \left\{ F(\omega)F^*(\omega+\Omega) - F^*(\omega)F(\omega-\Omega) \right\}. \tag{2}$$

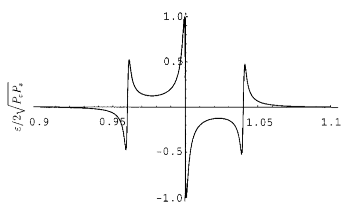

Figure 3: The strong sensitivity is represented by the sharp slope around 0.

Instead of using an expensive commercial laser lock box(around \$25,000), this project aims to reproduce the error signal through PDH and use it to lock a cavity with $Q \approx 100M$ at 976nm for at least an average time scale of minutes (This is 10^6 order of magnitude for a quantum bit operation, long enough for the quantum process to take place). While commercial lock boxes may offer locking capabilities for hours, their high cost is often unnecessary for quantum experiments, which typically operate on an extremely short timescale. Maintaining the performance of minute-scale cavity locking, our project presents a significant cost-saving opportunity and has the potential to be developed into a commercial product as a budget alternative.

2 Project Design

The schematics of our experiment setup would be as follows. We aim to invent an integrated device (orange box), where the generation of error signal (the LOCK block) and PID algorithm are all integrated.

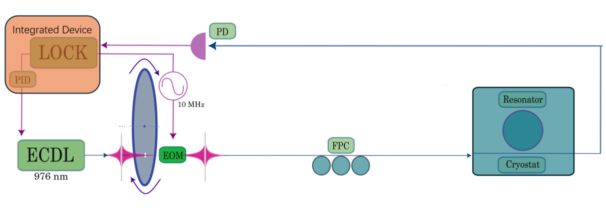

Figure 4: The external cavity diode laser(ECDL) would generate a laser at a frequency of around 976nm. The signal is modulated by the electro-optic modulator (EOM) at 10 MHz. The polarization controller (FPC) receives the signal. From here it is fed into the cryostat chamber to couple onto the resonator. The lock-in amplifier (LOCK) receives both the driving frequency from the EOM as a reference signal and the transmission signal from the resonator. Finally, the lock-in amplifier feeds the proportional-integral-derivative controller (PID) with the error signal which locks the laser at resonating frequency.

We follow this flow chart and break down each part to explain the project design, where the block "LOCK", the core of the experiment, will be specifically discussed later.

Electro-optic Modulator(EOM): Electro-optic modulators are designed to modulate the phase of electromagnetic radiation propagating through them. It is a free-space optics component that requires careful alignment. The material inside EOM is usually lithium niobate, whose induced birefringence is linearly proportional to the amplitude of the electric field (which will be experimentally controlled as the voltage applied to the EOM). The EOM has a BNC cable as input, which will be connected to a function generator producing a sine wave at 10 Mhz. After passing through the EOM, the phase will be modulated to:

$$E_0 e^{iwt} \to E_0 e^{i(wt+\beta \sin(\Omega t))} \tag{3}$$

where β and Ω are the amplitude and frequency of the sin wave generated by the function generator. Using Jacobi-Anger expansion[1], when the modulation depth is weak enough:

$$E_0 e^{i(wt+\beta \sin(\Omega t))} \approx E_0 e^{iwt}(1 + J_1(\beta)e^{i\Omega t} - J_1(\beta)e^{-i\Omega t}) \tag{4}$$

where J_1 is the first order Bessel function. This can be understood as a superposition of three signals (I will use ket to present them): $|w\rangle + |w+\Omega\rangle + |w-\Omega\rangle$

Polarization Controller(FPC): FPC is a device that controls the polarization of light within a fiber by applying mechanical strain on the fiber. It is used to tune the polarization until a clear resonance signal (a Gaussian peak) appears on the photondetector (PD)

Resonator: Resonator is a ring that can couple to the evanescent field from the fiber. If the wavelength of the light matches an integer multiple of the effective circumference (this is not the actual circumference as different polarization directions would receive a different circumference), a standing wave would form within the cavity, which means a significant of energy is stored inside the cavity and one would expect a dip in the output of photodetector (PD)

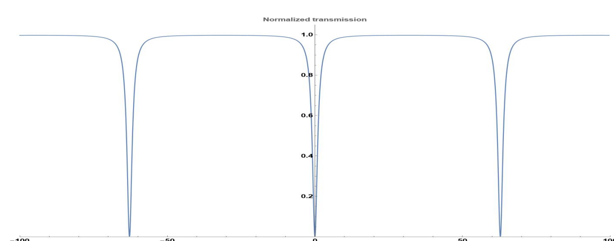

Figure 5: A theoretical calculation based on a Fabry Perot Cavity with $\delta w = 10s^-1$ and reflection $r = 0.95$[1]

The wavelengths where the peaks occur are the targets we want our laser wavelength to be locked to. However, due to the lack of a spectrometer, we do not have direct access to the wavelength. We can only measure the voltage applied on the piezo of the laser which will tune the wavelength of the output. **Therefore, our integrated device aims to continuously output a voltage that will lock the wavelength to one of the peaks. This output voltage should change to compensate for the laser fluctuations from all of the possible factors discussed above.**

Photondetector(PD): PD is a device that can measure the number of photons incident within a time window (δt), and outputs it as a voltage signal. The output bandwidth of the PD we are using, Newport 1811, goes up to 125M Hz, which is considered a high-speed signal that our integrated device must take care of to prevent signal interference.

LOCK: This is the $\boxed{core}$ of our project.

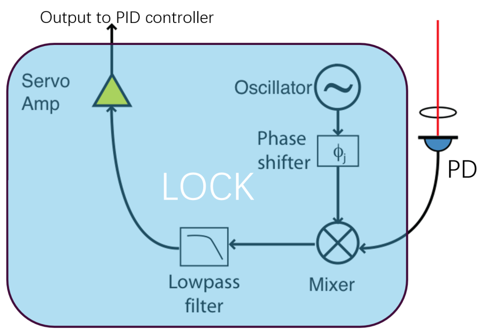

Figure 6: asdfasdfasd

Recall that the light fed into the cavity, after modulated by the EOM, becomes effective as $|input\rangle = |w\rangle + |w+\Omega\rangle + |w-\Omega\rangle$, where w is the resonant frequency and Ω is the modulation frequency. What we want is the output power measured on the PD side. Say the response function of the cavity to light at different frequencies is $\hat{F}(w) = \frac{E_{reflect}}{E_{input}}$[1]. The output voltage at the PD then is proportional to:

$$\begin{aligned} P_{\text{out}} &= |\hat{F}|input\rangle|^2 \\ &= P_c|F(\omega)|^2 + P_s\left\{|F(\omega+\beta)|^2 + |F(\omega-\beta)|^2\right\} + \\ 2\sqrt{P_cP_s}&\text{Re}\left\{\boxed{F(\omega)F^*(\omega+\beta) - F^*(\omega)F(\omega-\beta)}\right\}\cos\beta t + \\ &\text{Im}\left\{\boxed{F(\omega)F^*(\omega+\beta) - F^*(\omega)F(\omega-\beta)}\right\}\sin\beta t + \\ &(2\beta \text{ terms}). \end{aligned} \tag{5}$$

The complex function boxed in the equation is the error function, $\epsilon(w,\Omega)$ we are looking for PID control. In our project, we will only need the $\boxed{ImaginaryPart}$ for cavity locking. As you can see, the output signal at the PD side is already a modulated superposition of $|w\rangle \otimes |w \pm \Omega\rangle$. All of the difficulties reside in the extraction of this signal from P_{out}.

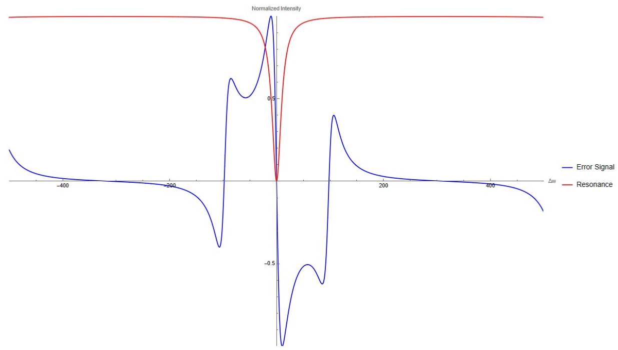

Figure 7: Calculated normalized resonance transmission signal and imaginary part of the error signal as a function of the deviation from the resonance frequency δw [1]. This should be experimentally observed on the oscilloscope.

The combination of an RF mixer (see difficulty part for detailed discussion) followed by a low pass filter (see difficulty part for detailed discussion) can lead to the extraction of the imaginary part if we demodulate the signal correctly (which means multiplying P_{out} by a sin wave that is correct in phase and has the same frequency as the modulation frequency), For this part, by choosing the correct electronic components, we will draw and design our own PCB board for the RF mixer and low pass filter, with additional necessary signal amplifiers and filters. We will figure out the correct phase by experimentally testing it. In reality, it is only within 0-180 degrees.

PID:

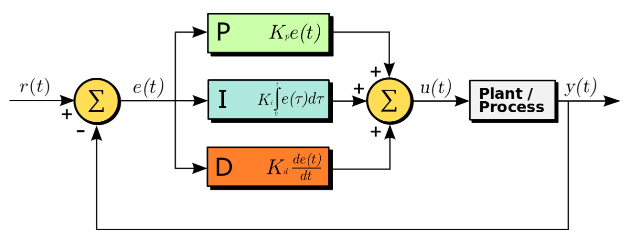

Figure 8: Caption

PID Stands for Proportional-Integral-Derivative algorithm, which uses controlled input to minimize the error between the measured output and the desired output, as shown in Figure 8. The three parameters in the algorithm (P, I, D) contribute to the controlled input which is described by:

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)d\tau + K_d\frac{de(t)}{dt} \tag{6}$$

$u(t)$ is the controlled input (in this case the controlled input is the voltage to the piezo in the laser cavity), and $e(t)$ is the error between the measured output and the desired output. K_p determines the ratio of $u(t)$ response to $e(t)$, which is a term that only accounts for the magnitude of the error signal. On the other hand, the second term is proportional to the both the magnitude and the time of the error. By integrating the error with respect to time, the second term compensates for the first term by helping to reduce the error during a period of time[2]. However, the integral will potentially cause an overshoot because it collects the errors cumulatively. The third term is thus existed to reduce the magnitude of overshoot by predicting the system's future behavior using rate of change of the error. Although, it improves the system's settling time, it also increases noise in the error signal.

Metric: The metric to calibrate the stability of the system is the average time the cavity remained locked. Due to the high Q of the cavity, the result is binary–it either remains locked or unlocked. The resonance is treated as a delta function that the transient behavior happens too fast which is negligible and not of our interest. When the cavity is in resonance, all of the energy is dumped into the cavity. Therefore, a drop in the transmission signal is expected. When the resonance is off, the energy recovers back to its initial level.

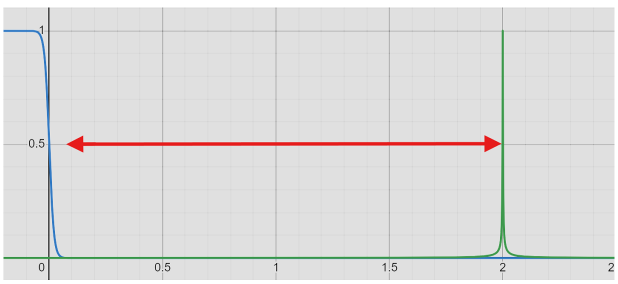

Figure 9: This is an example: A normalized graph of what would appear on the oscilloscope. The sudden drop represents the beginning of a successful locking, and the narrow Lorentzian peak represents a quick off-resonance. The duration time is taken as the horizontal distance between two spikes.

At the end of our project, we will perform multiple measurements and report the average duration time, which is expected to be on the order of **60s**, which is way longer (10^6 times) than most quantum processes.

In addition, for a backup metric to measure the long-term behavior, we propose another metric to define the performance of locking: Given a fixed period T

$$e = \frac{\delta t}{T} \tag{7}$$

where δt is the time the cavity stays on lock. The long-term behavior is important but not necessary for this project, as reaching a continuous locking for minutes is the basis for conducting quantum experiments on resonance. We still expect this ratio to be larger than 80%, but achieving this metric is not the main goal of our project.

3 Anticipated Challenges:

I will divide our challenges into two parts: hardware&software

Hardware:

1. **Low pass filter(Week3-5)**: This is a compromise between a fast, responsive error signal or a smooth error signal with larger latency.

$$V_{out} = V_{in} \frac{\frac{1}{iwc}}{R + \frac{1}{iwc}} \tag{8}$$

A strong and effective filter can suppress the high-speed noise but introduce a stronger phase delay according to Eq.8. This will introduce a large latency of the PID correction applied on the piezo, which will impact the performance of locking. A quick response filter, however, cannot effectively filter away the high-speed signal. As the minimum time scale of frequency drift in the laser could go down to 1 us, for effective cavity locking, we need a filter that imposes a delay of no longer than 1 us. Depending on the situation: (a)If achieving less than 1us delay does not filter out the signal(which means on the hardware side, that's the best it can achieve), we will impose a software filter in the PID algorithm using computational skill to remove it. For example, there is open source–Finite Impulse Response (FIR) Filters–that could perform real-time filtering. (b)If on the hardware side, this problem is solved, no more software correction is needed.

2. **RF Mixer(Week3-5)**: As the output bandwidth of the PD NewPort 1811 goes up to 125Mhz, and the local oscillator will oscillate at 10M hz, the mixer is operating at a range from radio frequency to high-speed range. Here are several considerations and solutions: (a) Conversion Loss: the output of a passive mixer can attenuate the error signal up to 5-9dB. Depending on the budget, we could either switch to an active mixer which can provide conversion gain, or use a combination of passive mixer + amplifying circuit(but this will introduce further latency to the signal). (b) Signal distortion: we must make sure that the mixer can handle this high-speed signal and also the impedance matches with the rest of the system(by placing negative back Op-amp as a buffer).

3. **ADC and DAC issue(Week5-7)**: TEENSY 4.1 will dynamically allocate the ADC sample rate. The default resolution is at 10 bits and a sample rate of 44.1kHz. We can increase the sample rate at the cost of decreasing the resolution. However, if this cannot afford a robust cavity locking, we will apply an external ADC module. In addition, the output of TEENSY can afford almost zero output current, which means it is a pure voltage signal. To address this issue, external DAC module can be adapted to TEENSY. Our solutions are: (a) for DAC, we choose MCP4725 DAC module which offers a 12-bit resolution and V_{dd} ranges from 2v to 5v. This can meet our requirement to drive the laser piezo, and it is communicated through I2C protocol with TEENSY. (b) for ADC, we choose LTC2380-24, an extremely powerful ADC module using SPI communication protocol

that offers 24-bit resolution and 1.5 M sample per second. *Note: External ADC& DAC might not be necessary but they might be a possible issue when dealing with signal processing. If these issues happen, these two (a) and (b) are the actions we will take.*

Software:

1. **Real Time Operating System(RTOS)(Week6-8):** To perform a real-time PID, we plan to use Interrupt to handle the correction on piezo given the error signal. However, since we are all physics majors, coding in C/C++ about embedded systems requires extra learning and time. Developing a sound and stable RTOS for cavity locking takes time and effort.
2. **Determining the P, I, and D(Week6-8):** To achieve a stable cavity locking, the coefficients P, I, and D must be carefully chosen. A wrong value not only leads to a failure of locking but also can destroy the laser. We will perform extensive experiments to test which set of parameters gives the best performance. For the safety of the laser, we will implement Schottky diodes or transient voltage suppressors (TVS) near the piezo actuator connection to protect against static discharge and voltage spikes.

4 Summary

Confident in our collective expertise and the preparatory groundwork already laid, I(Hongrui Yan), with Zhaozhong Cao and Alexander Nazeeri, propose this 10-week PHYS13CL project in implementing, verifying, and inventing an integrated device that can perform Pound-Drever Hall frequency locking on a cavity whose Q is around 100M at 976nm. This device should, in the short term, provide a cavity locking on the timescale of 60s. Within the locking period, the wavelength drift should be within $10^{-}5$nm. In the long term, the proportion of time it stays on the lock is expected to be larger than 80%.

5 Reference

[1] Black, Eric D. "An introduction to Pound–Drever–Hall laser frequency stabilization." American journal of physics 69.1 (2001): 79-87.
[2] Ang, Kiam Heong, Gregory Chong, and Yun Li. "PID control system analysis, design, and technology." IEEE transactions on control systems technology 13.4 (2005): 559-576.

6 Parts List/Budget:

Truly thanks to Prof. Dirk Bouwmeester who is willing to support our project with a laser, EOM, and PD. Thank you!

Item	**Part #**	**Vendor**	**Qty**	**Price/Unit**	**Total Cost**	**Lead Time**
Op-Amp	TL072	DigiKey	5	$3.52	$17.60	In Stock
DAC	MCP4725	Mouser	5	$1.11	$5.55	In Stock
SAR ADC	ADS8866	TI	5	$2.31	$11.55	In Stock
Micro	TEENSY 4.1	SparkFun	2	$31.50	$63.00	In Stock
Opt Coupler	LTV-1009	LCSC	5	$0.37	$1.85	In Stock
RF Mixer	ADE-1ASK+	Mouser	2	$13.60	$27.20	In Stock
TVS	SMBJ400CA	Mouser	5	$0.63	$3.15	In Stock
Cap	100 uF 0805	Mouser	10	$1.01	$10.10	In Stock
PCB	Custom	DigiKey	5	$40	$200	5 days
					Total Cost with Tax (15%)	$422.15
*PCB cost pending quote from the manufacturer; budget cap at $200.						

Table 1: Components for Low-Pass Filter and PID Controller

7 Milestone/Project timeline:

Task	**Week 1**	**Week 2**	**Week 3**	**Week 4**	**Week 5**	**Week 6**	**Week 7**	**Week 8**	**Week 9**	**Week 10**
Writing Proposal										
Familiarizing PDH Theory										
Waiting for orders										
Aligning free space optics for EOM to modulate laser										
Generating Resonance From the cavity										
Implementing PID algorithm on microcontroller										
Demodulating the cavity response through the Mixer to generate the error signal										
Tuning the PID parameters until we reach a locking										
Integrating the necessary filters, signal amplifier, and mixer(if we decide to build our mixer) into one PCB board.										
Drawing the schematics and PCB file. Send the Gerber file to the manufacturer										
Soldering(if we are rich enough, we will just let the company do it)										
Refining and Testing the integrated device to make sure the stability of locking(need to lock for hours)										

Figure 10: The table here already accounts for any variations in delivery time or unexpected errors that may cause a delay in our project. For the core parts, we expect on average 3 weeks to overcome the difficulty and refine our algorithm/integrated device. As you can see, there are two parts to our project—hardware that generates the error signal and software that applies PID on the error signal. We plan to proceed in parallel: While we are working to obtain the error signal, the PID algorithm could initially be studied and implemented on the microcontroller (the exact values of P, I, and D are not of interest at this point). Hongrui will be mainly responsible for the hardware part, and Johnson and Alex will be in charge of the software algorithm.