ECE471/571 – Homework #1 (Due 01/25)

Note: UG: 100+10, G: 100

Problem 1 (100/70): In a 1-D, 3-class problem, the density functions of all classes are adequately represented by univariate Gaussians, with $\mu_1 = 4, \sigma_1 = 2, \mu_2 = 6, \sigma_2 = 3, \mu_3 = 5, \sigma_3 = 2$.

(1) (15/10) Sketch the three density functions on the same figure using pencil and paper (i.e., without MATLAB or any other software package). Assume equal prior probability, predict how many decision regions there would be.

(2) (40/20) Assume equal prior probability,

a. (10/5) If x=4.7, which class does x belong to? Use the MAP method. Show detailed steps.

b. (15/10) Find the decision boundary using analytical methods instead of the sketch.

c. (15/5) Solve for the overall probability of error.

(3) (20/15) Assume that $P(\omega_1) = 0.6, P(\omega_2) = 0.2, P(\omega_3) = 0.2$, and zero-one loss.

a. (10/5) Use MATLAB to draw the pdf and the posteriori probability. Comment on the difference.

b. (5/5) Redo question 2 (a)

c. (5/5) Comment on the impact of prior probability.

(4) (10/10) What combinations of the standard deviations would generate just two decision regions or one decision region?

(5) (15/15) Write an expression for the probability of error $p(error \mid \omega_1)$ that an error occurs given that the truth is class 1. (Note: Be as specific as you can, but no need to solve the expression)

Problem 2 (+10/30): The probability densities representing a two-class pattern are

$$p(y \mid \omega_1) = \begin{cases} \exp(y-2) when (y \le 2) \\ 0 when (y > 2) \end{cases}$$

$$p(y \mid \omega_2) = \begin{cases} \exp(-(y-b)) when (y > b) \\ 0 otherwise \end{cases}$$

The prior probabilities are $P(\omega_1) = P(\omega_2) = 0.5$

(a) (+10/15) Sketch the two densities on the same figure for b<2. Show the regions corresponding to the decision rule that minimizes the probability of error.

(b) (0/10) What is $P(error \mid \omega_1)$ (conditional probability of error when we decide ω_2 but actually it should be ω_1) in terms of *b*? (Consider all values of *b* from –inf to inf)

(c) (0/5) What is the value of *b* that maximizes $P(error \mid \omega_1)$?