PHOTOELECTRIC INSTABILITY IN OPTICAL FIBRE RESULTING IN SPONTANEOUS SECOND HARMONIC GENERATION

Indexing terms: Optics, Optical fibres, Harmonic generation

The self-organised second harmonic generation (SHG) in optical fibres is explained as a consequence of a convective instability leading to spontaneous formation of the $\chi^{(2)}$ grating due to the growth of small fluctuations. We show that the observed SHG should be interpreted as amplification of noise.

Introduction: Österberg and Margulis[1] discovered that the pumping of an optical fibre by intense laser radiation for several hours produces effective second harmonic generation (SHG). After the pump is switched off the memory of this process persists for a long time (several weeks) manifesting itself in an immediate appearance of the second harmonic if probe radiation with the same wavelength as the initial pump light is injected into the fibre. This phenomenon is rather unusual because it is not normally possible for SHG to appear in the amorphous material of the fibre because of the centrosymmetric structure of the fibre. Thus a photo-induced transition of the fibre to a noncentrosymmetric state with a nonzero second-order susceptibility $\chi^{(2)}$ must occur. This transition is called 'selfpreparation'. The preparation time may be reduced to several minutes if, along with the pumping light, the fibre is seeded by its second harmonic.[2] Irradiation by sufficiently short-wavelength light erases the susceptibility $\chi^{(2)}$ in a previously prepared fibre.[3]

Baranova and Zel'dovich[4] and Stolen and Tom[5] suggested that $\chi^{(2)}$ needed for SHG is produced through the joint action of the first and second harmonic fields, E_1 and E_2, so that $\chi^{(2)} \simeq E_1^2 E_2^*$. The susceptibility $\chi^{(2)}$ should thus be modulated along the fibre with a period $2\pi/q$, where $q = k_2 - 2k_1$, k_1 and k_2 being the wavevectors of the corresponding harmonics. This automatically provides phase matching in the SHG process. The existence of such a $\chi^{(2)}$ grating in a prepared fibre was established experimentally.[5]

A theoretical description of the stationary propagation of the first and second harmonics, based on the above assumption concerning the origin of $\chi^{(2)}$, was proposed in Reference 2. However, no satisfactory explanation of the selfpreparation effect has emerged. The main problem is that the stationary theory[2] does not show amplification of a weak second harmonic signal which could appear in the fibre due, for example, to the small quadrupol nonlinearity considered by Payne.[6]

Dianov *et al.*[7] have stressed the importance of taking into account the dynamics of $\chi^{(2)}$ formation. They have shown that under nonstationary conditions strong amplification of the initial second harmonic signal is possible.

We propose an explanation of the selfpreparation phenomenon as a consequence of an instability leading to the spontaneous formation of the $\chi^{(2)}$ grating due to the temporal growth of small fluctuations. We show that the stationary states obtained in Reference 2 are unstable and that the observed SHG should be interpreted as amplification of noise.

Basic equations: The SHG is described by the usual coupled-mode equation

$$\frac{\partial E_2}{\partial z} = i\frac{2\pi\omega}{n_2 c}\chi^{(2)}E_1^2 \exp(-iqz) \tag{1}$$

and by the following phenomenological equation determining the dynamics of $\chi^{(2)}$ writing and relaxation:[7]

$$\frac{\partial \chi^{(2)}}{\partial t} + \frac{\chi^{(2)}}{\tau} = \beta(E_1^*)^2 E_2 \exp(iqz) + \text{c.c.} \tag{2}$$

Here E_1 and E_2 are the complex amplitudes of the first and second harmonics, n_2 is the refraction index at frequency 2ω, τ is the relaxation time, β determines the rate of $\chi^{(2)}$ writing. The coefficient β is, generally speaking, complex. However, if both ω and 2ω are far away from the resonance frequencies of the medium, the imaginary part of β is small. We shall consider β to be real and positive. Following Reference 7, eqn. 2 can be understood by assuming that $\chi^{(2)}$ is proportional to a static electric field arising as a result of charge separation due to an interference current $j \simeq (E_1^*)^2 E_2$. τ should be regarded as the Maxwell relaxation time.

We assume that the dark time τ is very long, which accounts for the memory effect. Under irradiation τ is presumably reduced depending on the first and second harmonic intensities. This explains the difference between the erasure time and the preparation times with and without the seeding second harmonic light.

Assuming that E_2 and the amplitude of the $\chi^{(2)}$ grating change smoothly on the scale of the period $2\pi/q$ we may neglect the second term in the right-hand side of eqn. 2. We can then rewrite eqns. 1 and 2 as follows:

$$\frac{\partial E_2}{\partial z} = \frac{i}{l}\mathscr{E} \qquad \frac{1}{l} = \frac{2\pi\omega}{n_2 c}|E_1|^4\beta\tau \tag{3}$$

$$\frac{\partial \mathscr{E}}{\partial t} = \frac{1}{\tau}(E_2 - \mathscr{E}) \tag{4}$$

where the complex quantity $\mathscr{E}$ determining the amplitude and the phase of the $\chi^{(2)}$ grating is defined by

$$\chi^{(2)} = (1/2)|E_1|^2\beta\tau[\mathscr{E}\exp(iqz) + \text{c.c.}] \tag{5}$$

Instability of stationary state: The stationary solution of eqns. 3 and 4, $\mathscr{E} = E_2 = E_2(0)\exp(iz/l)$ coincides with the one obtained in Reference 2 if $ql \gg 1$. However this solution is unstable. Indeed, putting $E_2, \mathscr{E} \simeq \exp(\lambda t + ipz)$ we obtain

$$\lambda = \frac{1}{\tau}\left(\frac{1}{pl} - 1\right) \tag{6}$$

i.e. fluctuations with $p < l^{-1}$ grow with time. Thus if initially $\chi^{(2)} = 0$ and $E_2 = 0$, these quantities should spontaneously appear due to the growth of small fluctuations.

To understand this result, suppose that initially there was a fluctuation of $\chi^{(2)}$ with a space period $2\pi/q$; the second harmonic amplitude E_2 and consequently the writing rate, would be proportional to $\chi^{(2)}$ and to the distance z, whereas the relaxation rate, also proportional to $\chi^{(2)}$, is z-independent. Therefore at large enough distances ($z > l$) the writing rate would exceed the relaxation rate and the fluctuation would grow. Note that the instability is of convective nature.

We emphasise that if the pump light is monochromatic and the fibre is infinitely long, the instability should occur for arbitrary low pump intensity, i.e. no threshold exists. In a finite sample substantial growth of fluctuations is possible only if the fibre length is greater than the characteristic length l which becomes very large at low pump intensity (see eqn. 3). It may be also shown that for pulsed pumping an instability threshold appears determined by the condition $l < L$, where L is the walk-off distance.

By solving eqns. 3 and 4 it can be shown that an initially localised fluctuation of $\chi^{(2)}$ transforms into a grating with a period close to $2\pi/q$. At distances $z > l$, the grating amplitude $\mathscr{E}$, and E_2, change in time as

$$\begin{aligned} E_2 &\simeq \mathscr{E} \\ &\simeq \Phi(z, t) \\ &= \exp\left[\left(\frac{2zt}{l\tau}\right)^{1/2} - \frac{t}{\tau}\right] \end{aligned} \tag{7}$$

reaching a maximum $\sim\exp(z/2l)$ at $t = \tau_{tr} = \tau z/2l$.

For t near τ_{tr}, eqn. 7 may be rewritten in the form

$$\Phi(z, t) = \exp\left[\frac{z}{2l} - \frac{(t - \tau_{tr})^2}{\tau_c^2}\right] \qquad \tau_c = \tau(2z/l)^{1/2} \tag{8}$$

Thus for a fibre of length z the transient time for the onset of SHG is τ_{tr} and τ_c determines the duration of the harmonic output signal induced by the initial fluctuation at $z = 0$. For strong amplification ($z \gg l$), $\tau \ll \tau_c \ll \tau_{tr}$.

Response to switching on: The theory predicts a similar character of the transient process arising after simultaneous switching on the pump and the seeding second harmonic signal. At distances $z \gg l$ the time dependence of E_2 is governed mainly by the factor $\Phi(z, t)$. Solving eqns. 3 and 4 with the appropriate boundary and initial conditions, $\mathscr{E}(z, 0) = 0$, and $E_2(0, t) = E_{20}$, we derive the following expression for the harmonic intensity $I_2(z, t)$:

$$I_2(z, t) = I_{20} \frac{l}{\pi\sqrt{(2)}z} \Phi^2(z, t) \tag{9}$$

where I_{20} is the input harmonic intensity. Eqn. 9 is valid for $|t - \tau_{tr}| \ll \tau_{tr}, z \gg l$.

Amplification of noise: If only the fundamental pump light is injected into the fibre the second harmonic should appear spontaneously because of the amplification of noise. This noise may be caused, for instance, by pump intensity fluctuations producing a weak time-dependent seeding harmonic signal due to small quadrupol nonlinearity. It may be shown that the main contribution to the output harmonic signal comes from the noise frequency band of width $\Delta = \tau_c^{-1}$ around the frequency τ^{-1}. The output intensity should show strong fluctuations with a correlation time τ_c. The delay of the output harmonic signal after switching on the pumping is determined by the transient time τ_{tr}. The average harmonic intensity should be distributed along the fibre as $\exp(z/l)$.

Fluctuations in the SHG intensity (with an amplitude scale of the order of the average intensity) were reported by Valk *et al.*[8] without any interpretation. The correlation time of these fluctuations (a few minutes) was several times shorter than the observed delay time, in accordance with our results.

Discussion: The above results are valid for constant β and τ, in which case eqns. 1 and 2 are linear in E_2 and $\chi^{(2)}$. We believe that the nonlinearity is mainly due to the reduction of the relaxation time τ with increasing harmonic intensity. This reduction should lead to an increase of the characteristic length l. Consequently, the amplification should saturate at such intensity when l becomes comparable to the fibre length or to the walk-off distance. Presumably these nonlinear effects should also reduce the amplitude of the output intensity fluctuations.

Finally we note the close analogy of the SHG phenomenon in fibres to the photoelectric instability in ruby, which also leads to spontaneous breaking of the inversion symmetry and to the appearance of a constant electric field under illumination.[9,10]

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References

1 ÖSTERBERG, U., and MARGULIS, W.: 'Dye laser pumped by Nd : YAG laser pulses frequency doubled in a glass optical fibre', *Opt. Lett.*, 1986, **11**, pp. 516–518

2 STOLEN, R. H., and TOM, H. W. K.: 'Self-organized phase matching harmonic generation in optical fibers', *Opt. Lett.*, 1987, **12**, pp. 585–587

3 OUELLETTE, F., HILL, K. O., and JOHNSON, D. C.: 'Light induced erasure of self-organized $\chi^{(2)}$ gratings in optical fibers', *Opt. Lett.*, 1988, **13**, pp. 515–517

4 BARANOVA, N. B., and ZEL'DOVICH, B. YA.: 'Expansion of holography to multifrequency fields', *Zh. Eksp. Teor. Fiz., Pis'ma*, 1987, **45**, pp. 562–565 (in Russian)

5 FARRIES, M. C., RUSSELL, P. ST. J., FERMANN, M. E., and PAYNE, D. N.: 'Second harmonic generation in optical fibre by self-written $\chi^{(2)}$ grating', *Electron. Lett.*, 1987, **23**, pp. 322–324

6 PAYNE, F. P.: 'Second-harmonic generation in single-mode optical fibres', *Electron. Lett.*, 1987, **23**, pp. 1215–1216

7 DIANOV, E. M., KZAZANSKII, P. G., and STEPANOV, D. YU.: 'A mechanism of arising of the efficient photo-induced SHG in fiber-optic waveguides', *Kvantovaya Elektronika*, 1990, **17**, pp. 926–927 (in Russian)

8 VALK, B., KIM, E. M., and SALOUR, M. M.: 'Second-harmonic generation in germanium-doped fibers with a mode-locked Kr laser', *Appl. Phys. Lett.*, 1987, **51**, pp. 722–724

9 DYAKONOV, M. I.: 'Photoinduced electrical domains in ruby', *Sov. Phys.-JETP Lett.*, **39**, pp. 185–187

10 DYAKONOV, M. I., and FURMAN, A. S.: 'Photoelectric domain structure theory', *Sov. Phys.-JETP*, 1984, **60**, pp. 1191–1197

NEW RESULTS IN SPECTRAL ESTIMATION OF DECIMATED PROCESSES

Indexing terms: Spectral analysis, Mathematical techniques, Modelling

The aim of this Letter is to derive the spectral density of decimated processes, and to establish their model when they are generated from ARMA processes. The optimal conditions of decimation are also discussed.

Introduction: The need for efficient translating between various sampling frequencies is one of considerable importance in many varied and diverse applications of digital techniques in communications and signal processing: the process of sampling rate reduction is often called decimation. An example of such an application is the conversion of digital signal code formats, i.e. conversion from delta modulation to pulse-code modulation which inherently operate at different sampling rates.[1] Another example is the transmission of speech via analysis-synthesis techniques such as linear predictive coding (LPC) or more recently multipulse LPC where a sampling rate reduction is required for efficient transmission. Yet another application is in digital transmission of broadcast television signals with rates of 140 Mbit/s or 34 Mbit/s where sampling rate conversion of the video signal components is an important means of data compression.

It is generally recognised that when performing spectral estimation via the modelling approach, autoregressive moving average (ARMA) models can provide estimates of higher resolution than conventional FFT spectral estimation.[2–4] In this Letter, we consider the problem of parametric spectral analysis of discrete time random processes when decimated by a certain ratio.

Parametric modelling: Given a discrete-time wide stationary process $\{y(m)\}$ generated by sampling a continuous-time process $\{y(t)\}$ with a sampling period of Δt, namely $y(m) = y(t)$; $t = 0, \Delta t, \ldots, m\,\Delta t, \ldots, (N-1)\,\Delta t$, as the original process. Decimating $\{y(m)\}$ by a ratio of M produces $\{x(m)\}$, where

$$x(m) = y(mM) \tag{1}$$

Decimation can thus be regarded as a decrease in the sampling frequency (it is equivalent to increase the sampling period from Δt to $M\,\Delta t$) or as the problem of periodically missing observations.

The autocorelation of $x(m)$ is defined by

$$\begin{aligned} C_x(\tau) &= E\{x(m)x(m-\tau)\} \\ &= E\{y(mM)y[(m-\tau)M]\} \\ &= C_y(\tau M) \end{aligned} \tag{2}$$

and its spectral density is defined by

$$S_x(z) = \sum_{\tau=-\infty}^{+\infty} C_x(\tau)z^{-\tau} = \sum_{\tau=-\infty}^{+\infty} C_y(\tau M)z^{-\tau} \tag{3}$$