No. 1.
Use the Divergence Theorem to show that

$$|\partial B(0,1) = n\alpha(n)|.$$

No. 2.
Let $u \in C^4(\bar{B}_1)$, where $B_1 := \{x \in \mathcal{R}^n, |x| < 1\}$. Suppose that

$$\Delta^2 u := \Delta(\Delta u) \quad in \quad B_1, \quad u = |Du| = 0 \quad on \quad \partial B_1$$

$= 0$

Show that $u = 0$ in B_1.

No. 3.
Suppose that $u \in C^\infty(\bar{B}_2)$

$$\Delta u = 0 \quad in \quad B_2, \quad u > 0$$

Show that $|D(\ln u)| \leq N$ in B_1, with a constant N depending only on n.

No. 4.
Let Φ be the fundamental solution of Laplace equation and $u = \Phi * f$. Show that

$$-\Delta u = f \quad in \quad \mathcal{R}^n$$

No. 5.
Suppose that $u \in C^2(\Omega)$ and $\Delta u = 0$ in Ω. Show that $u(x) = \frac{1}{|\partial B(x,r)|}\int_{\partial B(x,r)} u d\sigma$ for each ball $B(x,r) \subset \Omega$

No. 6.
Give a direct proof that if Ω is bounded and $u \in C_1^2(\Omega_T) \cap C(\bar{\Omega}_T)$ solves the heat equation, then

$$max_{\bar{\Omega}_T} u = max_{\Gamma_T} u$$

No. 7. Let $\Gamma(x,t)$ be the fundamental solution of $u_t - \Delta u = 0$ in $\mathcal{R}^n \times (0,\infty)$, show that

$$\int_{\mathcal{R}^n} \Gamma(x,t) dx = 1$$

No. 8.
1) Show the general solution of the PDE $u_{xy} = 0$ is $u(x,y) = F(x) + G(y)$ for arbitrary functions F, G.

2) Using the change of variables $\xi = x + t, \eta = x - t$, show $u_{tt} - u_{xx} = 0$ if and only if $u_{\xi\eta} = 0$.

3) Use 1) and 2) to derive d'Alembert's formula.

No. 9.
Let u solve the initial-value problem for the wave equation in one dimension: $u_{tt} - u_{xx} = 0$ in $\mathcal{R} \times (0,\infty)$ and $u = g, u_t = h$ on $\mathcal{R} \times (t = 0)$. Suppose g, h have compact support. The kinetic energy is $k(t) = \frac{1}{2}\int_{-\infty}^{\infty} u_t^2(x,t)dx$ and the potential energy is $p(t) = \frac{1}{2}\int_{-\infty}^{\infty} u_x^2(x,t)dx$. Prove

1) $k(t) + p(t)$ is constant in t

2) $k(t) = p(t)$ for all large enough times t.