MATH 472 COMPUTING PROJECT # 3

March 22, 2018, Due March 29, 2018

The object of this project is to implement and study the convergence rates of three iterative methods: Jacobi, Gauss-Seidel and SOR. These methods will be applied to the system $A\mathbf{x} = \mathbf{b}$ where A is the $n \times n$ tridiagonal matrix $A = [-1 \quad 2 \quad -1]$ and we want to apply these methods to different system sizes $n = 25, 50, 100, 200$.

For a given n, let $h = \frac{1}{n+1}$. The vector $\mathbf{b}$ is given by

$$b_i = 2h^2, \ i = 1, \ldots, n,$$

and the solution $\mathbf{x}$ is given by

$$x_i = ih(1 - ih), \ i = 1, \ldots, n.$$

Implement each of the three methods paying special attention **not the store the matrix in general form and to avoid multiplication by zeros**. In other words you need to exploit the sparsity of the matrix.

For the SOR method use the following values for the relaxation parameter ω:

n	25	50	100	200
ω	1.78486	1.88402	1.93968	1.96922

For each of the 12 tasks

(1) Start the iteration with $\mathbf{x}^{(0)} = \mathbf{0}$.
(2) Stop the iteration when $\|\mathbf{x}^{(\mathbf{k})} - \mathbf{x}\|_\infty < \mathbf{10^{-6}}$. Print the number k of iterations.
(3) Calculate and print the experimental estimation of $\rho(T)$ using the formula

$$\rho(T) \approx \exp\left(\frac{1}{k}\log\left(\|\mathbf{x}^{(\mathbf{k})} - \mathbf{x}\|_\infty / \|\mathbf{x}^{(\mathbf{0})} - \mathbf{x}\|_\infty\right)\right).$$