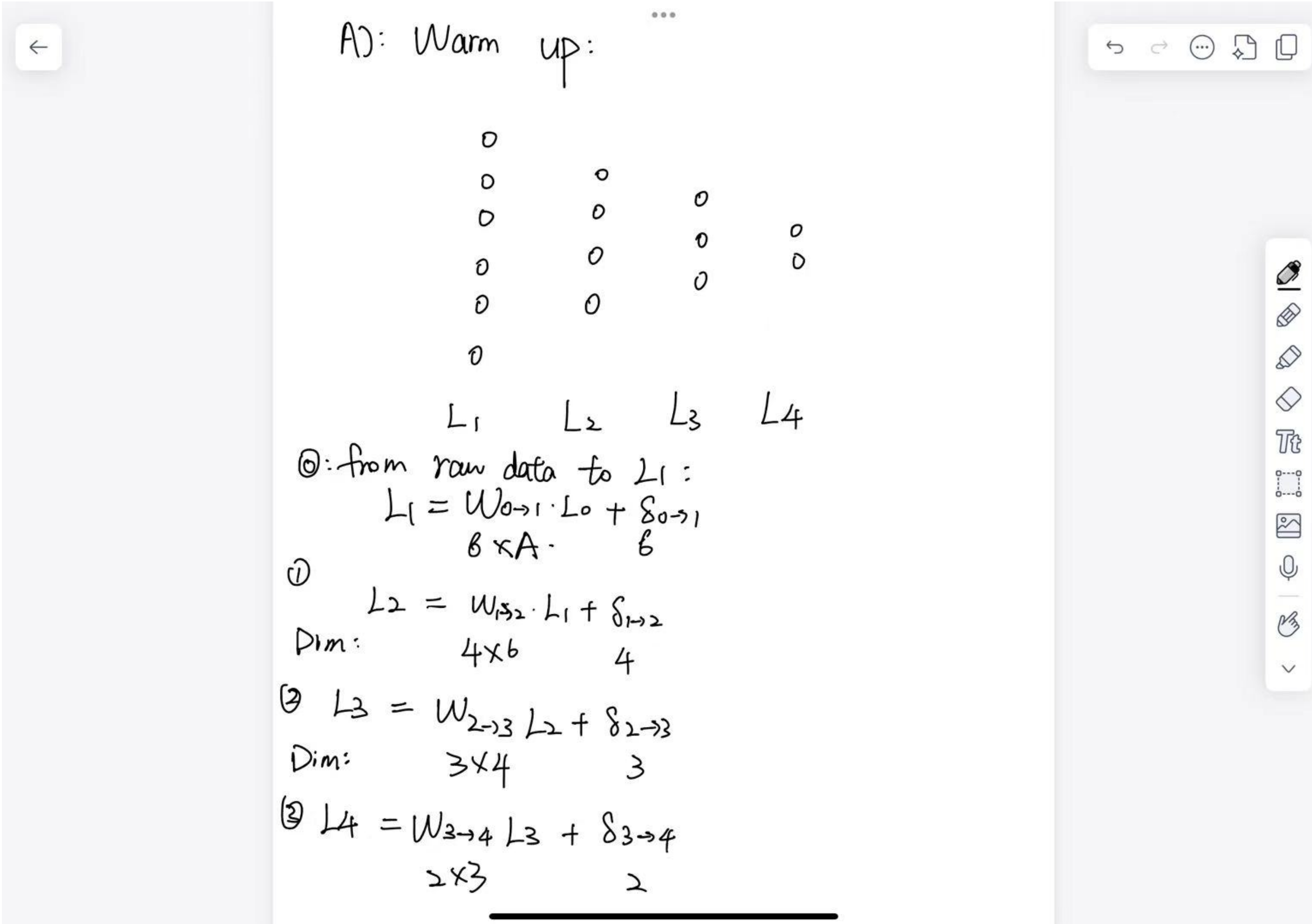
In [1]:
```python
import numpy as np

def relu(x):
    return np.maximum(0, x)

def relu_deriv(x):
    return (x > 0).astype(float)

def sigmoid(x):
    return 1 / (1 + np.exp(-x))

def sigmoid_deriv(x):
    s = sigmoid(x)
    return s * (1 - s)

def forward(x, parameters, activation='relu'):
    caches = []
    A = x
    L = len(parameters) // 2

    for l in range(1, L + 1):
        W = parameters["W" + str(l)]
        b = parameters["b" + str(l)]
        Z = np.dot(W, A) + b
        caches.append((A, Z))
        if activation == 'relu':
            A = relu(Z)
        elif activation == 'sigmoid':
            A = sigmoid(Z)
        else:
            raise ValueError("Unsupported activation function")

    AL = A
    return AL, caches

def backward(dAL, caches, parameters, activation='relu'):

    grads = {}
    L = len(caches)

    A_prev, Z = caches[-1]
    if activation == 'relu':
        dZ = dAL * relu_deriv(Z)
    elif activation == 'sigmoid':
        dZ = dAL * sigmoid_deriv(Z)
    else:
        raise ValueError("Unsupported activation function")


    grads["dW" + str(L)] = np.einsum('ik,jk->ij', dZ, A_prev)
    grads["db" + str(L)] = np.sum(dZ, axis=1, keepdims=True)
    dA_prev = np.dot(parameters["W" + str(L)].T, dZ)

    for l in range(L - 1, 0, -1):
        A_prev, Z = caches[l - 1]
        if activation == 'relu':
            dZ = dA_prev * relu_deriv(Z)
        elif activation == 'sigmoid':
            dZ = dA_prev * sigmoid_deriv(Z)
        grads["dW" + str(l)] = np.einsum('ik,jk->ij', dZ, A_prev)
        grads["db" + str(l)] = np.sum(dZ, axis=1, keepdims=True)
        if l > 1:
            dA_prev = np.dot(parameters["W" + str(l)].T, dZ)
```

```
    return grads
```



In [2]:

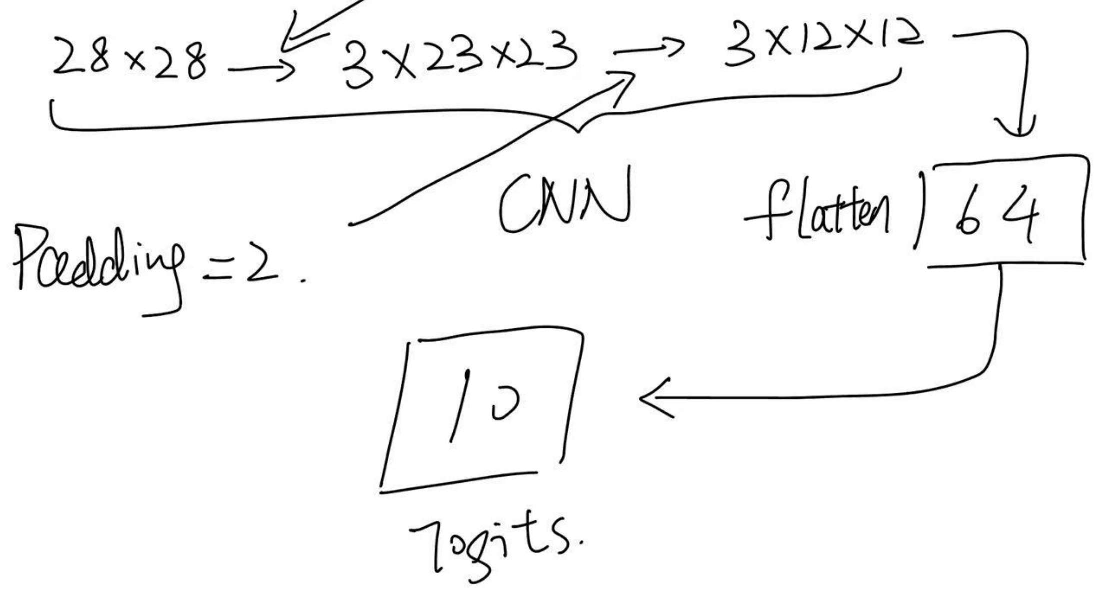
```python
# --- Example Usage ---

# Define network architecture: input 6, then layers with 6, 4, 3, and 2 neurons.
layer_dims = [6, 6, 4, 3, 2]
parameters = {}
np.random.seed(42)
L = len(layer_dims) - 1

for l in range(1, L + 1):
    parameters["W" + str(l)] = np.random.randn(layer_dims[l], layer_dims[l - 1]) * 0.01
    parameters["b" + str(l)] = np.zeros((layer_dims[l], 1))

x = np.random.randn(6, 1)

# Forward pass
AL, caches = forward(x, parameters, activation='sigmoid')
print("Forward propagation output (AL):\n", AL)

# Assume a dummy derivative from the loss (e.g., from mean squared error)
# For our output layer with 2 neurons, dAL has shape (2, 1)
dAL = np.random.randn(2, 1)

# Backward pass to compute gradients
grads = backward(dAL, caches, parameters, activation='sigmoid')

# Print gradients for each layer
for l in range(1, L + 1):
    print(f"\nGradients for Layer {l}:")
    print("dW" + str(l) + ":\n", grads["dW" + str(l)])
    print("db" + str(l) + ":\n", grads["db" + str(l)])
```

```
Forward propagation output (AL):
```

```
 [[0.49863851]
 [0.50075413]]

Gradients for Layer 1:
dW1:
 [[ 1.26488930e-09 -2.73979226e-08 -3.02809797e-09  4.92266836e-09
    2.03722379e-08 -7.14416856e-09]
 [ 4.16995430e-10 -9.03225959e-09 -9.98271557e-10  1.62285364e-09
    6.71610557e-09 -2.35521451e-09]
 [ 1.06951551e-09 -2.31660615e-08 -2.56038038e-09  4.16231695e-09
    1.72255583e-08 -6.04068600e-09]
 [ 3.36203080e-10 -7.28227044e-09 -8.04857674e-10  1.30842775e-09
    5.41486840e-09 -1.89889460e-09]
 [ 4.95193484e-10 -1.07260554e-08 -1.18547479e-09  1.92718311e-09
    7.97555915e-09 -2.79688168e-09]
 [ 1.07490250e-11 -2.32827455e-10 -2.57327664e-11  4.18328192e-11
    1.73123209e-10 -6.07111204e-11]]
db1:
 [[1.37846403e-08]
 [4.54437555e-09]
 [1.16554758e-08]
 [3.66390840e-09]
 [5.39657032e-09]
 [1.17141827e-10]]

Gradients for Layer 2:
dW2:
 [[-4.64793566e-08 -4.60100809e-08 -4.69072904e-08 -4.70964132e-08
   -4.62203650e-08 -4.58394542e-08]
 [-2.48009333e-06 -2.45505324e-06 -2.50292747e-06 -2.51301888e-06
   -2.46627379e-06 -2.44594877e-06]
 [ 1.04794593e-06  1.03736541e-06  1.05759433e-06  1.06185838e-06
    1.04210658e-06  1.03351838e-06]
 [ 2.06265479e-06  2.04182933e-06  2.08164558e-06  2.09003844e-06
    2.05116129e-06  2.03425728e-06]]
db2:
 [[-9.27281473e-08]
 [-4.94788390e-06]
 [ 2.09069341e-06]
 [ 4.11507755e-06]]

Gradients for Layer 3:
dW3:
 [[-0.00011922 -0.0001192  -0.00011935 -0.00012044]
 [-0.00040692 -0.00040683 -0.00040736 -0.00041107]
 [ 0.00070526  0.00070511  0.00070603  0.00071246]]
db3:
 [[-0.00023963]
 [-0.00081787]
 [ 0.00141751]]

Gradients for Layer 4:
dW4:
 [[-0.10031159 -0.10184285 -0.10147129]
 [-0.06225442 -0.06320474 -0.06297415]]
db4:
 [[-0.2021219 ]
 [-0.12543898]]
```

Task 2

B): CNN Using a 6x6 Kernel, 3 chan output



In [ ]:
```
import numpy as np

def relu(x):
    return np.maximum(0, x)

def relu_deriv(x):
    return (x > 0).astype(float)

class CNN:
    def __init__(self, kernel_size=3, num_filters=8, pool_size=2, num_classes=10):
        """
        Initializes a simple CNN for MNIST.
        - Convolutional layer: 3x3 filters, 'num_filters' filters.
        - Pooling layer: 2x2 max pooling.
        - Fully connected layer: flattens and outputs 'num_classes' scores.

        The input image is assumed to be 28x28x1.
        """
        self.kernel_size = kernel_size
        self.num_filters = num_filters
        self.pool_size = pool_size
        self.num_classes = num_classes

        # Initialize convolutional layer parameters.
        # Weight shape: (kernel_size, kernel_size, input_channels, num_filters)
        # For MNIST, input_channels = 1.
        self.W_conv = np.random.randn(kernel_size, kernel_size, 1, num_filters) * 0.01
        self.b_conv = np.zeros((1, 1, 1, num_filters))

        # After convolution:
        #   Output height/width = 28 - 3 + 1 = 26.
        # After 2x2 pooling with stride 2:
        #   Output height/width = 26 // 2 = 13.
        self.conv_out_height = 26
```

```
        self.conv_out_width  = 26
        self.pool_out_height = self.conv_out_height // pool_size
        self.pool_out_width  = self.conv_out_width  // pool_size

        # Fully connected layer parameters.
        self.fc_input_dim = self.pool_out_height * self.pool_out_width * num_filters  #
        self.W_fc = np.random.randn(num_classes, self.fc_input_dim) * 0.01
        self.b_fc = np.zeros((num_classes, 1))

    def forward(self, X):
        """
        Forward propagation for the CNN.

        Arguments:
        - X: Input image, a numpy array of shape (28, 28, 1).

        Returns:
        - out_fc: Output of the fully connected layer (before any softmax),
                  shape (num_classes, 1).
        Also stores intermediate values needed for backpropagation.
        """
        # Store input for backpropagation.
        self.X = X  # shape: (28,28,1)

        # ---- Convolutional Layer Forward ----
        H, W, C = X.shape  # (28, 28, 1)
        out_conv = np.zeros((H - self.kernel_size + 1, W - self.kernel_size + 1, self.nu
        # Naive convolution (no padding, stride=1)
        for i in range(out_conv.shape[0]):
            for j in range(out_conv.shape[1]):
                # Extract the current 3x3 patch from input.
                patch = X[i:i+self.kernel_size, j:j+self.kernel_size, :]  # shape: (3,3,
                for f in range(self.num_filters):
                    conv_sum = np.sum(patch * self.W_conv[:, :, :, f]) + self.b_conv[0,
                    out_conv[i, j, f] = conv_sum
        # Apply ReLU activation
        self.conv_out = relu(out_conv)  # shape: (26,26,num_filters)

        # ---- Pooling Layer Forward (Max Pooling) ----
        H_conv, W_conv, F = self.conv_out.shape  # (26,26,num_filters)
        H_pool = H_conv // self.pool_size  # 13
        W_pool = W_conv // self.pool_size  # 13
        out_pool = np.zeros((H_pool, W_pool, F))
        # To backpropagate through max pooling, record the mask.
        self.pool_mask = np.zeros_like(self.conv_out)
        for f in range(F):
            for i in range(H_pool):
                for j in range(W_pool):
                    h_start = i * self.pool_size
                    h_end = h_start + self.pool_size
                    w_start = j * self.pool_size
                    w_end = w_start + self.pool_size
                    patch = self.conv_out[h_start:h_end, w_start:w_end, f]
                    max_val = np.max(patch)
                    out_pool[i, j, f] = max_val
                    mask = (patch == max_val)
                    self.pool_mask[h_start:h_end, w_start:w_end, f] = mask
        self.pool_out = out_pool

        self.flat = self.pool_out.reshape(-1, 1)
        Z_fc = np.dot(self.W_fc, self.flat) + self.b_fc
        self.fc_out = relu(Z_fc)
        return self.fc_out

    def backward(self, dZ_fc, learning_rate=0.01):
        """
```

```
        Backward propagation for the CNN.

        Arguments:
        - dZ_fc: Gradient of the loss with respect to the FC layer output (shape: (num_c
        - learning_rate: Learning rate for parameter updates.

        Returns:
        - Gradients for parameters (for illustration).
        """
        dW_fc = np.dot(dZ_fc, self.flat.T)
        db_fc = dZ_fc
        dflat = np.dot(self.W_fc.T, dZ_fc)
        dpool = dflat.reshape(self.pool_out.shape)

        dconv_out = np.zeros_like(self.conv_out)  # shape: (26,26,num_filters)
        H_pool, W_pool, F = self.pool_out.shape
        for f in range(F):
            for i in range(H_pool):
                for j in range(W_pool):
                    h_start = i * self.pool_size
                    h_end = h_start + self.pool_size
                    w_start = j * self.pool_size
                    w_end = w_start + self.pool_size
                    dconv_out[h_start:h_end, w_start:w_end, f] += self.pool_mask[h_start


        dconv_out[self.conv_out <= 0] = 0

        dW_conv = np.zeros_like(self.W_conv)
        db_conv = np.zeros_like(self.b_conv)
        H, W, C = self.X.shape
        H_conv, W_conv, F = self.conv_out.shape  # (26,26,num_filters)
        for f in range(F):
            for i in range(H_conv):
                for j in range(W_conv):
                    patch = self.X[i:i+self.kernel_size, j:j+self.kernel_size, :]  # sha
                    dW_conv[:, :, :, f] += patch * dconv_out[i, j, f]
                    db_conv[0, 0, 0, f] += dconv_out[i, j, f]

        # ---- Parameter Update ----
        self.W_fc -= learning_rate * dW_fc
        self.b_fc -= learning_rate * db_fc
        self.W_conv -= learning_rate * dW_conv
        self.b_conv -= learning_rate * db_conv

        return dW_fc, db_fc, dW_conv, db_conv

np.random.seed(42)
cnn = CNN(kernel_size=3, num_filters=8, pool_size=2, num_classes=10)

X = np.random.randn(28, 28, 1)
out = cnn.forward(X)
print("CNN forward output (FC layer activations):\n", out)


dZ_fc = np.random.randn(10, 1)
grads = cnn.backward(dZ_fc, learning_rate=0.01)

print("\nParameter gradients computed during backward propagation:")
print("dW_fc shape:", grads[0].shape)
print("dW_conv shape:", grads[2].shape)
```

Task 3

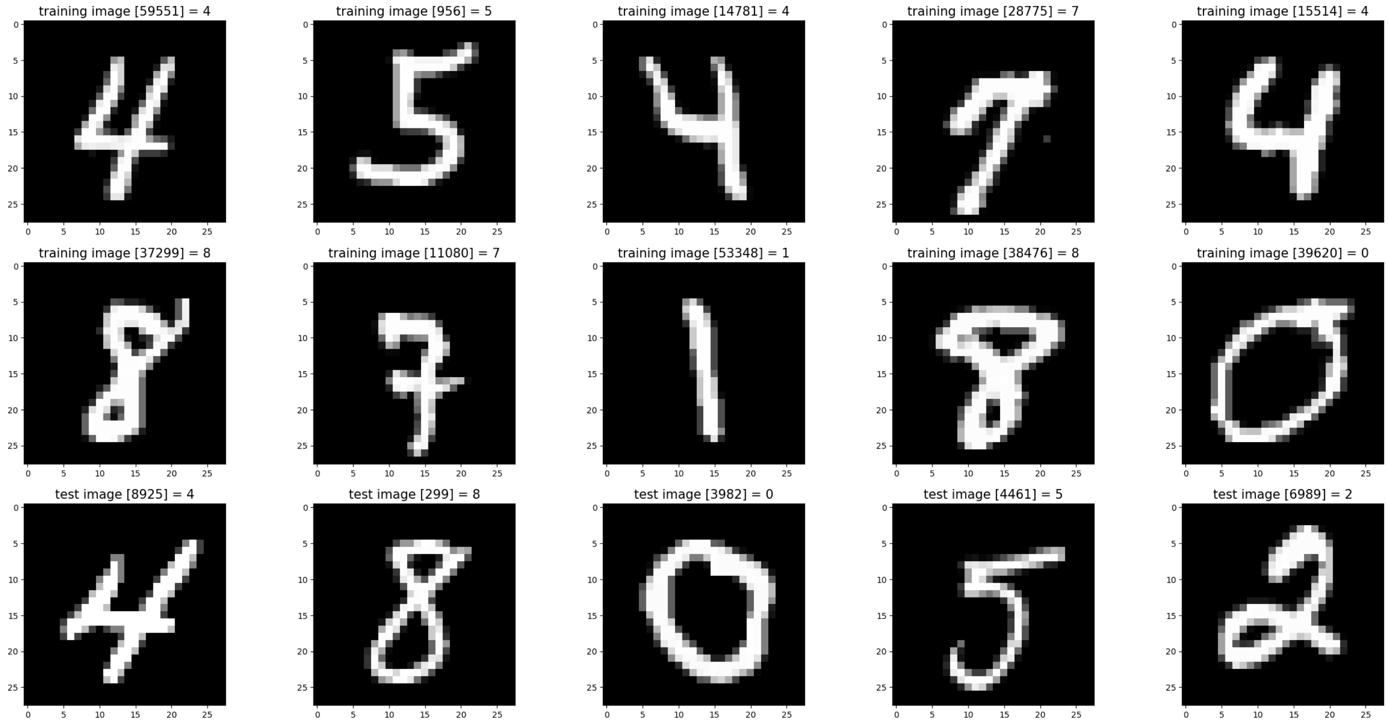
In [4]:
```python
import random
import matplotlib.pyplot as plt
import kagglehub
import numpy as np
import struct
from array import array
from os.path  import join
import torch
import torch.nn as nn
import torch.optim as optim
from torch.utils.data import TensorDataset, DataLoader
from torchvision import transforms
from tqdm import tqdm  # Standard tqdm package
import numpy as np
from matplotlib import pyplot as plt
from sklearn.metrics import confusion_matrix
```

In [ ]:
```python
#####################################
## Install the training material
#####################################
path = kagglehub.dataset_download("hojjatk/mnist-dataset")
print("Path to dataset files:", path)
data_set_location = r"C:\Users\Eric\.cache\kagglehub\datasets\hojjatk\mnist-dataset\vers
```

```
Downloading from https://www.kaggle.com/api/v1/datasets/download/hojjatk/mnist-dataset?d
ataset_version_number=1...
100%|██████████| 22.0M/22.0M [00:00<00:00, 65.0MB/s]
Extracting files...

Path to dataset files: C:\Users\Eric\.cache\kagglehub\datasets\hojjatk\mnist-dataset\ver
sions\1
```

In [5]:
```python
#########################################################################################
## Define the loading function for the training material, obtained from the example give
#########################################################################################
class MnistDataloader(object):
    def __init__(self, training_images_filepath,training_labels_filepath,
                 test_images_filepath, test_labels_filepath):
        self.training_images_filepath = training_images_filepath
        self.training_labels_filepath = training_labels_filepath
        self.test_images_filepath = test_images_filepath
        self.test_labels_filepath = test_labels_filepath

    def read_images_labels(self, images_filepath, labels_filepath):
        labels = []
        with open(labels_filepath, 'rb') as file:
            magic, size = struct.unpack(">II", file.read(8))
            if magic != 2049:
                raise ValueError('Magic number mismatch, expected 2049, got {}'.format(m
            labels = array("B", file.read())

        with open(images_filepath, 'rb') as file:
            magic, size, rows, cols = struct.unpack(">IIII", file.read(16))
            if magic != 2051:
                raise ValueError('Magic number mismatch, expected 2051, got {}'.format(m
            image_data = array("B", file.read())
        images = []
        for i in range(size):
            images.append([0] * rows * cols)
        for i in range(size):
            img = np.array(image_data[i * rows * cols:(i + 1) * rows * cols])
            img = img.reshape(28, 28)
            images[i][:] = img

        return images, labels
```

```python
    def load_data(self):
        x_train, y_train = self.read_images_labels(self.training_images_filepath, self.t
        x_test, y_test = self.read_images_labels(self.test_images_filepath, self.test_la
        return (x_train, y_train),(x_test, y_test)



#####################################################################################
## Example loading
#####################################################################################
input_path = r"C:\Users\Eric\.cache\kagglehub\datasets\hojjatk\mnist-dataset\versions\1"
training_images_filepath = join(input_path, 'train-images-idx3-ubyte/train-images-idx3-u
training_labels_filepath = join(input_path, 'train-labels-idx1-ubyte/train-labels-idx1-u
test_images_filepath = join(input_path, 't10k-images-idx3-ubyte/t10k-images-idx3-ubyte')
test_labels_filepath = join(input_path, 't10k-labels-idx1-ubyte/t10k-labels-idx1-ubyte')


def show_images(images, title_texts):
    cols = 5
    rows = int(len(images)/cols) + 1
    plt.figure(figsize=(30,20))
    index = 1
    for x in zip(images, title_texts):
        image = x[0]
        title_text = x[1]
        plt.subplot(rows, cols, index)
        plt.imshow(image, cmap=plt.cm.gray)
        if (title_text != ''):
            plt.title(title_text, fontsize = 15);
        index += 1


mnist_dataloader = MnistDataloader(training_images_filepath, training_labels_filepath, t
(x_train, y_train), (x_test, y_test) = mnist_dataloader.load_data()


images_2_show = []
titles_2_show = []
for i in range(0, 10):
    r = random.randint(1, 60000)
    images_2_show.append(x_train[r])
    titles_2_show.append('training image [' + str(r) + '] = ' + str(y_train[r]))

for i in range(0, 5):
    r = random.randint(1, 10000)
    images_2_show.append(x_test[r])
    titles_2_show.append('test image [' + str(r) + '] = ' + str(y_test[r]))

show_images(images_2_show, titles_2_show)
```



In [ ]:

```
# ---------------------------------------------------------------------------------
# Set up file paths for the MNIST dataset (adjust these paths as needed)
# ---------------------------------------------------------------------------------
input_path = r"C:\Users\Eric\.cache\kagglehub\datasets\hojjatk\mnist-dataset\versions\1"
training_images_filepath = join(input_path, 'train-images-idx3-ubyte/train-images-idx3-u
training_labels_filepath = join(input_path, 'train-labels-idx1-ubyte/train-labels-idx1-u
test_images_filepath = join(input_path, 't10k-images-idx3-ubyte/t10k-images-idx3-ubyte')
test_labels_filepath = join(input_path, 't10k-labels-idx1-ubyte/t10k-labels-idx1-ubyte')

# Load the MNIST data using your loader
mnist_dataloader = MnistDataloader(training_images_filepath, training_labels_filepath,
                                   test_images_filepath, test_labels_filepath)
(x_train, y_train), (x_test, y_test) = mnist_dataloader.load_data()
print("MNIST data loaded.")

# Convert the loaded images and labels to NumPy arrays
x_train = np.array(x_train)  # shape: (num_train, 28, 28)
x_test = np.array(x_test)    # shape: (num_test, 28, 28)
y_train = np.array(y_train)  # shape: (num_train,)
y_test = np.array(y_test)    # shape: (num_test,)




# ---------------------------------------------------------------------------------
# Convert the data to PyTorch tensors (and add a channel dimension)
# ---------------------------------------------------------------------------------
train_images_tensor = torch.tensor(x_train, dtype=torch.float32).unsqueeze(1)  # shape:
train_labels_tensor = torch.tensor(y_train, dtype=torch.long)
test_images_tensor = torch.tensor(x_test, dtype=torch.float32).unsqueeze(1)    # shape:
test_labels_tensor = torch.tensor(y_test, dtype=torch.long)

# Create a TensorDataset for training
train_dataset = TensorDataset(train_images_tensor, train_labels_tensor)

# ---------------------------------------------------------------------------------
# Define the CNN Model (using PyTorch modules)
# ---------------------------------------------------------------------------------
class CNN(nn.Module):
    def __init__(self):
        super(CNN, self).__init__()
        self.conv = nn.Conv2d(in_channels=1, out_channels=3, kernel_size=6)
```

```
        self.relu = nn.ReLU()
        self.pool = nn.MaxPool2d(kernel_size=2, padding=1)

        self.fc1 = nn.Linear(3 * 12 * 12, 60)
        self.sigmoid = nn.Sigmoid()
        self.fc2 = nn.Linear(60, 10)
        self.softmax = nn.Softmax(dim=1)

    def forward(self, x):
        x = self.conv(x)
        x = self.relu(x)
        x = self.pool(x)
        x = torch.flatten(x, 1)
        x = self.fc1(x)
        x = self.sigmoid(x)
        x = self.fc2(x)
        x = self.softmax(x)
        return x

    def train_all(self, dataset, batch_size, lr, epochs):
        loader = DataLoader(dataset, batch_size=batch_size, shuffle=True)
        crit = nn.CrossEntropyLoss()
        opt = optim.Adam(self.parameters(), lr=lr)
        losses = []
        total_steps = len(loader) * epochs
        with tqdm(total=total_steps, desc="CNN Training") as pbar:
            for epoch in range(epochs):
                self.train()
                epoch_loss = 0.0
                for images, labels in loader:
                    images, labels = images.to(device), labels.to(device)
                    opt.zero_grad()
                    outputs = self(images)
                    loss = crit(outputs, labels)
                    loss.backward()
                    opt.step()
                    epoch_loss += loss.item()
                    pbar.update(1)
                losses.append(epoch_loss)
                #print(f"Epoch {epoch+1}/{epochs}: Loss = {epoch_loss:.4f}")
        return losses

# ---------------------------------------------------------------------------------
# Define the MLP Model (using PyTorch modules)
# ---------------------------------------------------------------------------------
class MLP(nn.Module):
    def __init__(self):
        super(MLP, self).__init__()
        self.fc1 = nn.Linear(28 * 28, 500)
        self.relu = nn.ReLU()
        self.fc2 = nn.Linear(500, 300)
        self.sigmoid = nn.Sigmoid()
        self.fc3 = nn.Linear(300, 10)
        self.softmax = nn.Softmax(dim=1)

    def forward(self, x):
        x = torch.flatten(x, 1)
        x = self.fc1(x)
        x = self.relu(x)
        x = self.fc2(x)
        x = self.sigmoid(x)
        x = self.fc3(x)
        x = self.softmax(x)
        return x

    def train_all(self, dataset, batch_size, lr, epochs):
```

```python
        loader = DataLoader(dataset, batch_size=batch_size, shuffle=True)
        crit = nn.CrossEntropyLoss()
        opt = optim.Adam(self.parameters(), lr=lr)
        losses = []
        total_steps = len(loader) * epochs
        with tqdm(total=total_steps, desc="MLP Training") as pbar:
            for epoch in range(epochs):
                self.train()
                epoch_loss = 0.0
                for images, labels in loader:
                    images, labels = images.to(device), labels.to(device)
                    opt.zero_grad()
                    outputs = self(images)
                    loss = crit(outputs, labels)
                    loss.backward()
                    opt.step()
                    epoch_loss += loss.item()
                    pbar.update(1)
                losses.append(epoch_loss)
        return losses


def conf_matrix(model):
    model.eval()
    with torch.no_grad():
        outputs = model(test_images_tensor.to(device))
        preds = torch.argmax(outputs, dim=1).cpu().numpy()
    conf = confusion_matrix(y_test, preds)
    fig, ax = plt.subplots()
    im = ax.imshow(conf, cmap='Reds')
    ax.set_xticks(np.arange(conf.shape[1]))
    ax.set_yticks(np.arange(conf.shape[0]))
    ax.set_xticklabels(range(10))
    ax.set_yticklabels(range(10))
    plt.setp(ax.get_xticklabels(), rotation=45, ha="right", rotation_mode="anchor")

    thresh = conf.max() / 2.0
    for i in range(conf.shape[0]):
        for j in range(conf.shape[1]):
            color = "white" if conf[i, j] > thresh else "black"
            ax.text(j, i, f"{conf[i, j]:d}", ha="center", va="center", color=color)

    ax.set_xlabel('Predicted')
    ax.set_ylabel('Actual')
    plt.title("Confusion Matrix")
    plt.colorbar(im)
    plt.show()
```

```
MNIST data loaded.
```

In [50]:

```python
#################################################################
# Initialize the CNN and MLP instance
#################################################################
gpu = "cuda"
batch = 512
epoch = 100
iteration_step = 0.0001

device = torch.device(gpu)
cnn = CNN().to(device)

device = torch.device(gpu)
mlp = MLP().to(device)
```

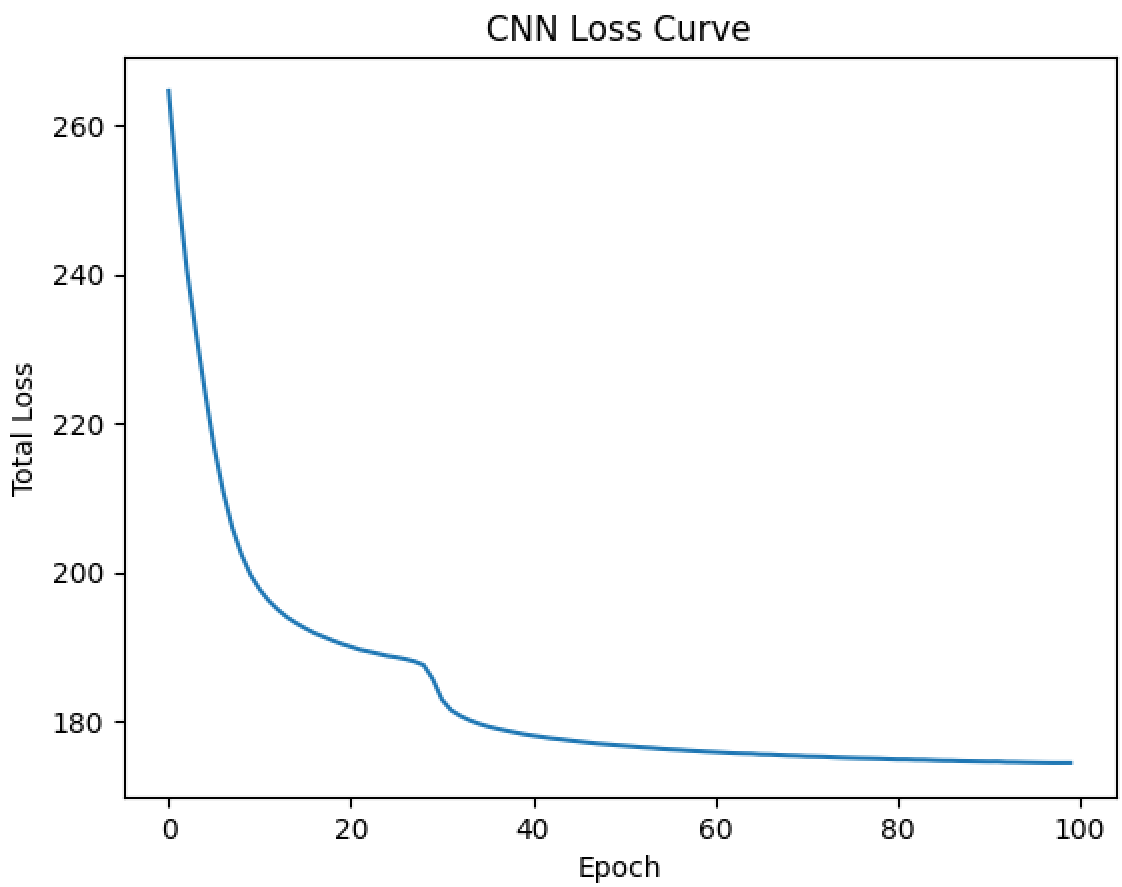
```
print(mlp.fc1.weight.shape)
print(mlp.fc1.bias.shape)
```

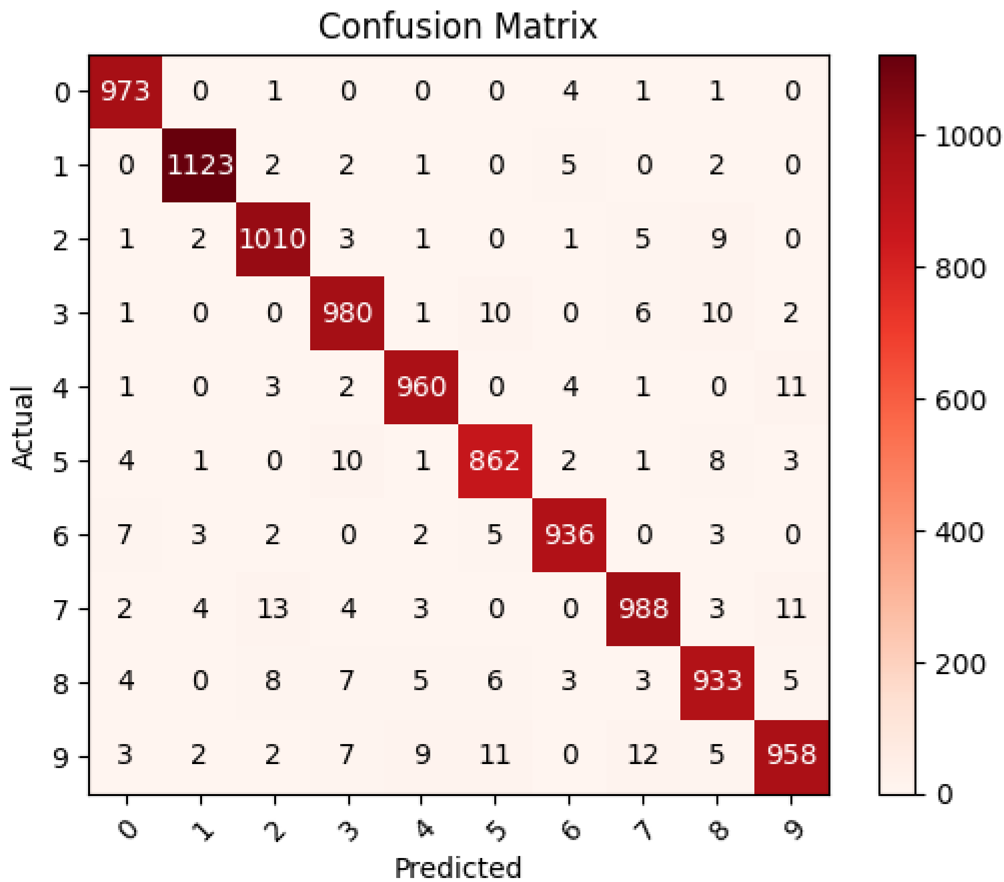
```
torch.Size([500, 784])
torch.Size([500])
```

In [ ]:

```
cnn_losses = cnn.train_all(train_dataset, batch_size=batch, lr=iteration_step, epochs=ep
plt.plot(cnn_losses)
plt.title("CNN Loss Curve")
plt.xlabel("Epoch")
plt.ylabel("Total Loss")
plt.show()
conf_matrix(cnn)
```

```
CNN Training: 100%|██████████| 11800/11800 [01:01<00:00, 192.43it/s]
```

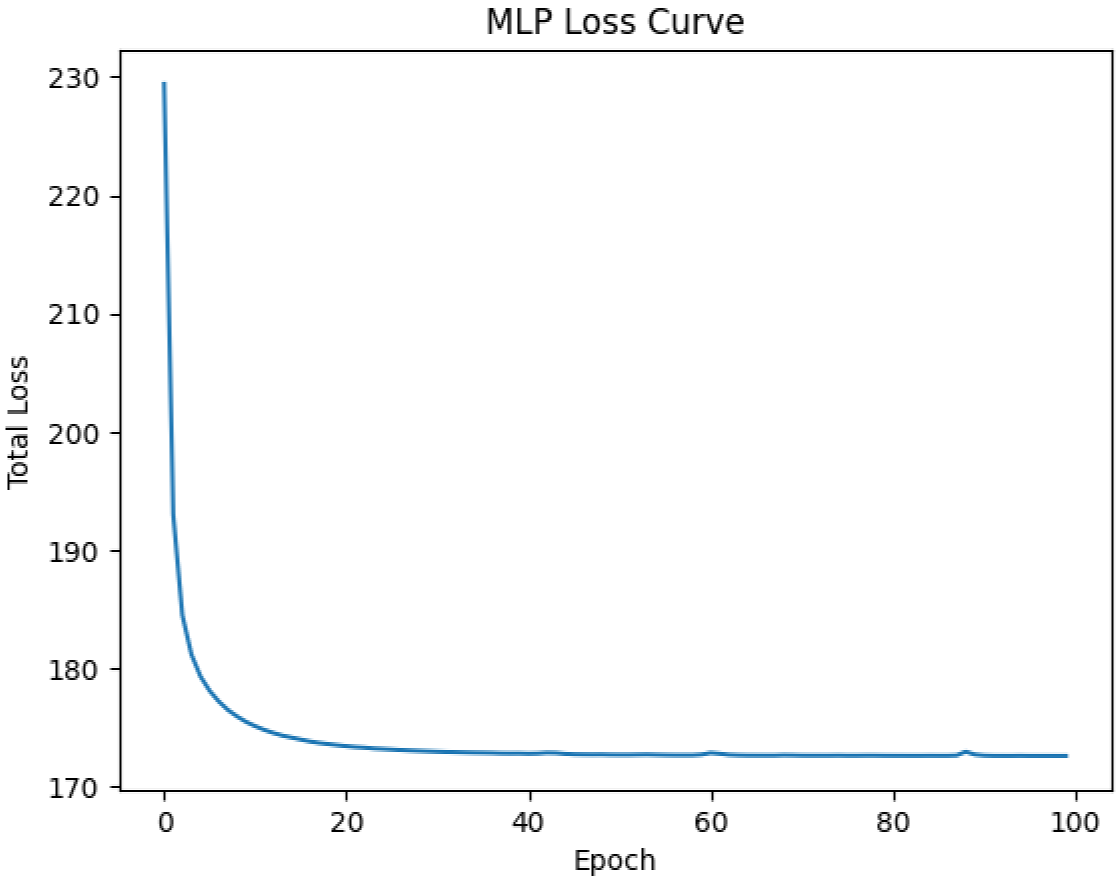


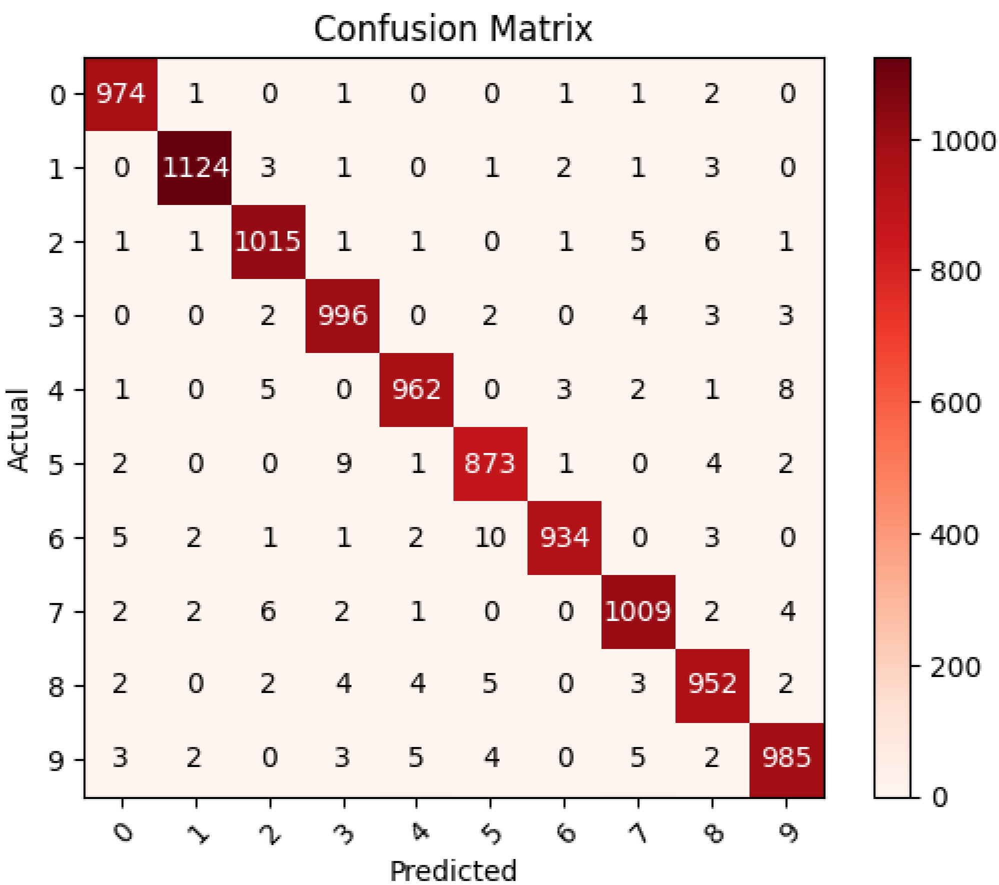


In [ ]:

```
mlp_losses = mlp.train_all(train_dataset, batch_size=batch, lr=iteration_step, epochs=ep
plt.plot(mlp_losses)
plt.title("MLP Loss Curve")
plt.xlabel("Epoch")
plt.ylabel("Total Loss")
plt.show()
conf_matrix(mlp)
```

```
MLP Training: 100%|██████████| 11800/11800 [00:56<00:00, 210.57it/s]
```


MLP Loss Curve
Total Loss
Epoch
230
220
210
200
190
180
170
0
20
40
60
80
100


Confusion Matrix
Actual
Predicted
974 1 0 1 0 0 1 1 2 0
0 1124 3 1 0 1 2 1 3 0
1 1 1015 1 1 0 1 5 6 1
0 0 2 996 0 2 0 4 3 3
1 0 5 0 962 0 3 2 1 8
2 0 0 9 1 873 1 0 4 2
5 2 1 1 2 10 934 0 3 0
2 2 6 2 1 0 0 1009 2 4
2 0 2 4 4 5 0 3 952 2
3 2 0 3 5 4 0 5 2 985
1000
800
600
400
200
0

In [ ]: