兰州大学
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习题五.

1. $P\{\xi \ge x\} = \cancel{\int_{-\infty}^{+\infty}} \int_{y \ge x} dF(y) \le e^{-ax}\int_{y \ge x} e^{ay}\, dF(y) \le e^{-ax}\int_{-\infty}^{+\infty} e^{ay}\, dF(y)$

设ξ的分布函数$F(x)$

$= e^{-ax} E e^{a\xi}$.

2. $P\{h(\xi) \ge c\}$，设 $h(\xi)$的分布函数$F_h(x)$.

$\forall c > 0$，$P\{h(\xi) \ge c\} = \int_{x \ge c} dF_h(x)$

$\le \frac{1}{c}\int_{x \ge c} x\, dF(x)$

$\le \frac{1}{c}\int_0^{\infty} x\, dF_h(x)$

$= \frac{1}{c} E[h(\xi)]$

4. 验证马尔可夫条件 $\frac{1}{n^2} D(\sum_{k=1}^{n} \xi_k) \to 0$.

已知 $\frac{D(\xi_n)}{n} \to 0$，则

$\frac{1}{n}\sum_{k=1}^{n}\frac{D(\xi_k)}{k} \to 0$ (数分)

$0 \le \frac{1}{n^2} D(\sum_{k=1}^{n}\xi_k) = \frac{1}{n}\sum_{k=1}^{n} \frac{D(\xi_k)}{n} \le \frac{1}{n}\sum_{k=1}^{n}\frac{D(\xi_k)}{k} \to 0$

6. (1) $D(X_k) = \frac{1}{2}\cdot 2^{2k} + \frac{1}{2}\cdot 2^{2k} = 2^{2k}$，$\frac{1}{n^2} D(\sum_{k=1}^{n} X_k) = \frac{1}{n^2}\sum_{k=1}^{n} 2^{2k} = \frac{4(1-4^n)}{n^2(1-4)} \not\to 0$ $(n \to \infty)$

不满足

(2) $E(X_k) = 0$，$D(X_k) = 2^{-(2k+1)}\cdot 2^{2k} + 2^{-(2k+1)}\cdot 2^{2k} = 1$.

$\frac{1}{n^2} D(\sum_{k=1}^{n} X_k) = \frac{1}{n^2}\cdot\sum_{k=1}^{n} 1 = \frac{1}{n} \to 0$ $(n \to \infty)$ 满足

(3) $E(X_k) = 0$. $D(X_k) = \frac{1}{2}k^{-\frac{1}{2}}\cdot k^2 + \frac{1}{2}k^{-\frac{1}{2}}\cdot k^2 = k^{\frac{3}{2}}$.

$\frac{1}{n^2} D(\sum_{k=1}^{n} X_k) = \frac{1}{n^2}\sum_{k=1}^{n} k^{\frac{3}{2}} \ge \frac{1}{n^2}\sum_{k=1}^{n} k = \frac{1}{n^2}\cdot\frac{n(n-1)}{2} \to 1$ $(n \to \infty)$

不满足

8. 由 $|i-j|\to\infty$. $r_{ij}\to 0$

$\forall \varepsilon>0$. $\exists N$, $|i-j|>N$ 时 $|r_{ij}|<\frac{\varepsilon}{C}$.

$$\frac{1}{n^2}D(\sum_{k=1}^{n}\xi_k)=\frac{1}{n^2}\sum_{k=1}^{n}D(\xi_k)+\frac{2}{n^2}\sum_{1\le i<j\le n}r_{ij}\sigma_i\sigma_j$$

$$\le\frac{1}{n^2}\cdot nC+\frac{2}{n^2}\sum_{i<j}|r_{ij}|\sigma_i\sigma_j$$

$$=\frac{C}{n}+\frac{2}{n^2}\sum_{0<j-i\le N}|r_{ij}|\sigma_i\sigma_j+\frac{2}{n^2}\sum_{j-i>N}|r_{ij}|\sigma_i\sigma_j$$

$$\le\frac{C}{n}+\frac{2}{n^2}nN+\frac{2}{n^2}\cdot\frac{n(n-1)}{2}\cdot C$$

$$\le\frac{C}{n}+\frac{2NC}{n}+\varepsilon\quad\to 0\quad(n\to\infty).$$

满足马尔可夫条件.

12. ~~P~~ $P\{\mu_n\ge 10\}\approx 1-\Phi(\frac{10-120\times0.05-0.5}{\sqrt{120\times0.05\times0.95}})=0.0725$.

17. $P\{|\frac{\mu}{6000}-\frac{1}{6}|<\frac{1}{100}\}=2\Phi(\frac{1}{100}\sqrt{\frac{6000}{\frac{1}{6}\times\frac{5}{6}}})-1=0.906$.

18. $P\{|\frac{\mu}{6000}-\frac{1}{6}|<\varepsilon\}=2\Phi(\varepsilon\sqrt{\frac{6000}{\frac{1}{6}\times\frac{5}{6}}})-1=0.99$,

$\varepsilon=0.014$. 915 - 1085.