UCSB, Physics 129AL, Computational Physics: Section Worksheet, Week 6B

Zihang Wang (UCSB), zihangwang@ucsb.edu

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Section Participation and Submission Guidelines

Section attendance is required, but you do not need to complete all the work during the section. At each section, the TA will answer any questions that you might have, and you are encouraged to work with others and look for online resources during the section and outside of sections. Unless otherwise stated, the work will be due one week from the time of assignment. The TA will give you 1 point for each task completed. You can see your grades on Canvas.

We will use GitHub for section worksheet submissions. By the due date, you should have a single public repository on GitHub containing all the work you have done for the section work. Finally, upload a screenshot or a .txt file to Canvas with your GitHub username and repository name so the TA knows who you are and which repository you are using for the section.

Remember: talk to your fellow students, work together, and use GPTs. You will find it much easier than working alone. Good luck! All work should be done in the Docker container, and don't forget to commit it to Git!

Task 1: Ito lemma and Stratonovich formulation

a) Conversion formula: Ito lemma and Stratonovich formulation

Use the idea we discussed in the lecture, demonstrate the conversion between Ito and Stratonovich stochastic integrator. Reminder, use mid point and left point summation rule of the stochastic integrator. Upload your proof to Github.

b) Geometric Brownian motion

Let's consider the stochastic process,

$$X_t = e^{\mu t + \sigma W_t},$$

where μ and σ are constants and W_t is a standard Wiener process. We want to determine the dynamics of X_t via Itô and Stratonovich stochastic integrator. Write a Python program that generates this stochastic random variable.

c) Ito differential form

Write down the Ito stochastic integrator for X_t^I. and in Python, simulate the trajectory of the following,

$$x^I(t) = \int_0^t dX_t^I. \tag{1}$$

with $t = 10$ and $N = 100$, $dt = t/N$.

d) Stratonovich differential form

Use the **same random seed** from the Ito case, write down the Stratonovich stochastic integrator for X_t^I. and in Python, simulate the trajectory of the following,

$$x^S(t) = \int_0^t dX_t^I, \tag{2}$$

with $t = 10$ and $N = 100$, $dt = t/N$.

e)

For each stochastic integrator, plot the statistics (mean and variance) of trajectories as a function of $N \in [10, 10^4]$ with **log spacing**. You should have total of 4 subplots.

f) Functional dynamics on Geometric Brownian motion

Let's consider the dynamics of a function,

$$f(X_t) = X_t^2. \tag{3}$$

Write down the Ito stochastic integrator and Stratonovich stochastic integrator, and calculate the stopping dynamics,

$$F^I(t) = \int_0^t f(X_t)dX_t^I, \quad F^S(t) = \int_0^t f(X_t)dX_t^S. \tag{4}$$

Do a similar analysis as above, and discuss the similarity and difference from the bare trajectory statistics you calculate in **c,d,e**.

g) Autocorrelation

Calculate the autocorrelation function for the stopping function $F(t)$ at various stopping time $t = 5, 10, 20, 30$ with $dt = 0.1$. Recall that the autocorrelation function is given by,

$$C(\tau) = \langle F(t)F(t+\tau) \rangle,$$

where τ is the time lag, and the ensemble average is given by the trajectory statistics. Plot the correlation functions.

Task 2: Langevin equation

Langevin equation has a noise term $\eta(t)$ as the derivative of a stochastic process X_t,

$$\eta(t) = \frac{dX_t}{dt}, \quad X_t = W^2(t).$$

The Langevin equation has the form,

$$\frac{dv}{dt} = -\gamma v(t) + \sqrt{2D}\,\eta(t),$$

with $v(t=0) = v_0$. Use the Ito stochastic integrator, find the solution $v(t)$ and calculate the mean and variance of the velocity.