Homework 1

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PROBLEM 1.

Let X be an infinite set.

(a) Show that

$$\mathcal{T}_1 = \{U \subseteq X : U = \emptyset \text{ or } X \setminus U \text{ is finite}\}$$

is a topology on X, called the **finite complement topology**.

(b) Show that

$$\mathcal{T}_2 = \{U \subseteq X : U = \emptyset \text{ or } X \setminus U \text{ is countable}\}$$

is a topology on X, called the **countable complement topology**.

(c) Let $p \in X$ be an arbitrary point in X. Show that

$$\mathcal{T}_3 = \{U \subseteq X : U = \emptyset \text{ or } p \in U\}$$

is a topology on X, called the **particular point topology**.

SOLUTION.

∎

PROBLEM 2.

Let (M, d) be a metric space, and let c be a positive real number. We define a new metric d' on M by

$$d'(x, y) = c \cdot d(x, y).$$

Prove that d and d' generate the same topology on M.

SOLUTION.

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PROBLEM 3.

Let X be a topological space and B be a subset of X. Prove the following set equalities.

(a)

$$\overline{X \setminus B} = X \setminus \operatorname{Int}(B).$$

(b)

$$\operatorname{Int}(X \setminus B) = X \setminus \overline{B}.$$

SOLUTION.

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PROBLEM 4.

Show that a subset of a topological space is closed if and only if it contains all of its limit points.

Solution.

$\square$

Problem 5.

Suppose X and Y are topological spaces, and $f : X \to Y$ is any map.

(a) f is continuous if and only if

$$f(\overline{A}) \subseteq \overline{f(A)} \quad \text{for all } A \subseteq X.$$

(b) f is continuous if and only if

$$f^{-1}(\mathrm{Int}(B)) \subseteq \mathrm{Int}(f^{-1}(B)) \quad \text{for all } B \subseteq Y.$$

Solution.

$\square$