HW 1 Graph Theory

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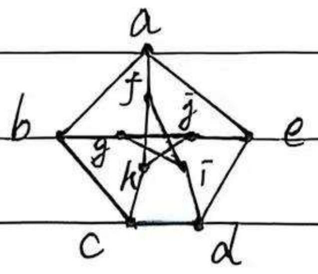
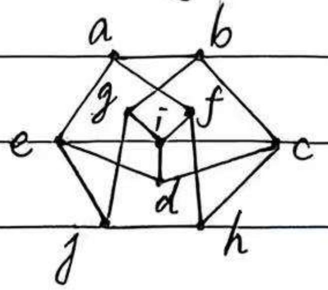
1.4 <Solution> Shown as follows.





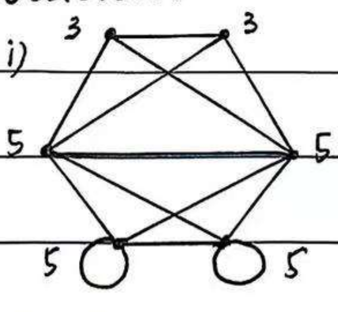
1.5. <Solution>

Notice that there are no vertices of degree 2 which are adjacent in the first graph. which the second graph not.

Since isomorphism preserves adjacent of vertices, the two graphs are not isomorphic.

1.9. <Solution> 29 and 23 respectively.

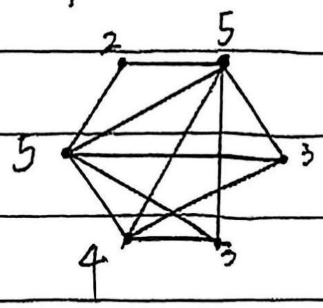
1.13. (i) <Solution>



There doesn't exist a simple graph with degree sequence (3, 3, 5, 5, 5, 5)

Suppose not, then there exists such a graph.
If four vertices with degree 5, then ~~eoh~~ they are connected with all other ~~of~~ five vertices, which implies that the other two vertices have degree 4 at least, contradicting with 3.

(ii) Yes. for example:

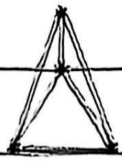


15. <proof> If not, then each vertex of G has different degree, which implies G with n vertices must have a vertex with degree n. ~~wt~~ This contradicts with a simple graph. Thus G must contain two or more vertices of the same degree

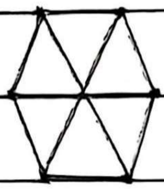
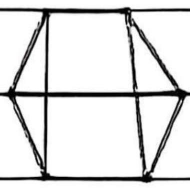
1.32 ‹Solution›

By the corollary of handshaking lemma we know that only graphs with even vertices can be cubic graphs.

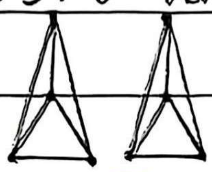
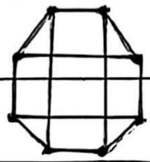
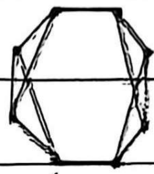
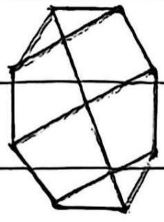
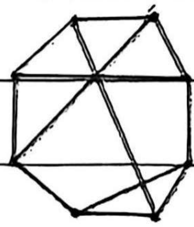
Case 1 4 vertices



Case 2: 6 vertices

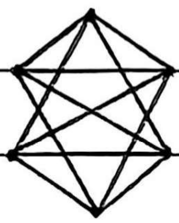



Case 3: 8 vertices

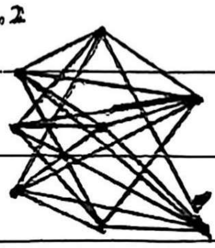






1.34 ‹Solution›

$K_{2,2,2}$



$K_{3,3,2}$



edges of $K_{3,4,5}$ $= (4+5)\times 3 + 4\times 5 = 27+20 = 47$

1.44 <proof> For n vertices, suppose the indeg of the i-th vertex is x_i. Since ~~it's~~ T is a tournament, the corresponding outdeg is $n-1-x_i$.

Apply hand-shaking lemma.

$$\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} (n-1)-x_i = n(n-1)-\sum_{i=1}^{n} x_i$$

thus $\sum_{i=1}^{n} x_i = \frac{1}{2} n(n-1)$

$$\sum indeg(v)^2 = \sum_{i=1}^{n} x_i^2$$

$$\sum outdeg(v)^2 = \sum_{i=1}^{n} (n-1-x_i)^2 = \sum_{i=1}^{n}[(n-1)^2-2(n-1)x_i+x_i^2]$$

$$= n(n-1)^2-2(n-1)\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} x_i^2$$

$$= \sum_{i=1}^{n} x_i^2 = \sum indeg(v)^2$$

1.46 <Solution>

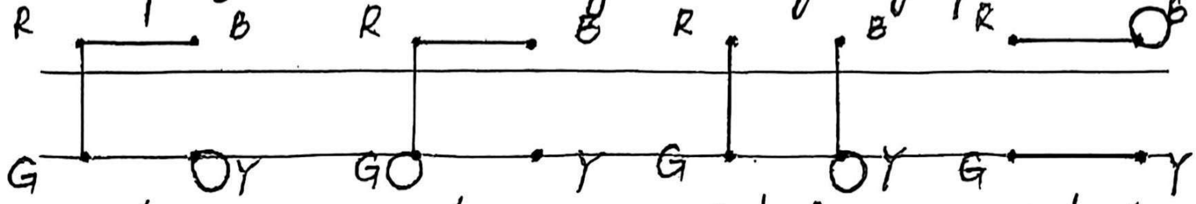
(i) $V(G)$: A central vertex v and countably infinite vertices $\{u_i\}$ $i \in \mathbb{N}$

$E(G)$: The central vertex v is connected to each u_i, $i \in \mathbb{N}$.

(ii) $V(G)$: All real numbers $\mathbb{R}$.

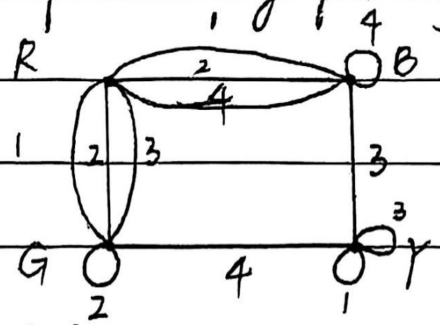
$E(G)$: All unordered pairs of distinct real numbers $\{(x,y) \mid x,y \in \mathbb{R}, x \neq y\}$.

1.50.<proof> Draw each ~~gro~~ cube by a graph



cube1 cube2 cube3 cube4

We next super~~mm~~impose these graphs to form a new graph G



Since GY. BY are ~~only~~ connected by one edge. this problem has no solution.

1.53 <proof>

(i) Step 1. Both K_3 and $K_{1,3}$ have 3 edges. thus their line graphs have 3 vertices

Step 2. Each edge in K_3 and $K_{1,3}$ have 2 adjacent edges. thus ~~their~~ vertices of line graphs .have 2 degrees. that is. their line graphs are both K_3.

(ii) Step 1. Tetrahedron has 6 edges. then it's line graph has 6 vertices

Step 2 each ~~ear~~edge in Tetrahedron has 4 edges adjacent. so ~~its~~ ~~deg~~ vert. of line graph has degree 4

So the line graph is the octahedron graph.

(iii). Notice that for each edge in G connect two vertices. Since G is regular of degree k, then each vertex have ~~k~~ k edges ~~inci~~ incident.
That implies each edge in G ~~have~~ has $2(k-1)$ adjacent edges, so the line graph is regular of degree $2k-2$

(iv) An edge in $L(G)$ is a 2-set $\{e_1, e_2\}$ of edges in G with which are adjacent to a common vertex v. This vertex v is uniquely determined by $\{e_1, e_2\}$. If a vertex v has degree d, there are C_d^2 2-sets $\{e_1, e_2\}$ s.t. e_1, e_2 are adjacent to v.
so the total number of edges in $L(G)$ is

$$\sum_{i=1}^{n} C_{d_i}^2 = \sum_{i=1}^{n} \frac{d_i(d_i-1)}{2}.$$

(v) K_5 has 10 edges, thus $L(K_5)$ has 10 vertices.
By (iii), each vertex of $L(K_5)$ has degree 6, it's the same as the complement of the Petersen graph.